

INTERDISCIPLINARY APPLIED MATHEMATICS

SYSTEMS AND CONTROL

CA 33

Nonlinear Systems

Analysis, Stability,
and Control

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There has been a great deal of excitement over the last ten years concerning the emergence of new mathematical techniques for the analysis and control of nonlinear systems: witness the emergence of a set of simplified tools for the analysis of bifurcations, chaos, and other complicated dynamical behavior and the development of a comprehensive theory of geometric nonlinear control. Coupled with this set of analytic advances has been the vast increase in computational power available both for the simulation of nonlinear systems as well as for the implementation in real time of sophisticated, real-time nonlinear control laws. Thus, technological advances have bolstered the impact of analytic advances and produced a tremendous variety of new problems and applications which are nonlinear in an essential way.

This book lays out in a concise mathematical framework the tools and methods of analysis which underlie this diversity of applications. The book covers analysis, stability theory, and geometric nonlinear control.

The material presented in this book is culled from different first-year graduate courses that the author has taught at MIT and at Berkeley.

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