

# Graduate Texts in Mathematics

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## **Topology and Geometry**



**Springer**

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This book is intended as a textbook for a first-year graduate course on algebraic topology, with a strong flavoring in smooth manifold theory. Starting with general topology, it discusses differentiable manifolds, cohomology, products and duality, the fundamental group, homology theory, and homotopy theory. It covers most of the topics all topologists will want students to see, including surfaces, Lie groups, and fibre bundle theory.

With a thoroughly modern point of view, it is the first truly new textbook in topology since Spanier, almost 25 years ago. Although the book is comprehensive, there is no attempt made to present the material in excessive generality, except where generality improves the efficiency and clarity of the presentation.

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