Discrete Mathematics and Theoretical Computer Science

INFORMATION ALGEBRAS

GENERIC STRUCTURES FOR INFERENCE

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