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Structured Matrices in Mathematics, Computer Science, and Engineering I

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Foreword

Many important problems in applied sciences, mathematics, and engineering can be reduced to matrix problems. Moreover, various applications often introduce a special structure into the corresponding matrices, so that their entries can be described by a certain compact formula. Among classical examples are Toeplitz matrices $[a_{i-j}]$, Hankel matrices $[a_{i+j}]$, Toeplitz-plus-Hankel matrices, Vandermonde matrices $[a_i^{j-1}]$, Cauchy matrices $[\frac{1}{a_i-b_j}]$, Pick matrices $[\frac{1-a_i a_j^*}{b_i+b_j^*}]$, and also Bezoutians, controllability and observability matrices and others. Though standard linear algebra methods are, of course, readily available, there are several reasons why they can be unattractive in many instances. Along with just the desire to find elegant structure-exploiting solutions, there are also practical computational considerations such as the storage limitations, the need in reducing computational complexity as well as in obtaining a better numerical accuracy. In many cases these goals can be achieved by solving the underlying problem in terms of only $O(n)$ of parameters defining a structured $n \times n$ matrix via a compact formula as in the above examples.

Structured matrices have been under close study for a long time, and in quite diverse (and seemingly unrelated) areas. Typically, not only an area of application gives rise to certain patterns of structure, but it often provides a technique to solve the associated matrix problems. As an illustration of this principle we mention the classical interpolation problems of Caratheodory-Toeplitz and of Nevanlinna-Pick. Not only are they related to positive definite Toeplitz and Pick matrices, resp., but the recursive algorithms of Schur and Nevanlinna for computing their solutions admit nice matrix interpretations. In fact, these classical interpolation algorithms can be understood as efficient structure-exploiting ways to compute the Cholesky decompositions for these matrices. As was mentioned above, structured matrices were studied from different points of view, in mathematics, computer science and engineering. For example, the same Toeplitz and Pick matrices were closely looked at using the methods of reproducing kernel Hilbert spaces, lifting-of-commutants, state-space methods, as well as the methods of system theory and signal processing, network theory, linear prediction, to mention just a few mathematical and engineering fields. Interestingly, in the latter a physical intuition often provides deep insights into structured matrix problems. An interplay between the techniques of engineers and mathematicians is reflected in these volumes. There are several other areas providing their own applications and their own languages to attack structured matrix problems. It can be quite difficult to survey all such connections; in fact, browsing through the papers of these volumes can give a flavor of the plethora of different techniques and approaches. It appears that the theory of structured matrices is positioned to bridge the gaps between these diverse areas.