

EIGENVALUES OF MATRICES

$$\begin{pmatrix} \alpha_0 & \beta_1 & 0 & 0 & \dots & 0 \\ \beta_1 & \alpha_1 & \beta_2 & 0 & \dots & 0 \\ 0 & \beta_2 & \alpha_2 & \beta_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \beta_n & \alpha_n \end{pmatrix}$$

Françoise Chatelin



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Eigenvalues of Matrices

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*With exercises by Françoise Chatelin and
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The calculation of eigenvalues of matrices is a problem of great practical and theoretical importance with many different types of application. This book provides a modern and complete guide to this subject, at an elementary level, by presenting in matrix notation the fundamental aspects of the theory of linear operators in finite dimensions.

This volume is a combination of two books; translations of Professor Chatelin's original and the corresponding book of exercises by Professor Ahués. The exercises are an indispensable complement to the main text. Solutions are furnished for some of the exercises.

The book will be of particular value to undergraduate students following courses on numerical analysis and for researchers and practitioners with an interest in this area.

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