

Eberhard Zeidler

**Applied  
Mathematical  
Sciences  
109**

# **Applied Functional Analysis**

Main Principles and  
Their Applications



Springer-Verlag

# Contents

<b>Preface</b>	<b>vii</b>
<b>Contents of AMS Volume 108</b>	<b>xiii</b>
<b>1 The Hahn–Banach Theorem Optimization Problems</b>	<b>1</b>
1.1 The Hahn–Banach Theorem . . . . .	2
1.2 Applications to the Separation of Convex Sets . . . . .	6
1.3 The Dual Space $C[a, b]^*$ . . . . .	10
1.4 Applications to the Moment Problem . . . . .	13
1.5 Minimum Norm Problems and Duality Theory . . . . .	15
1.6 Applications to Čebyšev Approximation . . . . .	19
1.7 Applications to the Optimal Control of Rockets . . . . .	20
<b>2 Variational Principles and Weak Convergence</b>	<b>39</b>
2.1 The $n$ th Variation . . . . .	43
2.2 Necessary and Sufficient Conditions for Local Extrema and the Classical Calculus of Variations . . . . .	45
2.3 The Lack of Compactness in Infinite-Dimensional Banach Spaces . . . . .	48
2.4 Weak Convergence . . . . .	49
2.5 The Generalized Weierstrass Existence Theorem . . . . .	53
2.6 Applications to the Calculus of Variations . . . . .	56
2.7 Applications to Nonlinear Eigenvalue Problems . . . . .	59
2.8 Reflexive Banach Spaces . . . . .	61

2.9	Applications to Convex Minimum Problems and Variational Inequalities . . . . .	66
2.10	Applications to Obstacle Problems in Elasticity . . . . .	71
2.11	Saddle Points . . . . .	72
2.12	Applications to Duality Theory . . . . .	73
2.13	The von-Neumann Minimax Theorem on the Existence of Saddle Points . . . . .	75
2.14	Applications to Game Theory . . . . .	81
2.15	The Ekeland Principle about Quasi-Minimal Points . . . . .	83
2.16	Applications to a General Minimum Principle via the Palais-Smale Condition . . . . .	86
2.17	Applications to the Mountain Pass Theorem . . . . .	87
2.18	The Galerkin Method and Nonlinear Monotone Operators . . . . .	93
2.19	Symmetries and Conservation Laws (The Noether Theorem) . . . . .	98
2.20	The Basic Ideas of Gauge Field Theory . . . . .	102
2.21	Representations of Lie Algebras . . . . .	107
2.22	Applications to Elementary Particles . . . . .	112
<b>3</b>	<b>Principles of Linear Functional Analysis</b>	<b>167</b>
3.1	The Baire Theorem . . . . .	169
3.2	Application to the Existence of Nondifferentiable Continuous Functions . . . . .	171
3.3	The Uniform Boundedness Theorem . . . . .	172
3.4	Applications to Cubature Formulas . . . . .	175
3.5	The Open Mapping Theorem . . . . .	178
3.6	Product Spaces . . . . .	180
3.7	The Closed Graph Theorem . . . . .	181
3.8	Applications to Factor Spaces . . . . .	183
3.9	Applications to Direct Sums and Projections . . . . .	188
3.10	Dual Operators . . . . .	199
3.11	The Exactness of the Duality Functor . . . . .	205
3.12	Applications to the Closed Range Theorem and to Fredholm Alternatives . . . . .	210
<b>4</b>	<b>The Implicit Function Theorem</b>	<b>225</b>
4.1	$m$ -Linear Bounded Operators . . . . .	227
4.2	The Differential of Operators and the Fréchet Derivative . . . . .	228
4.3	Applications to Analytic Operators . . . . .	233
4.4	Integration . . . . .	238
4.5	Applications to the Taylor Theorem . . . . .	243
4.6	Iterated Derivatives . . . . .	244
4.7	The Chain Rule . . . . .	247
4.8	The Implicit Function Theorem . . . . .	250
4.9	Applications to Differential Equations . . . . .	254
4.10	Diffeomorphisms and the Local Inverse Mapping Theorem . . . . .	258

4.11	Equivalent Maps and the Linearization Principle . . . . .	260
4.12	The Local Normal Form for Nonlinear Double Splitting Maps . . . . .	264
4.13	The Surjective Implicit Function Theorem . . . . .	268
4.14	Applications to the Lagrange Multiplier Rule . . . . .	270
<b>5</b>	<b>Fredholm Operators</b>	<b>281</b>
5.1	Duality for Linear Compact Operators . . . . .	284
5.2	The Riesz–Schauder Theory on Hilbert Spaces . . . . .	286
5.3	Applications to Integral Equations . . . . .	291
5.4	Linear Fredholm Operators . . . . .	292
5.5	The Riesz–Schauder Theory on Banach Spaces . . . . .	295
5.6	Applications to the Spectrum of Linear Compact Operators . . . . .	296
5.7	The Parametrix . . . . .	298
5.8	Applications to the Perturbation of Fredholm Operators . .	300
5.9	Applications to the Product Index Theorem . . . . .	301
5.10	Fredholm Alternatives via Dual Pairs . . . . .	303
5.11	Applications to Integral Equations and Boundary-Value Problems . . . . .	305
5.12	Bifurcation Theory . . . . .	309
5.13	Applications to Nonlinear Integral Equations . . . . .	313
5.14	Applications to Nonlinear Boundary-Value Problems . . . .	315
5.15	Nonlinear Fredholm Operators . . . . .	317
5.16	Interpolation Inequalities . . . . .	322
5.17	Applications to the Navier–Stokes Equations . . . . .	329
	<b>References</b>	<b>371</b>
	<b>List of Symbols</b>	<b>385</b>
	<b>List of Theorems</b>	<b>391</b>
	<b>List of Most Important Definitions</b>	<b>393</b>
	<b>Subject Index</b>	<b>399</b>

This is the second part of an elementary textbook which combines linear functional analysis, nonlinear functional analysis, numerical functional analysis, and their substantial applications with each other. The book addresses undergraduate students and beginning graduate students of mathematics, physics, and engineering who want to learn how functional analysis elegantly solves mathematical problems which relate to our real world and which play an important role in the history of mathematics. The book's approach begins with the question "what are the most important applications" and proceeds to try to answer this question. The applications concern integral equations, differential equations, bifurcation theory, the moment problem, Čebyšev approximation, the optimal control of rockets, game theory, symmetries and conservation laws (the Noether theorem), the quark model, and gauge theory in elementary particle physics. The presentation is self-contained. As for prerequisites, the reader should be familiar with some basic facts of calculus.

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