

Peitgen

Jürgens

Saupe

Chaos and Fractals

New Frontiers of Science

SECOND EDITION



Springer

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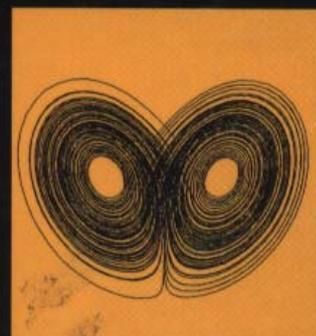
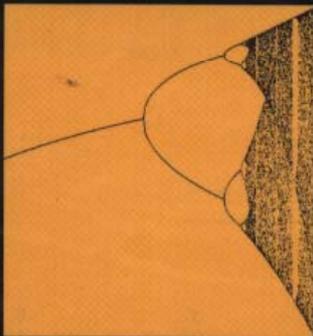
From the reviews of the first edition:

"Fascinating and authoritative, *Chaos and Fractals: New Frontiers of Science* is a truly remarkable book that documents recent discoveries in chaos theory with plenty of mathematical detail, but without alienating the general reader. In all, this text offers an extremely rich and engaging tour of this quite revolutionary branch of mathematical research. ...

There have been advances in the field since this book's publication in 1992, but *Chaos and Fractals* remains an authoritative general reference on chaos theory and fractals. A must for math students (and math enthusiasts), *Chaos and Fractals* also deserves a place on the bookshelf of any general reader or programmer who wants to understand how today's mathematicians and scientists make sense of our world using chaos theory." *Richard Dragan, Editor for Amazon.com*

"This book—which has been named "Most Outstanding Book in the Mathematics Category" by the Association of American Publishers in 1992—contains all one ever wanted to know about fractals, and more. Written by—next to Mandelbrot—the greatest popularizer of the concept of fractal geometry (plus two co-workers), it contains a wealth of information on nearly every angle of the topic. ..." *K. Brod for Zentralblatt*

The fourteen chapters of this book cover the central ideas and concepts of chaos and fractals as well as many related topics including: the Mandelbrot set, Julia sets, cellular automata, L-systems, percolation and strange attractors. This new edition has been updated in many places. The appendices of the original edition were taken out since more recent publications cover this material in more depths. Instead of the focussed computer programs in BASIC, the authors provide 10 interactive JAVA-applets for this second edition.



The computer-generated images of the Feigenbaum diagram, the Mandelbrot set, and the Lorenz attractor are the icons of chaos theory and fractal geometry. They will be remembered as milestones in the deciphering of the secrets of nature.

