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Computer Algebra Handbook



FOUNDATIONS
APPLICATIONS
SYSTEMS



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Foundations • Applications • Systems



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