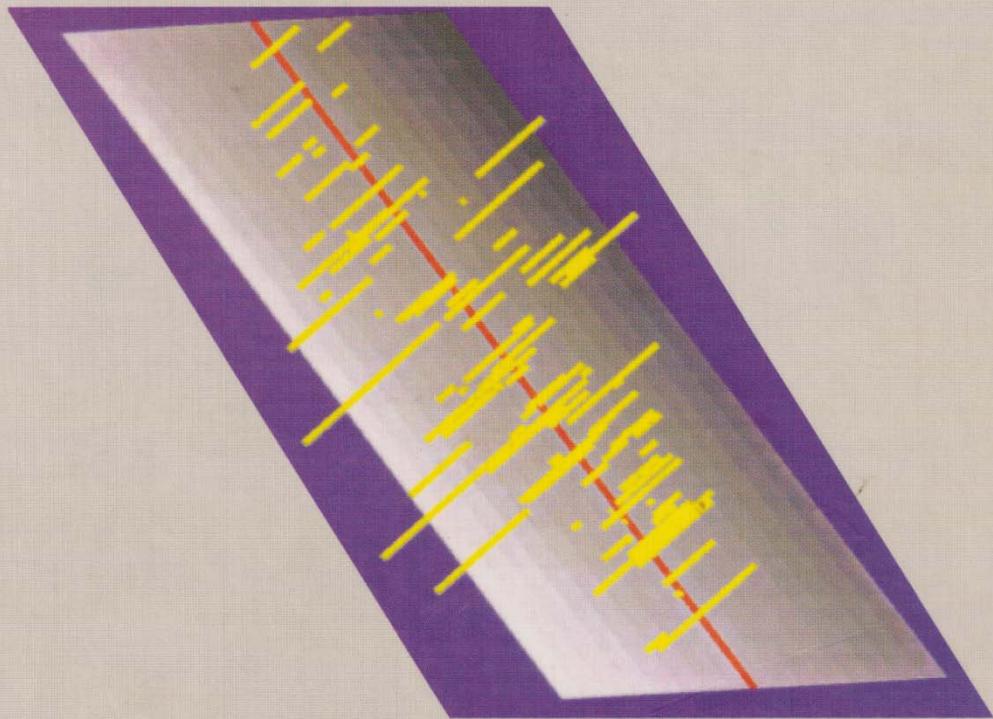


Walter Gander Jiří Hřebíček

Solving Problems  
in Scientific Computing  
Using **Maple** and



**MATLAB<sup>®</sup>**

*Third Edition*



Springer

-024-26-11

2-004-26-1

Walter Gander • Jiří Hřebíček

# Solving Problems in Scientific Computing Using Maple and MATLAB®



Third, Expanded and Revised Edition 1997

With 161 Figures and 12 Tables



Springer

# Contents

<b>Chapter 1. The Tractrix and Similar Curves</b> . . . . .	1
1.1 Introduction . . . . .	1
1.2 The Classical Tractrix . . . . .	1
1.3 The Child and the Toy . . . . .	4
1.4 The Jogger and the Dog . . . . .	6
1.5 Showing the Motions with MATLAB . . . . .	12
1.6 Jogger with Constant Velocity . . . . .	15
1.7 Using a Moving Coordinate System . . . . .	17
1.7.1 Transformation for Jogger/Dog . . . . .	18
1.7.2 Transformation for Child/Toy . . . . .	20
1.8 Examples . . . . .	22
References . . . . .	25
<b>Chapter 2. Trajectory of a Spinning Tennis Ball</b> . . . . .	27
2.1 Introduction . . . . .	27
2.2 MAPLE Solution . . . . .	29
2.3 MATLAB Solution . . . . .	33
2.4 Simpler Solution for MATLAB 5 . . . . .	35
References . . . . .	37
<b>Chapter 3. The Illumination Problem</b> . . . . .	39
3.1 Introduction . . . . .	39
3.2 Finding the Minimal Illumination Point on a Road . . . . .	40
3.3 Varying $h_2$ to Maximize the Illumination . . . . .	43
3.4 Optimal Illumination . . . . .	45
3.5 Conclusion . . . . .	50
References . . . . .	50
<b>Chapter 4. Orbits in the Planar Three-Body Problem</b> . . . . .	51
4.1 Introduction . . . . .	51
4.2 Equations of Motion in Physical Coordinates . . . . .	52
4.3 Global Regularization . . . . .	56
4.4 The Pythagorean Three-Body Problem . . . . .	62
4.5 Conclusions . . . . .	70
References . . . . .	71

<b>Chapter 5. The Internal Field in Semiconductors . . .</b>	<b>73</b>
5.1 Introduction . . . . .	73
5.2 Solving a Nonlinear Poisson Equation Using MAPLE	74
5.3 MATLAB Solution . . . . .	80
References . . . . .	81
<b>Chapter 6. Some Least Squares Problems . . . . .</b>	<b>83</b>
6.1 Introduction . . . . .	83
6.2 Fitting Lines, Rectangles and Squares in the Plane	83
6.3 Fitting Hyperplanes . . . . .	95
References . . . . .	101
<b>Chapter 7. The Generalized Billiard Problem . . . . .</b>	<b>103</b>
7.1 Introduction . . . . .	103
7.2 The Generalized Reflection Method . . . . .	103
7.2.1 Line and Curve Reflection . . . . .	104
7.2.2 Mathematical Description . . . . .	105
7.2.3 MAPLE Solution . . . . .	106
7.3 The Shortest Trajectory Method . . . . .	107
7.3.1 MAPLE Solution . . . . .	108
7.4 Examples . . . . .	108
7.4.1 The Circular Billiard Table . . . . .	108
7.4.2 The Elliptical Billiard Table . . . . .	113
7.4.3 The Snail Billiard Table . . . . .	116
7.4.4 The Star Billiard Table . . . . .	116
7.5 Conclusions . . . . .	119
References . . . . .	121
<b>Chapter 8. Mirror Curves . . . . .</b>	<b>123</b>
8.1 The Interesting Waste . . . . .	123
8.2 The Mirror Curves Created by MAPLE . . . . .	123
8.3 The Inverse Problem . . . . .	125
8.3.1 Outflanking Manoeuvre . . . . .	125
8.3.2 Geometrical Construction of a Point on the Pattern Curve . . . . .	126
8.3.3 MAPLE Solution . . . . .	127
8.3.4 Analytic Solution . . . . .	128
8.4 Examples . . . . .	128
8.4.1 The Circle as the Mirror Curve . . . . .	128
8.4.2 The Line as the Mirror Curve . . . . .	131
8.5 Conclusions . . . . .	132
References . . . . .	134

<b>Chapter 9. Smoothing Filters</b> . . . . .	135
9.1 Introduction . . . . .	135
9.2 Savitzky-Golay Filter . . . . .	135
9.2.1 Filter Coefficients . . . . .	136
9.2.2 Results . . . . .	139
9.3 Least Squares Filter . . . . .	139
9.3.1 Lagrange Equations . . . . .	141
9.3.2 Zero Finder . . . . .	143
9.3.3 Evaluation of the Secular Function . . . . .	144
9.3.4 MEX-Files . . . . .	146
9.3.5 Results . . . . .	150
References . . . . .	152
<b>Chapter 10. The Radar Problem</b> . . . . .	153
10.1 Introduction . . . . .	153
10.2 Converting Degrees into Radians . . . . .	154
10.3 Transformation into Geocentric Coordinates . . . . .	155
10.4 The Transformations . . . . .	158
10.5 Final Algorithm . . . . .	160
10.6 Practical Example . . . . .	160
References . . . . .	163
<b>Chapter 11. Conformal Mapping of a Circle</b> . . . . .	165
11.1 Introduction . . . . .	165
11.2 Problem Outline . . . . .	165
11.3 MAPLE Solution . . . . .	166
11.4 MATLAB Solution . . . . .	170
References . . . . .	172
<b>Chapter 12. The Spinning Top</b> . . . . .	173
12.1 Introduction . . . . .	173
12.2 Formulation and Basic Analysis of the Solution . . . . .	175
12.3 The Numerical Solution . . . . .	180
References . . . . .	182
<b>Chapter 13. The Calibration Problem</b> . . . . .	183
13.1 Introduction . . . . .	183
13.2 The Physical Model Description . . . . .	183
13.3 Approximation by Splitting the Solution . . . . .	186
13.4 Conclusions . . . . .	191
References . . . . .	192
<b>Chapter 14. Heat Flow Problems</b> . . . . .	193
14.1 Introduction . . . . .	193
14.2 Heat Flow through a Spherical Wall . . . . .	193
14.2.1 A Steady State Heat Flow Model . . . . .	194

14.2.2	Fourier Model for Steady State . . . . .	195
14.2.3	MAPLE Plots . . . . .	196
14.3	Non Stationary Heat Flow through an Agriculture Field . . . . .	197
14.3.1	MAPLE Plots . . . . .	201
References	. . . . .	201
<b>Chapter 15.</b>	<b>Modeling Penetration Phenomena . . . . .</b>	<b>203</b>
15.1	Introduction . . . . .	203
15.2	Short description of the penetration theory . . . . .	203
15.3	The Tate-Alekseevskii model . . . . .	205
15.3.1	Special case $R_t = Y_p$ . . . . .	207
15.3.2	Special case $\rho_p = \rho_t = \rho$ . . . . .	207
15.4	The eroding rod penetration model . . . . .	209
15.5	Numerical Example . . . . .	214
15.6	Conclusions . . . . .	218
References	. . . . .	218
<b>Chapter 16.</b>	<b>Heat Capacity of System     of Bose Particles . . . . .</b>	<b>219</b>
16.1	Introduction . . . . .	219
16.2	MAPLE Solution . . . . .	221
References	. . . . .	225
<b>Chapter 17.</b>	<b>Free Metal Compression . . . . .</b>	<b>227</b>
17.1	Introduction . . . . .	227
17.2	The Base expansion . . . . .	229
17.3	Base Described by One and Several Functions . . . . .	231
17.4	The Lateral Side Distortion . . . . .	235
17.5	Non-centered bases . . . . .	238
17.6	Three Dimensional Graphical Representation of the Distorted Body . . . . .	242
17.6.1	Centered base . . . . .	242
17.6.2	Non-centered, Segmented Base . . . . .	244
17.6.3	Convex Polygon Base . . . . .	246
17.7	Three Dimensional Animation . . . . .	248
17.8	Limitations and Conclusions . . . . .	250
References	. . . . .	250
<b>Chapter 18.</b>	<b>Gauss Quadrature . . . . .</b>	<b>251</b>
18.1	Introduction . . . . .	251
18.2	Orthogonal Polynomials . . . . .	252
18.3	Quadrature Rule . . . . .	266
18.4	Gauss Quadrature Rule . . . . .	267
18.5	Gauss-Radau Quadrature Rule . . . . .	268

18.6 Gauss-Lobatto Quadrature Rule . . . . .	271
18.7 Weights . . . . .	274
18.8 Quadrature Error . . . . .	275
References . . . . .	277
<b>Chapter 19. Symbolic Computation of Explicit Runge-Kutta Formulas . . . . .</b>	<b>281</b>
19.1 Introduction . . . . .	281
19.2 Derivation of the Equations for the Parameters . . . . .	283
19.3 Solving the System of Equations . . . . .	285
19.3.1 Gröbner Bases . . . . .	286
19.3.2 Resultants . . . . .	289
19.4 The Complete Algorithm . . . . .	291
19.4.1 Example 1: . . . . .	291
19.4.2 Example 2: . . . . .	292
19.5 Conclusions . . . . .	295
References . . . . .	296
<b>Chapter 20. Transient Response of a Two-Phase Half-Wave Rectifier . . . . .</b>	<b>297</b>
20.1 Introduction . . . . .	297
20.2 Problem Outline . . . . .	297
20.3 Difficulties in Applying Conventional Codes and Software Packages . . . . .	300
20.4 Solution by Means of MAPLE . . . . .	302
References . . . . .	308
<b>Chapter 21. Circuits in Power Electronics . . . . .</b>	<b>309</b>
21.1 Introduction . . . . .	309
21.2 Linear Differential Equations with Piecewise Constant Coefficients . . . . .	311
21.3 Periodic Solutions . . . . .	314
21.4 A MATLAB Implementation . . . . .	315
21.5 Conclusions . . . . .	320
References . . . . .	320
<b>Chapter 22. Newton's and Kepler's laws . . . . .</b>	<b>321</b>
22.1 Introduction . . . . .	321
22.2 Equilibrium of Two Forces . . . . .	321
22.3 Equilibrium of Three Forces . . . . .	322
22.4 Equilibrium of Three Forces, Computed from the Potential Energy . . . . .	324
22.5 Gravitation of the Massive Line Segment . . . . .	326
22.5.1 Potential and Intensity . . . . .	326
22.5.2 The Particle Trajectory . . . . .	329

22.6 The Earth Satellite . . . . .	330
22.7 Earth Satellite, Second Solution . . . . .	333
22.8 The Lost Screw . . . . .	335
22.9 Conclusions . . . . .	336
References . . . . .	337
<b>Chapter 23. Least Squares Fit of Point Clouds . . . . .</b>	<b>339</b>
23.1 Introduction . . . . .	339
23.2 Computing the Translation . . . . .	339
23.3 Computing the Orthogonal Matrix . . . . .	340
23.4 Solution of the Procrustes Problem . . . . .	341
23.5 Algorithm . . . . .	342
23.6 Decomposing the Orthogonal Matrix . . . . .	344
23.7 Numerical Examples . . . . .	345
23.7.1 First example . . . . .	345
23.7.2 Second example . . . . .	348
References . . . . .	349
<b>Chapter 24. Modeling Social Processes . . . . .</b>	<b>351</b>
24.1 Introduction . . . . .	351
24.2 Modeling Population Migration . . . . .	351
24.2.1 Cyclic Migration without Regulation . . . . .	353
24.2.2 Cyclic Migration with Regulation . . . . .	354
24.3 Modeling Strategic Investment . . . . .	356
References . . . . .	358
<b>Chapter 25. Contour Plots of Analytic Functions . . . . .</b>	<b>359</b>
25.1 Introduction . . . . .	359
25.2 Contour Plots by the <code>contour</code> Command . . . . .	359
25.3 Differential Equations . . . . .	362
25.3.1 Contour Lines $r = \text{const.}$ . . . . .	362
25.3.2 Contour Lines $\varphi = \text{const.}$ . . . . .	364
25.4 The Contour Lines $r = 1$ of $f = e_n$ . . . . .	366
25.5 The Contour Lines $\varphi = \text{const}$ of $f = e_n$ . . . . .	370
References . . . . .	372
<b>Chapter 26. Non Linear Least Squares: Finding the     most accurate location of an aircraft . . . . .</b>	<b>373</b>
26.1 Introduction . . . . .	373
26.2 Building the Least Squares Equations . . . . .	374
26.3 Solving the Non-linear System . . . . .	376
26.4 Confidence/Sensitivity Analysis . . . . .	378

<b>Chapter 27. Computing Plane Sundials</b> . . . . .	381
27.1 Introduction . . . . .	381
27.2 Astronomical Fundamentals . . . . .	381
27.2.1 Coordinate Systems . . . . .	382
27.2.2 The Gnomonic Projection . . . . .	384
27.3 Time Marks . . . . .	386
27.3.1 Local Real Time . . . . .	386
27.3.2 Mean Time . . . . .	387
27.3.3 Babylonian and Italic Hours . . . . .	392
27.4 Sundials on General Planes . . . . .	393
27.5 A Concluding Example . . . . .	394
References . . . . .	396
<b>Index</b> . . . . .	397
<b>Index of used MAPLE Commands</b> . . . . .	403
<b>Index of used MATLAB Commands</b> . . . . .	407