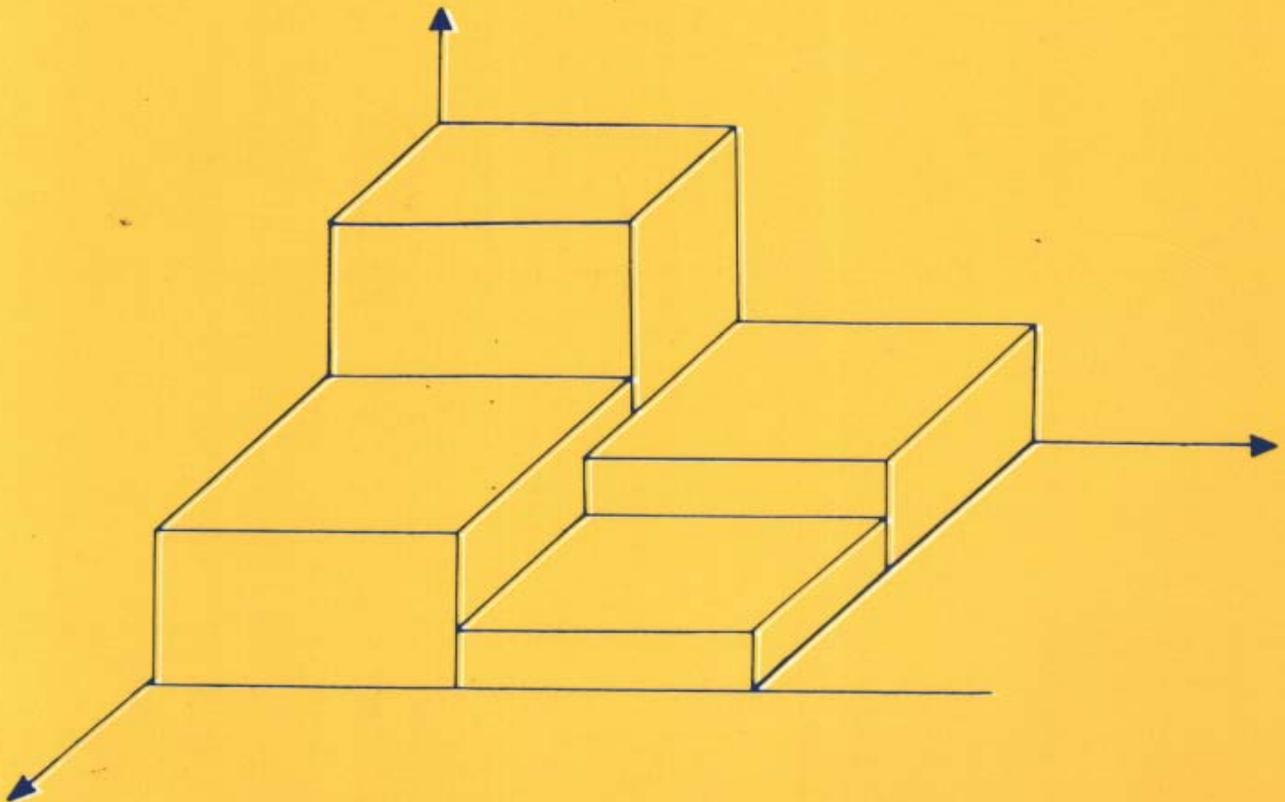


Soo Bong Chae

LEBESGUE INTEGRATION

SECOND EDITION



Springer-Verlag

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The fundamental concepts of Lebesgue integration provide a basis for approaching theoretical and applied problems in mathematics, engineering, physics, and many other sciences. *Lebesgue Integration* presents a logical development of the basic concepts of Lebesgue's integration theorems, proceeding from the study of topological concepts on the real line. This book employs F. Riesz's innovative methodology to motivate Lebesgue's theories, rather than taking the more traditional approach which employs abstract measure theory.

The book contains an in-depth study of the fundamental theorem of calculus for the Lebesgue integral, which adopts L.A. Rubel's proof of the differentiability of monotone functions in conjunction with F. Riesz's Rising Sun Lemma. The book also thoroughly reviews Riemann integration—and its deficiencies—to underline the need for Lebesgue integration. Historical material found throughout the book helps to explain the development of various concepts of integration from Cauchy to Lebesgue. Numerous examples and exercises also help to clarify the ideas.

This self-contained text defines all of the concepts it presents. *Lebesgue Integration* is intended for advanced undergraduates and beginning graduate students in mathematics, physics, and related scientific fields, and is also suitable for self-study by students preparing for the masters or doctoral preliminary examinations. The book can be used as a textbook for a course in real analysis, or as a supplementary text for a graduate course in abstract integration theory.

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