NONLINEAR PROGRAMMING

THEORY AND ALGORITHMS

SECOND EDITION

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Nonlinear Programming

Theory and Algorithms

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