AN INTRODUCTION TO

NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

J. DAVID LOGAN

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IN PURE AND APPLIED MATHEMATICS

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