

**Applied
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82**

Rainer Kress

Linear Integral Equations

Second Edition



Springer

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This book originated from the author's fascination with the mathematical beauty of integral equations. It is an attempt to combine theory, applications, and numerical methods, and cover each of these fields with the same weight. In order to make the book accessible to mathematicians, physicists, and engineers, the author has made the work as self-contained as possible, by requiring only a solid foundation in differential and integral calculus. The functional analysis which is necessary for an adequate treatment of the theory and the numerical solution of integral equations is developed within the book. Problems are included at the end of each chapter.

For the second edition, in addition to corrections and adjustments throughout the text, as well as an updated reference section, new topics have been added.

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