

CMS Books in Mathematics

Jonathan M. Borwein
Adrian S. Lewis

Convex Analysis and Nonlinear Optimization

Theory and Examples

$$g^*(y) = \log \sum_i \exp \langle a^i, y \rangle$$



Canadian Mathematical Society
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**Convex Analysis and Nonlinear Optimization:
Theory and Examples**

Ouvrages de mathématiques de la SMC

This book is a concise account of convex analysis, its applications and extensions, for a broad audience. Blurring as it does the distinctions between mathematical optimization and modern analysis, the elegant language of convexity and duality is indispensable both in computational optimization and for understanding variational properties of functions and multifunctions. Primarily aimed at first-year graduate students, the text consists of short, self-contained sections, each followed by an extensive set of exercises, many of which are guided. The book is thus appropriate either as a class text or for self-study.

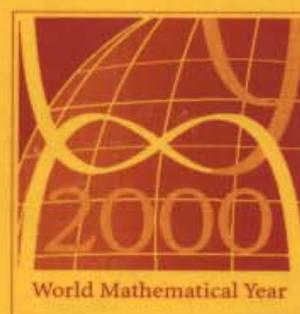
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Springer

ISBN 0-387-98940-4
www.springer-ny.com
www.cms.math.ca

