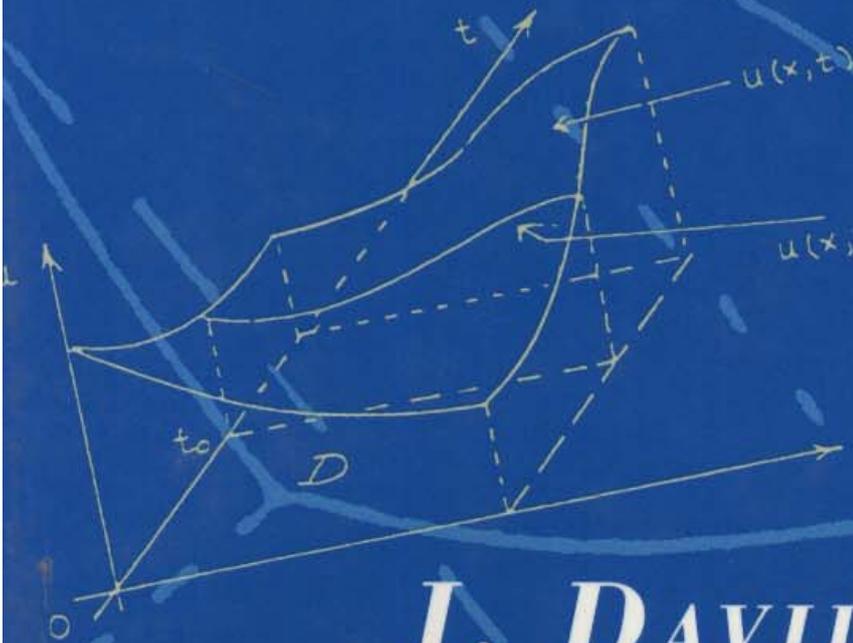


AN INTRODUCTION TO

**NONLINEAR
PARTIAL
DIFFERENTIAL
EQUATIONS**



J. DAVID LOGAN

*A VOLUME IN THE WILEY-INTERSCIENCE SERIES
IN PURE AND APPLIED MATHEMATICS*

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