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Comprehensive Mathematics for Computer Scientists 1

Sets and Numbers, Graphs and Algebra,
Logic and Machines, Linear Geometry



Springer

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Volume II

III Topology and Calculus

Limits and Topology, Differentiability, Inverse and Implicit Functions, Integration, Fubini and Changing Variables, Vector Fields, Fixpoints, Main Theorem of ODEs

IV Selected Higher Subjects

Numerics, Probability and Statistics, Splines, Fourier, Wavelets, Fractals, Neural Nets, Global Coordinates and Manifolds, Categories, Lambda Calculus



GUERINO MAZZOLA received his PhD in Mathematics at the University of Zurich. He first qualified as a professor in Mathematics and then also in Computational Science. In 2002 he published a major monograph on the topos-theoretic foundations of Music Theory and Music Informatics. His current interests include Gesture Theory and Robot Logic.



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This two-volume textbook *Comprehensive Mathematics for Computer Scientists* is a self-contained comprehensive presentation of mathematics including sets, numbers, graphs, algebra, logic, grammars, machines, linear geometry, calculus, ODEs, and special themes such as neural networks, Fourier theory, wavelets, numerical issues, statistics, categories, and manifolds. The concept framework is streamlined but defining and proving virtually everything. The style implicitly follows the spirit of recent topos-oriented theoretical computer science. Despite the theoretical soundness, the material stresses a large number of core computer science subjects, such as, for example, a discussion of floating point arithmetic, Backus-Naur normal forms, L-systems, Chomsky hierarchies, algorithms for data encoding, e.g., the Reed-Solomon code. The numerous course examples are motivated by computer science and bear a generic scientific meaning. This text is complemented by an online university course which covers the same theoretical content, however, in a totally different presentation. The student or working scientist who once gets involved in this text may at any time consult the online interface which comprises applets and other interactive tools.

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