

# EIGENVALUES OF MATRICES

$$\begin{pmatrix} \alpha_0 & \beta_1 & 0 & 0 & \dots & 0 \\ \beta_1 & \alpha_1 & \beta_2 & 0 & \dots & 0 \\ 0 & \beta_2 & \alpha_2 & \beta_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & \beta_n \\ 0 & \dots & \dots & 0 & \beta_n & \alpha_n \end{pmatrix}$$

Françoise Chatelin



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# Contents

<b>Preface</b>	<b>xi</b>
<b>Preface to the English Edition</b>	<b>xv</b>
<b>Notation</b>	<b>xvii</b>
<b>Chapter 1 Supplements from Linear Algebra</b>	<b>1</b>
1.1 Notation and definitions	1
1.2 The canonical angles between two subspaces	5
1.3 Projections	8
1.4 The gap between two subspaces	10
1.5 Convergence of a sequence of subspaces	14
1.6 Reduction of square matrices	18
1.7 Spectral decomposition	27
1.8 Rank and linear independence	31
1.9 Hermitian and normal matrices	32
1.10 Non-negative matrices	33
1.11 Sections and Rayleigh quotients	34
1.12 Sylvester's equation	35
1.13 Regular pencils of matrices	42
1.14 Bibliographical comments	43
Exercises	43
<b>Chapter 2 Elements of Spectral Theory</b>	<b>61</b>
2.1 Revision of some properties of functions of a complex variable	61
2.2 Singularities of the resolvent	63
2.3 The reduced resolvent and the partial inverse	73
2.4 The block-reduced resolvent	76
2.5 Linear perturbations of the matrix $A$	79
2.6 Analyticity of the resolvent	82
2.7 Analyticity of the spectral projection	84
2.8 The Rellich–Kato expansions	85
2.9 The Rayleigh–Schrödinger expansions	86
2.10 Non-linear equation and Newton's method	89
2.11 Modified methods	92
2.12 The local approximate inverse and the method of residual correction	95
2.13 Bibliographical comments	98
Exercises	98

<b>Chapter 3</b>	<b>Why Compute Eigenvalues?</b>	<b>111</b>
3.1	Differential equations and difference equations	111
3.2	Markov chains	114
3.3	Theory of economics	117
3.4	Factorial analysis of data	119
3.5	The dynamics of structures	120
3.6	Chemistry	122
3.7	Fredholm's integral equation	124
3.8	Bibliographical comments	126
	Exercises	126
<b>Chapter 4</b>	<b>Error Analysis</b>	<b>149</b>
4.1	Revision of the conditioning of a system	149
4.2	Stability of a spectral problem	150
4.3	<i>A priori</i> analysis of errors	165
4.4	<i>A posteriori</i> analysis of errors	170
4.5	<i>A</i> is almost diagonal	177
4.6	<i>A</i> is Hermitian	180
4.7	Bibliographical comments	190
	Exercises	191
<b>Chapter 5</b>	<b>Foundations of Methods for Computing Eigenvalues</b>	<b>205</b>
5.1	Convergence of a Krylov sequence of subspaces	205
5.2	The method of subspace iteration	208
5.3	The power method	213
5.4	The method of inverse iteration	217
5.5	The <i>QR</i> algorithm	221
5.6	Hermitian matrices	226
5.7	The <i>QZ</i> algorithm	226
5.8	Newton's method and the Rayleigh quotient iteration	227
5.9	Modified Newton's method and simultaneous inverse iterations	228
5.10	Bibliographical comments	235
	Exercises	235
<b>Chapter 6</b>	<b>Numerical Methods for Large Matrices</b>	<b>251</b>
6.1	The principle of the methods	251
6.2	The method of subspace iteration revisited	253
6.3	The Lanczos method	257
6.4	The block Lanczos method	266
6.5	The generalized problem $Kx = \lambda Mx$	270
6.6	Arnoldi's method	272
6.7	Oblique projections	279
6.8	Bibliographical comments	280
	Exercises	281
<b>Chapter 7</b>	<b>Chebyshev's Iterative Methods</b>	<b>293</b>
7.1	Elements of the theory of uniform approximation for a compact set in $\mathbb{C}$	293
7.2	Chebyshev polynomials of a real variable	299

## CONTENTS

ix

7.3	Chebyshev polynomials of a complex variable	300
7.4	The Chebyshev acceleration for the power method	304
7.5	The Chebyshev iteration method	305
7.6	Simultaneous Chebyshev iterations (with projection)	308
7.7	Determination of the optimal parameters	311
7.8	Least squares polynomials on a polygon	312
7.9	The hybrid methods of Saad	314
7.10	Bibliographical comments	316
	Exercises	316

## **Appendices** **323**

A	Solution to Exercises	323
B	References for Exercises	367
C	References	371

## **Index** **378**

# Eigenvalues of Matrices

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The calculation of eigenvalues of matrices is a problem of great practical and theoretical importance with many different types of application. This book provides a modern and complete guide to this subject, at an elementary level, by presenting in matrix notation the fundamental aspects of the theory of linear operators in finite dimensions.

This volume is a combination of two books; translations of Professor Chatelin's original and the corresponding book of exercises by Professor Ahués. The exercises are an indispensable complement to the main text. Solutions are furnished for some of the exercises.

The book will be of particular value to undergraduate students following courses on numerical analysis and for researchers and practitioners with an interest in this area.

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