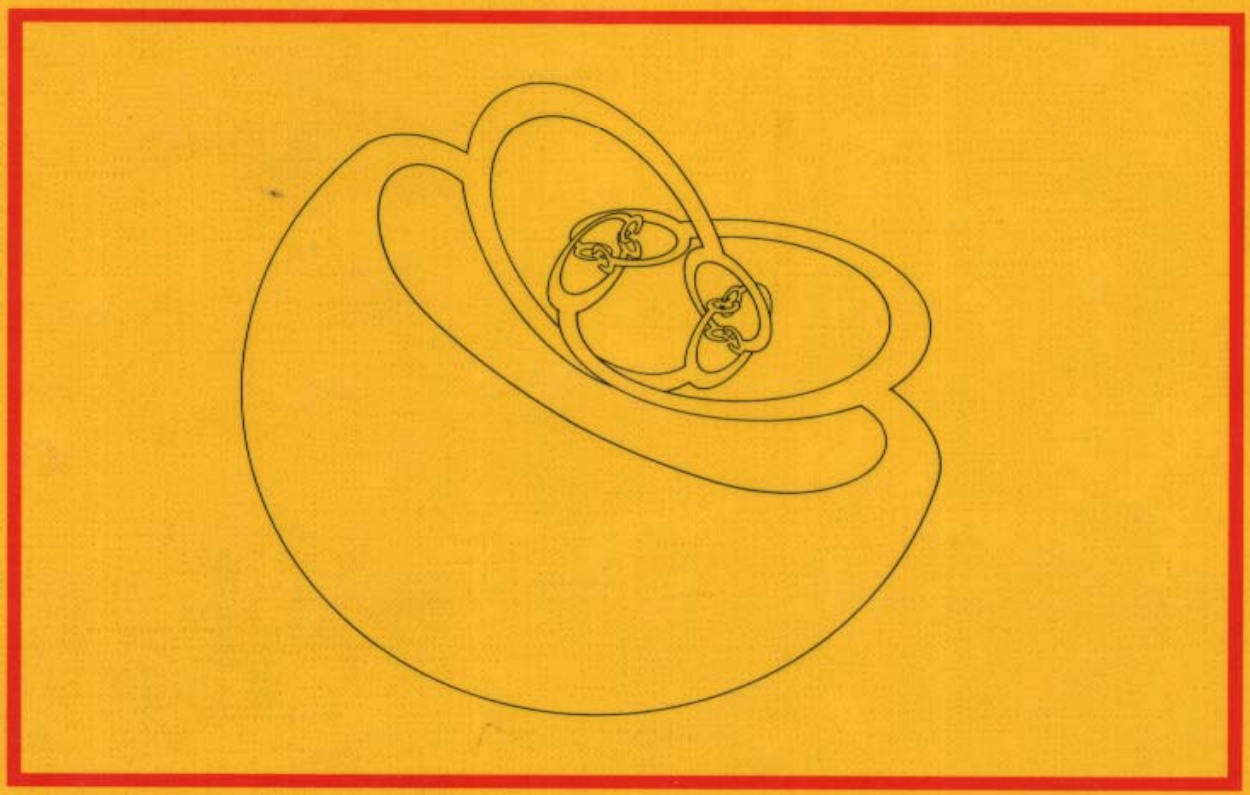
Undergraduate Texts in Mathematics

L. Christine Kinsey

# Topology of Surfaces



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This book aims to provide undergraduates with an understanding of geometric topology. Topics covered include a sampling from pointset, geometric, and algebraic topology. The presentation is pragmatic, and includes many line drawings to help the student visualize the concepts, and tries to avoid "that famous pedagogical method whereby one begins with the general and proceeds to the particular only after the student is too confused to understand even that anymore."

Exercises are an integral part of the text. Students taking the course are assumed to have some exposure to algebraic ideas, as would be acquired in a linear algebra course. All of the group theory necessary is developed in the appendix.

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