

SOLID-STATE SCIENCES

K. Binder  
D. W. Heermann

**Monte Carlo  
Simulation  
in Statistical  
Physics**  
An Introduction

Fourth Edition



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# Monte Carlo Simulation in Statistical Physics

An Introduction

Fourth, Enlarged Edition  
With 48 Figures



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# Contents

<b>1. Introduction: Purpose and Scope of this Volume, and Some General Comments</b> .....	1
<b>2. Theoretical Foundations of the Monte Carlo Method and Its Applications in Statistical Physics</b> .....	5
2.1 Simple Sampling Versus Importance Sampling .....	5
2.1.1 Models .....	5
2.1.2 Simple Sampling .....	6
2.1.3 Random Walks and Self-Avoiding Walks .....	8
2.1.4 Thermal Averages by the Simple Sampling Method ...	13
2.1.5 Advantages and Limitations of Simple Sampling .....	14
2.1.6 Importance Sampling .....	17
2.1.7 More About Models and Algorithms .....	20
2.2 Organization of Monte Carlo Programs, and the Dynamic Interpretation of Monte Carlo Sampling ...	23
2.2.1 First Comments on the Simulation of the Ising Model .....	23
2.2.2 Boundary Conditions .....	26
2.2.3 The Dynamic Interpretation of the Importance Sampling Monte Carlo Method .....	29
2.2.4 Statistical Errors and Time-Displaced Relaxation Functions .....	33
2.3 Finite-Size Effects .....	35
2.3.1 Finite-Size Effects at the Percolation Transition .....	35
2.3.2 Finite-Size Scaling for the Percolation Problem .....	39
2.3.3 Broken Symmetry and Finite-Size Effects at Thermal Phase Transitions .....	42
2.3.4 The Order Parameter Probability Distribution and Its Use to Justify Finite-Size Scaling and Phenomenological Renormalization .....	44
2.3.5 Finite-Size Behavior of Relaxation Times .....	54
2.3.6 Finite-Size Scaling Without “Hyperscaling” .....	57
2.3.7 Finite-Size Scaling for First-Order Phase Transitions .....	58

2.3.8	Finite-Size Behavior of Statistical Errors and the Problem of Self-Averaging . . . . .	65
2.4	Remarks on the Scope of the Theory Chapter . . . . .	68
<b>3.</b>	<b>Guide to Practical Work with the Monte Carlo Method . .</b>	<b>69</b>
3.1	Aims of the Guide . . . . .	72
3.2	Simple Sampling . . . . .	74
3.2.1	Random Walk . . . . .	74
3.2.2	Nonreversal Random Walk . . . . .	82
3.2.3	Self-Avoiding Random Walk . . . . .	83
3.2.4	Percolation . . . . .	87
3.3	Biased Sampling . . . . .	95
3.3.1	Self-Avoiding Random Walk . . . . .	95
3.4	Importance Sampling . . . . .	98
3.4.1	Ising Model . . . . .	98
3.4.2	Self-Avoiding Random Walk . . . . .	112
<b>4.</b>	<b>Some Important Recent Developments of the Monte Carlo Methodology . . . . .</b>	<b>115</b>
4.1	Introduction . . . . .	115
4.2	Application of the Swendsen–Wang Cluster Algorithm to the Ising Model . . . . .	117
4.3	Reweighting Methods in the Study of Phase Diagrams, First-Order Phase Transitions, and Interfacial Tensions . . . . .	122
4.4	Some Comments on Advances with Finite-Size Scaling Analyses . . . . .	127
<b>5.</b>	<b>Quantum Monte Carlo Simulations: An Introduction . . . . .</b>	<b>137</b>
5.1	Quantum Statistical Mechanics vs. Classical Statistical Mechanics . . . . .	137
5.2	The Path Integral Quantum Monte Carlo Method (PIMC) . . . . .	143
5.3	Quantum Monte Carlo for Lattice Models . . . . .	150
5.4	Concluding Remarks . . . . .	158
<b>Appendix</b>	. . . . .	<b>159</b>
A.1	Algorithm for the Random Walk Problem . . . . .	159
A.2	Algorithm for Cluster Identification . . . . .	160
<b>References</b>	. . . . .	<b>165</b>
<b>Bibliography</b>	. . . . .	<b>175</b>
<b>Subject Index</b>	. . . . .	<b>177</b>

Monte Carlo Simulation in Statistical Physics deals with the computer simulation of many-body systems in condensed-matter physics and related fields of physics, chemistry and beyond, to traffic flows, stock market fluctuations, etc.). Using random numbers generated by a computer, probability distributions are calculated, allowing the estimation of the thermodynamic properties of various systems. This book describes the theoretical background to several variants of these Monte Carlo methods and gives a systematic presentation from which newcomers can learn to perform such simulations and to analyze their results. This fourth edition has been updated and a new chapter on Monte Carlo simulation of quantum-mechanical problems has been added. To help students in their work a special web server has been installed to host programs and discussion groups (<http://wwwcp.tphys.uni-heidelberg.de>). Professor Binder was the winner of the Berni J. Alder CECAM Award for Computational Physics 2001.

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