

Texts in
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Mathematics
35

J. Kevorkian

Partial Differential Equations

Analytical Solution
Techniques

Second Edition



Springer

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This book contains a broad treatment of partial differential equations that arise in the sciences and engineering, particularly emphasizing the analytical solution techniques. In each chapter the author discusses the important applications that lead to the class of equations to be studied, presents different methods for tackling these equations, and concludes with a selection of instructive problems. A relevant bibliography has also been included at the end of this book.

This new edition has been substantially updated to include a wider scope of topics. New material has been added to all the chapters, and some of the derivations and discussions have been streamlined. Also, much of the needed background is reviewed in the Appendix.

Students and researchers in the sciences and engineering will find this book useful.

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