MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

UNIVERSITY of BLIDA 1

FACULTY OF TECHNOLOGY

ELECTRONICS DEPARTMENT

Course

Fundamental Electrotechnics

Intended for 2nd year undergraduate students in Electrical Engineering

Realized par

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Foreword

This course begins with a review of fundamental mathematical concepts, particularly complex numbers, and the basic laws of electricity. It then progresses to the analysis of electrical circuits in direct current (DC), single-phase alternating current (AC), and three-phase AC systems.

Magnetic circuits and electrical transformers are presented in a structured and pedagogical manner, helping students build a strong understanding of their principles and applications. The course concludes with technological descriptions and practical insights into various types of electrical machines.

The content of this course aligns with the curriculum taught in the Department of Electronics and is designed for second-year undergraduate students (LMD system) specializing in electrical engineering.

Introduction

This course provides a foundation in electrotechnics, covering the fundamental principles, mathematical tools, and practical applications essential for understanding electricity and simple analysis of electrical machines. It is structured in six chapters, ensuring that students build both theoretical knowledge and applied skills step by step.

The course begins with a review of mathematical concepts, particularly complex numbers, which are used for analyzing alternating current (AC) circuits and advanced electrical phenomena. It then moves into the core laws of electricity, providing the groundwork for studying resistive, capacitive, and inductive elements in both direct current (DC) and AC regimes. These basics are reinforced by practical methods for circuit analysis and power calculations.

Building on these foundations, the course explores circuits and electrical power, introducing single-phase and three-phase systems. Students learn how to evaluate active, reactive, and apparent powers, understand the significance of the power factor, and perform accurate power measurements in practical scenarios.

Subsequent chapters introduce magnetic circuits and transformers, focusing on the principles of magnetism, energy transfer, and voltage transformation. These sections highlight real-world applications in energy distribution and industrial systems, preparing students for more advanced topics in power engineering.

Finally, the course culminates with an introduction to electrical machines, explaining their structure, operating principles, and energy conversion processes in both motor and generator modes. Efficiency analysis and power balance are emphasized to help students understand performance optimization and practical implementation.

CHAPTER 1

Mathematical Reminders About Complex Numbers

Chapter 1: Mathematical Reminders About Complex Numbers

1.1. Objective

The objective of this chapter is to provide students with a foundational understanding of complex numbers and their properties. By revisiting key concepts and operations involving complex numbers, the chapter aims to ensure that students have the necessary skills to use the complex numbers in electricity and alternative courant circuits.

1.2. Prerequisites

To effectively understand and engage with the content of this chapter, students should have the following background knowledge and skills:

- Understanding of algebraic expressions and equations.
- Awareness of the basic definition of a complex number.
- Understanding of the Cartesian coordinate system.
- Ability to plot points on the complex plane.
- Knowledge of sine, cosine, and tangent functions.
- Understanding of the unit circle and radian measure.

1.3. Introduction

Complex numbers extend the concept of one-dimensional real numbers to the two-dimensional complex plane by incorporating an imaginary component. This chapter revisits fundamental aspects of complex numbers, providing a solid mathematical foundation necessary for various advanced topics. Understanding complex numbers is important not only for mathematics but also for practical applications in engineering, physics, and particularly in electrical engineering where they are used to analyze AC circuits and signal processing.

Throughout this chapter, we will explore the properties and operations of complex numbers, including their representation in different forms such as Cartesian, trigonometric, and exponential. We will also delve into key theorems like Moivre's and Euler's, and highlight real-world applications to ensure a comprehensive grasp of this essential mathematical concept.

1.4. Complex number

Complex numbers are important in mathematics and others technology fields as the electrical engineering, where they are used for solving the complex equations and helping to analyze the alternating current (AC) circuits. Their mathematical notation reveals both the modulus and the argument. These numbers are used to solve various mathematical equations and can be represented in various forms, such as algebraic, vector, or exponential.

1.4.1. Algebraic form

Complex numbers are numbers that include both a real part and an imaginary part. They are typically defined by the addition between " x " and " yj " with i is the imaginary unit with the property $j^2 = -1$.

Just as $\mathfrak{R}$ is the set of real numbers, $\mathfrak{C}$ is the set of complex numbers. If " z " is a complex number, the algebraic form of a complex number is expressed as

$$z = x + y\,j \tag{1.1}$$

1.4.2. Real and imaginary parts

In the algebraic form of a complex number: $z = x + y\,j$,

We can find two parts, the first is the real part noted by Re{ z }, with Re{ z } = a, and the second ones is the imaginary noted Im{ z }, with Im{ z } = b.

For $z = 3 - j$, the real part is Re{ z } = 3 and imaginary is Im{ z } = -1.

1.4.3. Cartesian form

Since the complex number " z " is made up of two numbers, we can take these two values as coordinates in the Cartesian plane.

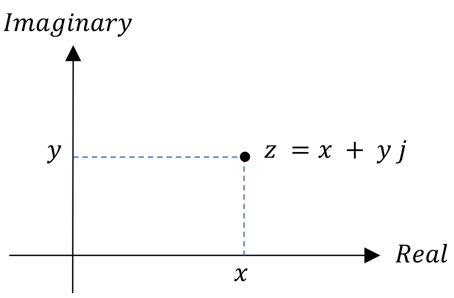


Figure 1.1. Cartesian form of complex number " z "

1.4.4. Conjugate

The conjugate of a complex number is obtained by changing the sign of its imaginary part. If a complex number is given in the form $z = x + y\,j$, its conjugate is noted by $\bar{z} = x - y\,j$.

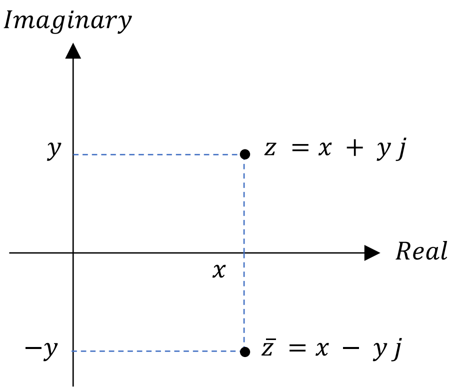


Figure 1.2. Representation of complex number " z " and its conjugate " $\bar{z}$ "

1.4.5. Modulus

The modulus (or absolute value) of a complex number " $x + y j$ " is a measure of its distance from the origin in the complex plane. It is noted by various representation: r, $mod(z)$, $|z|$ and $|x + y j|$.

By using the Pythagoras theorem $|z|^2 = x^2 + y^2$, the modulus of complex number z is calculated by the next formula:

$$|z| = \sqrt{x^2 + y^2} \tag{1.2}$$

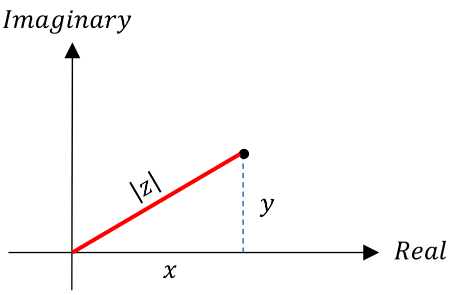


Figure 1.3. Modulus of complex number " z "

1.4.6. Argument

The argument of a complex number z is the angle θ that the line representing the complex number makes with the positive real axis in the complex plane. It can be calculated using the arctangent function, taking into account the signs of x and y to determine the correct quadrant. The argument θ is given by:

$$\theta = tan^{-1}\left(\frac{y}{x}\right) \tag{1.3}$$

However, since the arctangent function only gives values in the range $-\frac{\pi}{2}$ et $\frac{\pi}{2}$, we need to adjust the angle based on the quadrant in which the complex number lies:

$$\text{if } x = 0 \text{ and } y < 0, \theta = -\frac{\pi}{2} \tag{1.4}$$

$$\text{if } x = 0 \text{ and } y > 0, \theta = \frac{\pi}{2} \tag{1.5}$$

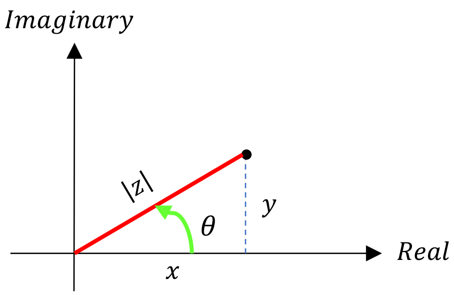


Figure 1.4. Argument θ of complex number " z "

1.5. Trigonometric form

The trigonometric form of a complex number " z " is a way of expressing the number using its modulus and argument, emphasizing the trigonometric functions cosine and sine.

The trigonometric form of the complex number " $z = x + y\,j$ " is:

$$z = r\big(cos(\theta) + j\ sin(\theta)\big) \tag{1.6}$$

We recall here that, $r = \sqrt{x^2 + y^2}$ and $\theta = tan^{-1}\left(\frac{y}{x}\right)$.

So, for the complex number:

$$z = \sqrt{x^2 + y^2}\left(cos\left(tan^{-1}\left(\frac{y}{x}\right)\right) + j\ sin\left(tan^{-1}\left(\frac{y}{x}\right)\right)\right) \tag{1.7}$$

1.6. Exponential form

The exponential form of a complex number $z = x + y\,j$ relates complex exponentials to trigonometric functions.

$$e^{i\theta} = cos(\theta) + j\ sin(\theta) \tag{1.8}$$

By using the recently characteristic of Equation (8), the complex number z can be expressed in exponential form as:

$$z = r\ e^{j\theta} \tag{1.9}$$

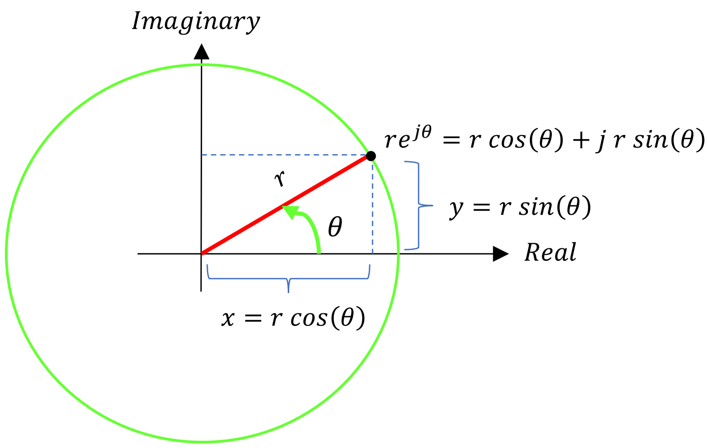


Figure 1.5. Exponential and trigonometric forms of complex number " z "

This Exponential form presents the relationship between the modulus and argument (angle) of the complex number z in the complex plane. As we can write z by:

$$z = \sqrt{x^2 + y^2}\, e^{j\left(tan^{-1}\left(\frac{y}{x}\right)\right)} \tag{1.10}$$

1.7. Operation on complex number

Operations on complex numbers include addition, subtraction, multiplication and division. To add or subtract complex numbers, we can simply combine their real parts and their imaginary parts separately. Multiplication involves using the distributive property and simplifying with $j^2 = -1$, while division requires multiplying the numerator and denominator by the conjugate of the denominator and then simplifying.

1.7.1. Addition

Adding two complex numbers involves combining their real parts and their imaginary parts separately. This operation is straightforward and follows basic algebraic principles.

Suppose we have two complex numbers:

$$\begin{cases} z_1 = x_1 + y_1\, j \\ z_2 = x_2 + y_2\, j \end{cases} \tag{1.11}$$

The addition between these two complex numbers is given by:

$$z_1 + z_2 = (x_1 + x_2) + (y_1 + y_2)\, j \tag{1.12}$$

1.7.2. Subtraction

Subtracting complex numbers involves combining their real parts and their imaginary parts separately, just like addition, but with subtraction. We use the two complex numbers z_1 and z_2 presented previously in subsection 1.7.1.

The subtraction between these two complex numbers is given by:

$$z_1 - z_2 = (x_1 - x_2) + (y_1 - y_2)\, j \tag{1.13}$$

1.7.3. Multiplication

For multiplying two complex numbers, we involve expanding the product using the distributive property (similar to polynomial multiplication) and then simplifying the expression using the fact that $j^2 = -1$. To multiply the two complex numbers z_1 and z_2,

$$z_1 \times z_2 = (x_1 + y_1\, j) \times (x_2 + y_2\, j) \tag{1.14}$$

$$z_1 \times z_2 = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + y_1 x_2)\, j \tag{1.15}$$

In exponential form,

$$\begin{cases} z_1 = r_1\, e^{j\,\theta_1} \\ z_2 = r_2\, e^{j\,\theta_2} \end{cases} \tag{1.16}$$

$$z_1 \times z_2 = (r_1 \times r_2)\, e^{j\,(\theta_1+\theta_2)} \tag{1.17}$$

1.7.4. Division

For dividing complex numbers involves multiplying the numerator and the denominator by the conjugate of the denominator to eliminate the imaginary part from the denominator. To calculate z_1/z_2, we can use the next step:

$$\frac{z_1}{z_2} = \frac{x_1 + y_1\, j}{x_2 + y_2\, j} \tag{1.18}$$

$$\frac{z_1}{z_2} = \frac{x_1 + y_1\, j}{x_2 + y_2\, j} \times \frac{x_2 - y_2\, j}{x_2 - y_2\, j} \tag{1.19}$$

After more mathematical developments, we obtain:

$$\frac{z_1}{z_2} = \left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}\right) + \left(\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}\right)\; j \tag{1.20}$$

In exponential form,

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right)\, e^{j\,(\theta_1-\theta_2)} \tag{1.21}$$

1.7.5. Inverse

To find the inverse (or reciprocal) of a complex number $z = x + y\,j$, we can follow a similar process as described previously in division.

$$z^{-1} = \frac{1}{z} \tag{1.22}$$

After more mathematical developments, we obtain:

$$\frac{1}{z} = \left(\frac{x}{x^2+y^2}\right) - \left(\frac{y}{x^2+y^2}\right) j \tag{1.23}$$

In exponential form,

$$\frac{1}{z} = \left(\frac{1}{r}\right)\, e^{-j\,\theta} \tag{1.24}$$

1.8. De Moivre's theorem

De Moivre's theorem is an important theorem in complex number theory that links complex numbers and trigonometry. It provides a formula for raising complex numbers to integer powers. The theorem is particularly useful when dealing with complex numbers in polar form. For any complex number in trigonometric form, $z = r\big(cos(\theta) + j\; sin(\theta)\big)$ and any integer n, De Moivre's theorem states:

$$(z)^n = \Big(r\big(cos(\theta) + j\; sin(\theta)\big)\Big)^n \tag{1.26}$$

$$(z)^n = (r)^n\big(cos(n\theta) + j\ sin(n\theta)\big) \tag{1.27}$$

We recall here that r is the modulus of the complex number, and θ is the argument.

1.9. Euler's theorem

Euler's Theorem, often referred to as Euler's formula, is a fundamental equation in complex analysis that establishes a deep relationship between trigonometric functions and the exponential function. The theorem states that for any real number θ:

$$e^{i\theta} = cos(\theta) + j\ sin(\theta) \tag{1.28}$$

where,

$$\begin{cases} cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2} \\ sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2\ j} \end{cases} \tag{1.29}$$

1.10. Applications of complex numbers to electricity

Complex numbers are important part in electricity, particularly in the analysis and design of alternating current (AC) circuits. A circuit is said to be sinusoidal if each excitation source is sinusoidal. By representing sinusoidal waveforms as complex numbers, engineers can simplify calculations and gain deeper insights into circuit behavior.

The complex numbers approach is essential for tasks such as calculating impedance and analyzing power. The use of complex numbers streamlines the study of circuits with resistors, capacitors, and inductors, making it easier to design efficient and effective electrical systems.

Consider a sinusoidal alternating voltage whose expression as a function of time is given as follows:

$$u(t) = U_{max}\cos(\omega\ t\ +\ \varphi_u) \tag{1.30}$$

U_{max} : the maximal value [in V] of $u(t)$
ω : the pulsation [in rad/s], $\omega = 2\pi f$
f : the frequency [in Hertz], $f = \frac{1}{T}$
T : the period of $u(t)$
φ_u : the phase of the voltage signal at the origin.

As a simple representation, the expression for this sinusoidal voltage can be written in a complex form as follows:

$$\overline{\mathrm{U}} = [U_{max}, \varphi_u] = U_{max}\left(cos(\varphi_u) + j\ sin(\varphi_u)\right) \tag{1.31}$$

For a sinusoidal alternating current, the same principle applies:

$$\overline{\mathrm{I}} = [I_{max}, \varphi_i] = I_{max}\big(cos(\varphi_i) + j\ sin(\varphi_i)\big) \tag{1.32}$$

φ_i : the phase of the current signal at the origin.

CHAPTER 2

Fundamental laws of electricity

Chapter 2: Fundamental laws of electricity

2.1. Objective

The objective of this chapter is to provide a comprehensive review of the fundamental laws of electricity. This review aims to collect the foundational knowledge necessary for understanding more complex electrical concepts.

After this chapter, students will reinforce their understanding of electric dipoles, resistive, capacitive, and inductive components in both direct and alternating current regimes, as well as the behavior of circuits in transitional states.

2.2. Prerequisites

Before diving into this chapter, students should have a basic understanding of the following concepts:

- Basic principles of electricity and magnetism,
- Ohm's Law and Kirchhoff's laws,
- Fundamental circuit components: resistors, capacitors, and inductors,
- Basic calculus of complex numbers,
- Understanding of sinusoidal functions and their properties,
- Familiarity with the concepts of voltage, current, and power in electrical circuits.

2.3. Introduction

In this chapter, we will revisit the fundamental laws of electricity that form the backbone of electrical engineering. Understanding these basic principles is important for analyzing and designing electrical circuits. We will explore the behavior of electric dipoles and the characteristics of resistive, capacitive, and inductive components in direct current (DC) and alternating current (AC) regimes.

Additionally, we will examine the transitional behavior of circuits containing resistors, inductors, and capacitors. This chapter provides a thorough review of the fundamental laws of electricity, it is essential and presents the first concepts for understanding in the electrotechnical field.

2.4. Direct regime

A continuous regime refers to a steady-state condition where electrical quantities such as voltage, current, and power remain constant over time. In this regime, the circuit parameters do not change, and the system operates under stable conditions without any transient fluctuations.

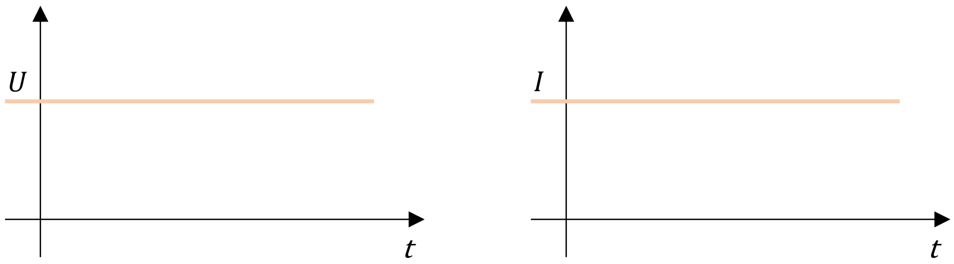


Figure 2.1. Representation of continuous regime of voltage U and current I

2.4.1. Electric dipoles

The electric dipole is defined as a component or set of components that has two terminals (A and B). In the context of electrical circuits, it typically refers to any element that exhibits a potential difference between A and B, such as a resistor, capacitor, or inductor. This concept is fundamental in analyzing the behavior of electrical components and understanding how they interact within a circuit.

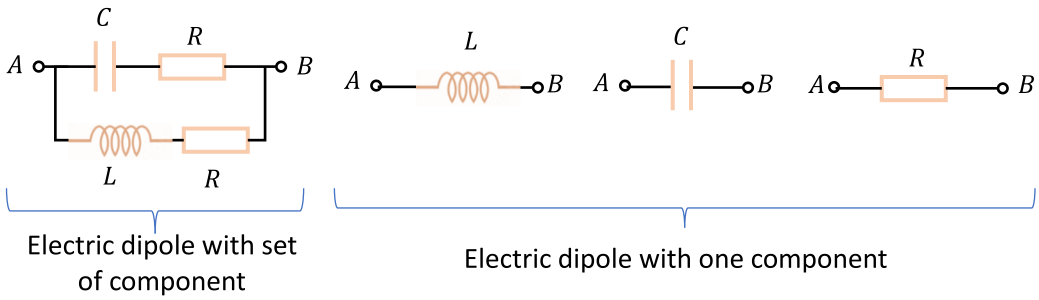


Figure 2.2. Representation of electric dipole

In electrical circuits, we can find the two types of convention, receiver and generator, that are used to describe the direction of current flow and the associated voltage drops or rises in different components of a circuit.

i) *Receiver Convention:* it is used for components that consume electrical energy, such as resistors, capacitors, and inductors in a load configuration. In this case, current and voltage are in opposite directions

ii) *Generator Convention:* it is used for components that supply electrical energy, such as batteries, power supplies, and generators. For a generator, current and voltage are in the same direction

Figure 2.3. Types of electric dipole, Generator et Receiver

2.4.2. Resistive dipole

A resistive dipole is a basic electrical component characterized obeying Ohm's Law. It is a passive element that dissipates electrical energy in the form of heat. The relationship between the voltage across the terminals and the current flowing through the dipole is linear, meaning the voltage drop (V) across the resistor is directly proportional to the current (I) passing through it, with the proportionality constant being the resistance (R) with its unit is Ohm [Ω].

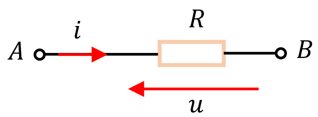


Figure 2.4. Resistive dipole

Empirically, we can see that the resistance depends on three parameters, namely:

$$R = \rho \frac{L}{S} \quad (2.1)$$

R is the resistance of the conductor [Ω],

ρ is the resistivity of the material [Ω m],

L is the length of the conductor [m],

S is the cross-sectional area of the conductor [m^2].

We also define the inverse of the resistance which is the conductivity σ [Ω^{-1}]:

$$\sigma = \frac{1}{\rho} \quad (2.2)$$

The Ohm's Law describes the relationship between voltage, current, and resistance in an electrical circuit. It states that the current passing through a conductor between two points is directly proportional to the voltage across the two points and inversely proportional to the resistance between them.

$$u = R\, i \quad (2.3)$$

In resistive circuits, we can find two types of association in series or in parallel:

i) When resistors are connected in series, they are connected end-to-end, such that the same current flows through each resistor. The equivalent resistance of the series combination is the sum of the individual resistances.

$$R_{eq} = \sum R_i = R_1 + R_2 \quad (2.4)$$

ii) In parallel, the resistors are connected such that each resistor is directly connected to the same two points, meaning the voltage across each resistor is the same. In this case, the equivalent resistance is found using the reciprocal of the sum of the reciprocals of the individual resistances.

$$\frac{1}{R_{eq}} = \sum \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} \quad (2.5)$$

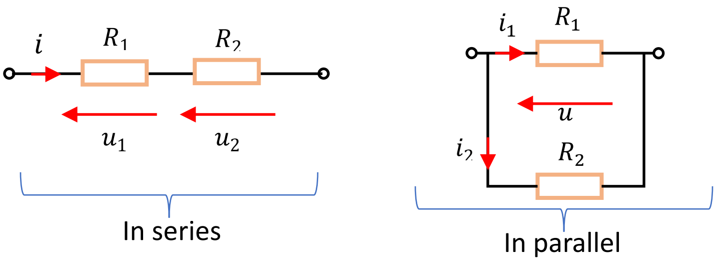


Figure 2.5. Resistors association

2.4.3. Capacitive dipole

A capacitive dipole stores electrical energy in an electric field. This electrical component consists of two conductive plates separated by an insulating material called a dielectric. The component is characterized by its capacitance C in Farad (F).

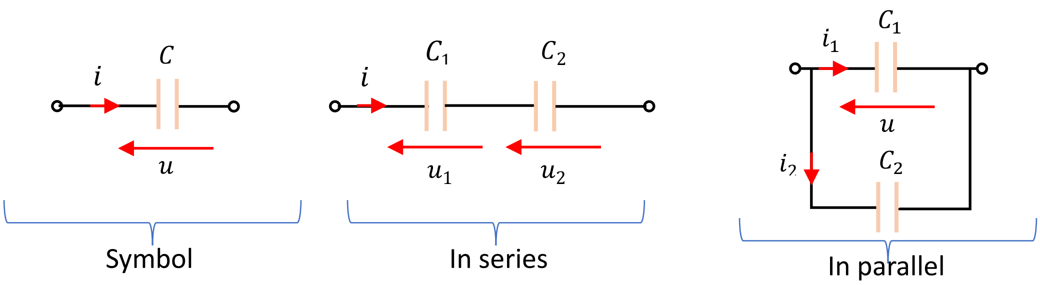


Figure 2.6. Association of capacitive dipoles

The equivalent capacitors in parallel is the sum of the two individual capacitors C_1 and C_2.

$$C_{eq} = \sum C_i = C_1 + C_2 \quad (2.6)$$

When capacitors are connected in series, the equivalent capacitor is found using the reciprocal of the sum of the reciprocals of the individual capacitors.

The equivalent capacitor C_{eq} is less than the smallest individual capacitance.

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} \quad (2.7)$$

2.4.4. Inductive dipole

An inductive dipole is a passive electrical component that stores energy in a magnetic field when electrical current flows through it. It is characterized by its inductance L. The unit of inductance is the Henry (H).

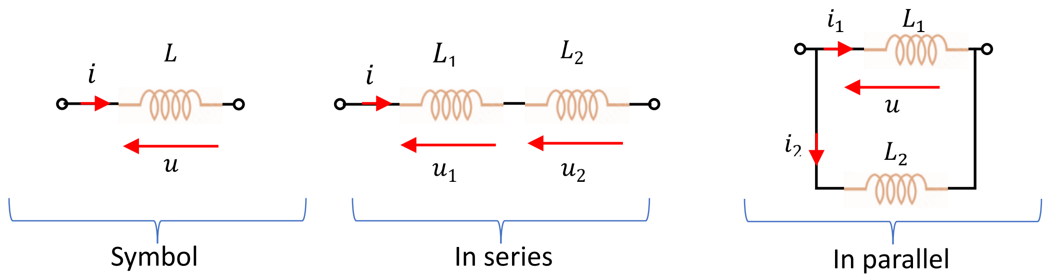


Figure 2.7. Association of inductive dipoles

When inductors are connected in series, the equivalent inductance is the sum of the individual inductances. In the other case, when the inductors are connected in parallel, the equivalent inductance is calculated as the equivalent resistor.

$$\begin{cases} L_{eq} = \sum L_i = L_1 + L_2 & \text{Association in series} \\ \frac{1}{L_{eq}} = \sum \frac{1}{L_i} = \frac{1}{L_1} + \frac{1}{L_2} & \text{Association in parallel} \end{cases} \quad (2.8)$$

2.5. Alternative regime

The alternative regime refers to the behavior of electrical circuits when alternating current (AC) flows through them. Unlike direct current (DC), which flows in one direction, AC periodically reverses direction.

The sinusoidal regime, widely used for the production, transport, and distribution of electrical energy, involves studying the operation of linear circuits powered by sinusoidal generators. This includes analyzing electrical networks composed of linear elements—resistors, capacitors, and inductors—under sinusoidal voltage or current sources.

2.5.1. Periodic sinusoidal signal

A periodic signal is one that repeats itself at regular intervals over time. In the context of AC, the most common periodic signal is the sinusoidal waveform, which can be presented in Figure 2.8 and described by the equation (2.9).

$$u(t) = U_{max} \sin(\omega\, t\; +\; \varphi_u) \tag{2.9}$$

U_{max} : the maximal value [in V] of $u(t)$

ω : the pulsation [in rad/s], $\omega = 2\pi f$

f : the frequency [in Hertz], $f = \frac{1}{T}$

T : the period of $u(t)$

φ_u : the phase of the voltage signal at the origin

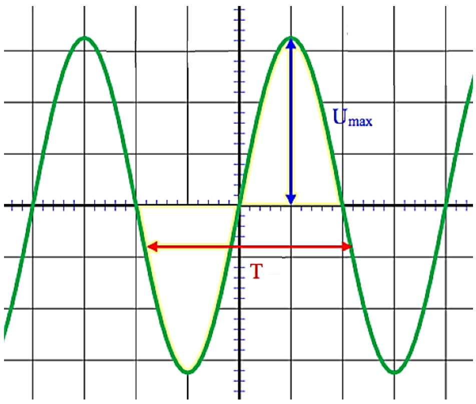


Figure 2.8. Example of sinusoidal alternating voltage

As we can using the next representation by the exponential form,

$$\bar{U} = U_{max}\, e^{j(\omega\, t\, +\, \varphi_u)} \tag{2.10}$$

$$\bar{I} = I_{max}\, e^{j(\omega\, t\, +\, \varphi_i)} \tag{2.11}$$

All quantities in an electrical circuit vary with the same frequency, so we can use the next simple form,

$$\bar{U} = U_{max}\, e^{j\varphi_u} \tag{2.12}$$

$$\bar{I} = I_{max}\, e^{j\varphi_i} \tag{2.13}$$

2.5.2. Average values of sinusoidal current

The average value of a sinusoidal current (or voltage) over a complete cycle is zero, since the positive and negative halves of the cycle cancel each other out. However, the average of the absolute value, or the rectified signal, is useful in power calculations.

We use the next sinusoidal current: $i(t) = I_{max} \sin(\omega\, t)$

We note the average value of a sinusoidal current by I_{moy} and defined as:

$$I_{moy} = \frac{1}{T}\int_{t=0}^{T} i(t)\, dt = \frac{1}{T}\int_{t=0}^{T} I_{max} \sin(\omega\, t)\ dt \tag{2.14}$$

$$I_{moy} = \frac{1}{T}\left[\frac{-\cos(\omega\, t)}{\omega}\right]_0^T = -\frac{1}{T\omega}\left[\cos(\omega\, t)\right]_0^T \tag{2.15}$$

$$I_{moy} = -\frac{1}{2\pi}\left[\cos(\omega\, T) - \cos(0)\right] = -\frac{1}{2\pi}[1-1] \tag{2.16}$$

So $I_{moy} = 0$

2.5.3. Root-Mean-Square values of sinusoidal current

The RMS value of a sinusoidal current is denoted I_{eff} and it is defined as a measure of the effective value given by:

$$I_{eff}^2 = \frac{1}{T}\int_{t=0}^{T}[i(t)]^2\, dt = \frac{2}{T}\int_{t=0}^{T/2} I_{max}^2\ \sin^2(\omega\, t)\ dt = \frac{2\, I_{max}^2}{T}\int_{t=0}^{T/2T} \sin^2(\omega\, t)\ dt \tag{2.17}$$

$$I_{eff}^2 = \frac{2\, I_{max}^2}{T}\int_{t=0}^{T/2}\left[\frac{1-\cos(2\,\omega\, t)}{2}\right] dt = \frac{2\, I_{max}^2}{2\, T}\left[t - \frac{\sin(2\,\omega\, t)}{2\,\omega}\right]_0^{T/2} \tag{2.18}$$

$$I_{eff}^2 = \frac{I_{max}^2}{2} \rightarrow I_{eff} = \frac{I_{max}}{\sqrt{2}} \tag{2.19}$$

Using the same steps, the RMS value of a sinusoidal voltage is defined by,

$$U_{eff} = \frac{U_{max}}{\sqrt{2}} \tag{2.20}$$

2.5.4. Complex representation of derivate and integrated operations

Considering the sinusoidal alternating voltage $i(t) = I_{max}\, sin(\omega\, t\ +\ \varphi_i)$, complex number is $\bar{I} = I_{max}\, e^{j\varphi_i}$.

a) Complex representation of $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = \omega\, I_{max}\, cos(\omega\, t\ +\ \varphi_i) = \omega\, I_{max}\, sin\left(\omega\, t\ +\ \varphi_i + \frac{\pi}{2}\right) \tag{2.21}$$

So, the complex number of this later is,

$$\frac{di(t)}{dt} = \omega\, I_{max}\, e^{j\left(\varphi_i + \frac{\pi}{2}\right)} = \omega\, I_{max}\, e^{j\varphi_i}\, e^{j\frac{\pi}{2}} \tag{2.22}$$

with, $e^{j\frac{\pi}{2}} = j$

Finally, we note that:

$$\frac{di(t)}{dt} = j\,\omega\, I_{max}\, e^{j\varphi_i} = j\,\omega\,\bar{I} \tag{2.23}$$

We conclude that, $j\,\omega$ is the derivate operation.

b) Complex representation of $\int i(t)\,dt$

$$\int i(t)\,dt = \int I_{max}\, sin(\omega\, t \;+\; \varphi_i)\;dt = -\frac{I_{max}}{\omega}\, cos(\omega\, t \;+\; \varphi_i)$$

$$= \frac{I_{max}}{\omega}\, sin\left(\omega\, t \;+\; \varphi_i - \frac{\pi}{2}\right) \quad (2.24)$$

So, the complex number of this later is,

$$\int i(t)\,dt = \frac{I_{max}}{\omega}\, e^{j\left(\varphi_i - \frac{\pi}{2}\right)} = \frac{I_{max}}{\omega}\, e^{j\varphi_i}\, e^{-j\frac{\pi}{2}} \quad (2.25)$$

with, $e^{-j\frac{\pi}{2}} = j$

Finally, we note that:

$$\int i(t)\,dt = -j\frac{I_{max}}{\omega}\, e^{j\varphi_i} \quad (2.26)$$

$$\int i(t)\,dt = \frac{1}{j\omega}\bar{I} \quad (2.27)$$

We conclude that, $\frac{1}{j\omega}$ is the integrate operation.

2.5.5. Fresnel representation

Fresnel representation involves using phasors to represent AC quantities in a plane, making it easier to add, subtract, and analyze AC voltages and currents.

$$\begin{cases} \bar{\text{U}} = \;|U|\; e^{j\varphi_u} \\ \bar{\text{I}} = \;|I|\; e^{j\varphi_i} \end{cases} \quad (2.28)$$

Figure 2.9. Fresnel representation of alternating current and voltage

2.5.6. Impedance and admittance

The complex impedance is a concept used in electrical engineering to describe the opposition that a circuit presents to the flow of alternating current. It generalizes the idea of resistance to include both resistive and reactive (inductive and capacitive) components. Complex impedance is usually denoted by Z and is expressed as a complex number:

$$\bar{Z} = \frac{\bar{\text{U}}}{\bar{\text{I}}} = \frac{U_{max}\, e^{j\varphi_u}}{I_{max}\, e^{j\varphi_i}} = \frac{U_{max}}{I_{max}}\; e^{j(\varphi_u - \varphi_i)} \quad (2.29)$$

This complex form of Ohm's law allows for the analysis of circuits with sinusoidal sources using phasors, where voltages and currents are represented as complex numbers.

The complex impedance $\bar{Z}$ is presented respectively, by a modulus and a phase angle:

$$|\bar{Z}| = \frac{|\bar{U}|}{|\bar{I}|} \quad (2.30)$$

$$arg(\bar{Z}) = \varphi_u - \varphi_i \quad (2.31)$$

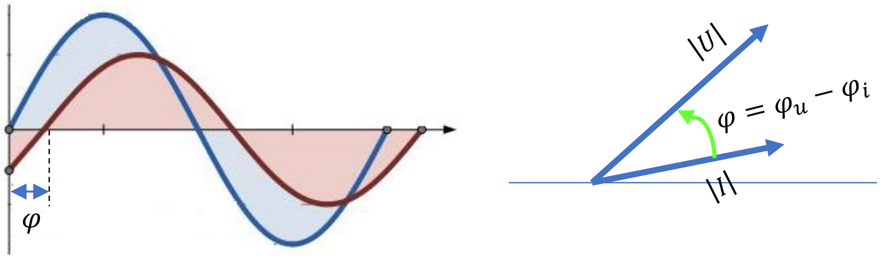


Figure 2.10. The phase angle φ of the impedance in function of φ_u and φ_i

Important remarque

We can observe three situations of the circuits:

The current in advance to Voltage: when the current reaches its maximum value before the voltage does. This is typically observed in capacitive circuits. Mathematically, if the phase angle of the current φ_i is greater than the phase angle of the voltage φ_u, the current is said to be leading.

$$\varphi_u - \varphi_i < 0$$

Current Lagging Voltage: when the current reaches its maximum value after the voltage does. This is typically observed in inductive circuits. If the phase angle of the current φ_i is less than the phase angle of the voltage φ_u, the current is said to be lagging.

$$\varphi_u - \varphi_i < 0$$

In a purely resistive circuit, the voltage and current are in phase. This means that the phase angle difference between the voltage and the current is zero.

$$\varphi_u - \varphi_i = 0$$

The complex admittance Y is the reciprocal of the complex impedance Z. It also has both a magnitude and a phase angle, providing a measure of how easily a circuit allows current to flow.

The modulus of the admittance is the reciprocal of the magnitude of the impedance:

$$|\bar{Y}| = \frac{1}{|\bar{Z}|} = \frac{|\bar{I}|}{|\bar{U}|} \quad (2.32)$$

The phase angle of the admittance is the negative of the impedance phase angle:

$$arg(\bar{Y}) = -\ arg(\bar{Z}) = \varphi_i - \varphi_u \tag{2.33}$$

Finally, the admittance Y is defined by:

$$\mathrm{Y} = \frac{|\bar{\mathrm{I}}|}{|\bar{\mathrm{U}}|}\ e^{-j(\varphi_u - \varphi_i)} \tag{2.34}$$

2.5.7. Resistance and reactance

Complex impedance Z can be expressed as following form:

$$Z = R\ +\ X\,j \tag{2.35}$$

- R is the resistance, which is the real part of the impedance.
- X is the reactance, which is the imaginary part of the impedance.

We note that the resistance and reactance can be calculated by,

$$\begin{cases} R = |\bar{Z}|\,cos(\varphi_u - \varphi_i) \\ X = |\bar{Z}|\,sin(\varphi_u - \varphi_i) \end{cases} \tag{2.36}$$

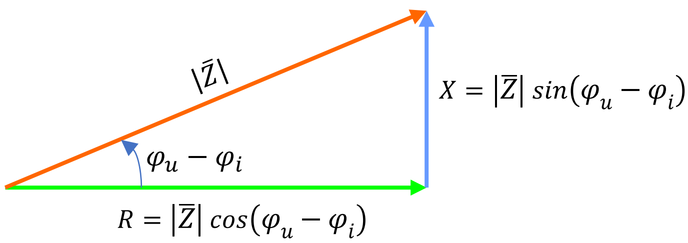


Figure 2.11. Complex impedance representation

When the modulus $|Z|$ and the argument φ of Z are given by:

$$\begin{cases} |Z| = \sqrt{R^2 + X^2} \\ \varphi = \varphi_u - \varphi_i = tan^{-1}\left(\frac{X}{R}\right) \end{cases} \tag{2.37}$$

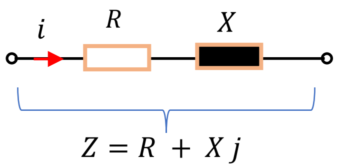


Figure 2.12. Impedance Z of an electric circuit with R is real part and X is imaginary part

In this circuit, we can find generally three types:

Resistive: A resistive circuit consists solely of resistors. The imaginary part of the circuit equal to 0, so $Z = R$.

Inductive: it contains inductors and may also include resistors. In this type, the current lags behind the voltage by 90 degrees in a purely inductive circuit, $Z = R + X\,j$ with the imaginary part is positive ($X > 0$).

Capacitive: contains capacitors and may also include resistors. In this type of circuit, the current leads the voltage by 90 degrees in a purely capacitive circuit, $Z = R + X\,j$ with the imaginary part is negative ($X < 0$).

2.5.8. Resistance impedance

In this case, the impedance is purely real and equal to the resistance, $Z_R = R$

The modulus of Z_R is $|Z_R| = R$. The voltage and current are in phase, the argument is $\varphi = 0$.

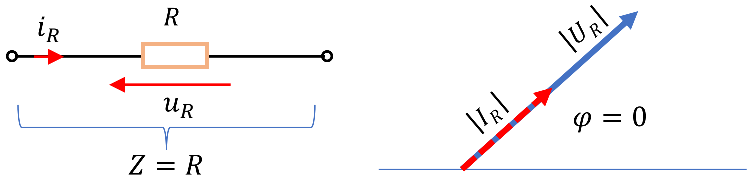


Figure 2.12. Impedance Z_R and Fresnel representation

2.5.9. Inductive impedance

Inductive impedance is a measure of the opposition that an inductor presents to the alternating current. it is the impedance of an inductor when subjected to an AC voltage or current. It is given as, $Z_L = j\,L\omega$

The modulus of Z_L is $|Z_L| = L\omega$

The phase angle of the inductive impedance is $+90^{\circ}$ or $+\frac{\pi}{2}$ radians. This indicates that the current is late to the voltage by 90° in a purely inductive circuit.

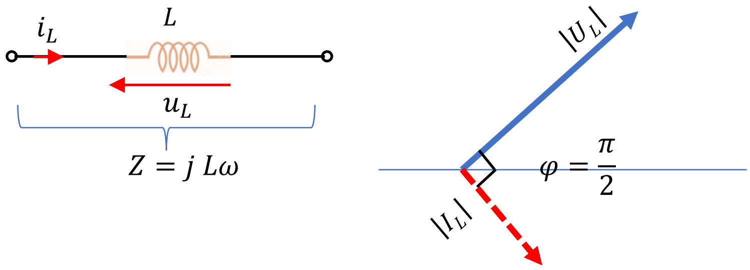


Figure 2.13. Impedance Z_L and Fresnel representation

2.5.10. Capacitive impedance

Capacitive impedance Z_C is the impedance of a capacitor. It is given by: $Z_C = \frac{1}{j\,C\omega} = -j\frac{1}{C\omega}$

The modulus of Z_C is $|Z_C| = \frac{1}{C\omega}$

The phase angle of the capacitive impedance is -90° or $-\frac{\pi}{2}$ radians. This indicates that the current in advance to the voltage by 90° in a purely capacitive circuit.

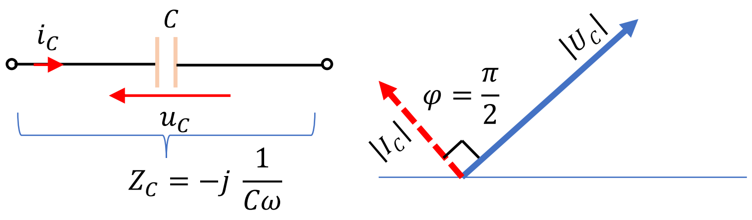


Figure 2.14. Impedance Z_C and Fresnel representation

2.5.11. RLC circuit

An RLC circuit is a general type of AC circuit that includes a resistor (R), an inductor (L), and a capacitor (C). These components can be connected in series or parallel. The RLC circuit is fundamental in electrical engineering as it exhibits resonant behavior and can be used in filters, oscillators, and various signal processing applications.

In a series RLC circuit, the resistor, inductor, and capacitor are connected end-to-end in a single path for the current.

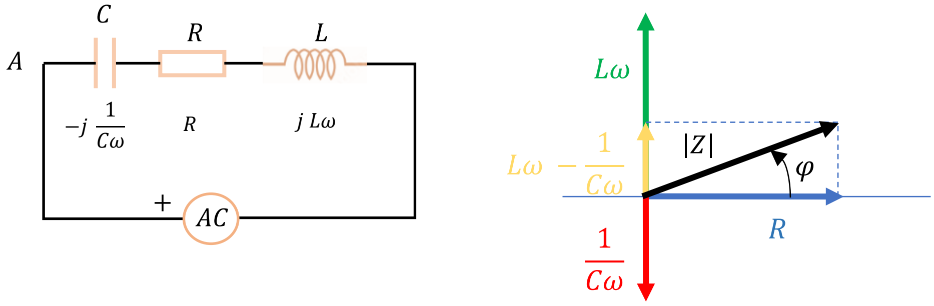


Figure 2.15. Series RLC circuit and impedance diagram

The impedance Z of series RLC circuit is:

$$Z = R + j\left(L\omega - \frac{1}{C\omega}\right) \tag{2.38}$$

The modulus and argument of Z are calculated as in complex number,

$$\begin{cases} |Z| = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} \\ \varphi = tan^{-1}\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right) \end{cases} \tag{2.39}$$

Resonance case: Resonance occurs when the inductive reactance $L\omega$ equals the capacitive reactance $\frac{1}{C\omega}$, resulting in a purely resistive impedance:

$$L\omega_0 = \frac{1}{C\omega_0} \tag{2.40}$$

The resonant pulsation ω_0 is:

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (2.41)$$

The corresponding resonant frequency in hertz is:

$$f_0 = \frac{1}{2\,\pi\,\sqrt{LC}} \quad (2.42)$$

2.5.12. Powers

In AC circuits, power can be categorized into three types: active, reactive, and apparent. Understanding these different types is essential for analyzing the electrical systems.

i) Active Power (P): known as real power or true power, is the power that actually performs work in the circuit. It is measured in watts (W).

$$P = U_{eff}\, I_{eff} \cos(\varphi) \quad (2.43)$$

ii) Reactive Power (Q): the power that oscillates between the source and the reactive components (inductors and capacitors) in the circuit. It does not perform any actual work but is necessary for maintaining the electric and magnetic fields in these components. It is measured in volt-amperes reactive (VAR).

$$Q = U_{eff}\, I_{eff} \sin(\varphi) \quad (2.44)$$

iii) Apparent Power (S)**:** the combination of active power and reactive power. It represents the total power supplied to the circuit. It is measured in volt-amperes (VA).

$$S = U_{eff}\, I_{eff} \quad (2.45)$$

Apparent power can also be expressed as a complex number:

$$S = P + j\,Q \quad (2.46)$$

The magnitude of apparent power is:

$$S = \sqrt{P^2 + Q^2} \quad (2.47)$$

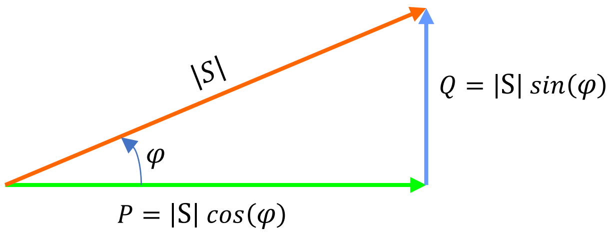


Figure 2.16. Complex power representation

2.5.13. Power Factor

The power factor (PF) is a measure of how effectively the electrical power is being converted into useful work output. It is defined as the ratio of active power to apparent power.

$$\cos(\varphi) = \frac{P}{S} \tag{2.48}$$

A power factor of 1 means all the power is being effectively converted into useful work, which occurs in purely resistive circuits. A lower power factor indicates a larger proportion of the power is reactive, common in inductive or capacitive circuits.

2.5.14. Boucherot's theorem

For an AC circuit with N branches, each branch having its own complex power $(S_1, S_2, \dots, S_N)$, the total complex power S_T is:

$$S_T = S_1 + S_2 + \dots + S_N$$

$$S_T = (P_1 + j\,Q_1) + (P_2 + j\,Q_2) + \dots + (P_N + j\,Q_N)$$

$$S_T = (P_1 + P_2 + \cdots + P_N) + j(Q_1 + Q_2 + \cdots + Q_N)$$

We can observe the total active power P_T equal to,

$$P_T = (P_1 + P_2 + \cdots + P_N) \tag{2.49}$$

And the reactive one is,

$$Q_T = (Q_1 + Q_2 + \cdots + Q_N) \tag{2.50}$$

The magnitude of total apparent power is:

$$S_T = \sqrt{P_T^2 + Q_T^2} \tag{2.51}$$

Finally, the total power factor is,

$$\cos(\varphi) = \frac{P_T}{S_T} \tag{2.52}$$

2.6. Transitional regime

The transitional regime denotes to the period during which a circuit is transitioning from one steady-state to another. This typically occurs when a switch is turned on or off, causing rapid changes in voltage or current. The behavior of the circuit during this period is determined by the transient response of its components.

2.6.1. RL circuit

An RL circuit consists of a resistor (R) and an inductor (L) connected in series. The transient response of an RL circuit is characterized by the time it takes for the current to change after a rapid change in voltage.

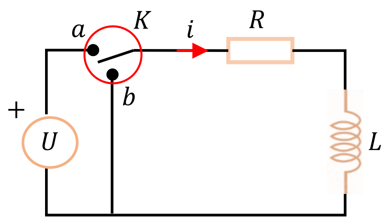


Figure 2.17. Series RL circuit

Switch K in position (a):

The equation of the mesh is written:

$$U = U_R + U_L \quad (2.53)$$

$$U = i\,R + L\,\frac{d\,i}{dt} \quad (2.54)$$

$$\frac{U}{R} = i + \frac{L}{R}\,\frac{d\,i}{dt} \quad (2.55)$$

The equation (2.55) is a first order linear differential equation with constant coefficients and the second member. The solution to this equation is the sum of a general solution without a second member and a particular solution with a second member:

- The solution of the general equation without a second member $\left[i\,R + L\,\frac{d\,i}{dt}\right]$ is $i = Ae^{-\frac{1}{\tau}t}$, with $\tau = \frac{L}{R}$ represents the time constant.
- The solution of the particular equation with a second member when the current is constant ($i(t) = cst\ \rightarrow \frac{d\,i}{dt} = 0$) is the next, $i = \frac{U}{R}$

Finally, the total solution is,

$$i = \frac{U}{R} + Ae^{-\frac{1}{\tau}t} \quad (2.56)$$

Taking into account the initial conditions, at $t = 0\ \rightarrow i(0) = 0 \rightarrow i(0) = \frac{U}{R} + A$

So $A = -\frac{U}{R}$

$$i = \frac{U}{R} - \frac{U}{R}e^{-\frac{1}{\tau}t} = \frac{U}{R}\left(1 - e^{-\frac{1}{\tau}t}\right) \quad (2.57)$$

The voltage across the resistor is given as follows:

$$U_R = R\,i = U\left(1 - e^{-\frac{1}{\tau}t}\right) \quad (2.58)$$

The voltage across the resistor is given as follows:

$$U_L = L\,\frac{d\,i}{dt} = Ue^{-\frac{1}{\tau}t} \quad (2.59)$$

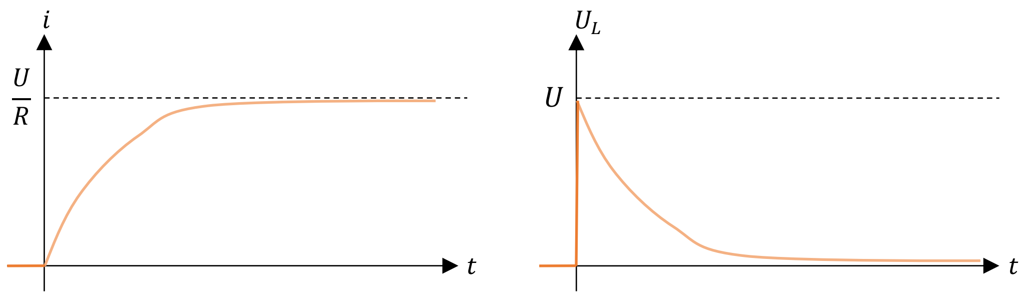


Figure 2.18. Transitional current i and voltage U_L in RL circuit with Switch K in position (a)

Switch K in position (b):

The equation of the mesh is written: $U_R + U_L = 0 \rightarrow i\,R + L\,\frac{d\,i}{dt} = 0$

$$i + \frac{L}{R}\frac{d\,i}{dt} = 0 \quad (2.60)$$

it is a first order linear differential equation with constant coefficients without second member. The solution to this equation is,

$$i = Ae^{-\frac{1}{\tau}t} \quad (2.61)$$

Taking into account the initial conditions, at $t = 0 \ \rightarrow i(0) = \frac{U}{R} \rightarrow i(0) = A$

So, $A = \frac{U}{R}$

Finally, the current is

$$i = \frac{U}{R}e^{-\frac{1}{\tau}t} \quad (2.62)$$

The voltage across the resistor is given as follows:

$$U_R = R\,i = Ue^{-\frac{1}{\tau}t} \quad (2.63)$$

The voltage across the resistor is given as follows:

$$U_L = L\,\frac{d\,i}{dt} = -Ue^{-\frac{1}{\tau}t} \quad (2.64)$$

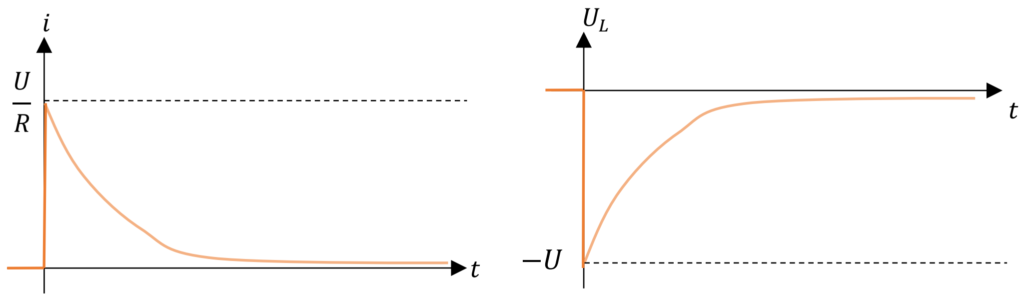


Figure 2.19. Transitional current i and voltage U_L in RL circuit with Switch K in position (b)

2.6.2. RC circuit

The series RC circuit corresponds to the case of a capacitor placed in series with a resistor, the whole being powered by a source of voltage U.

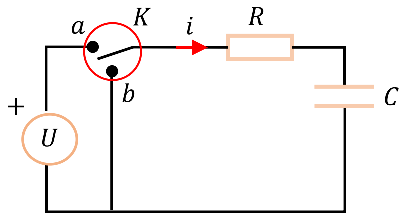


Figure 2.20. Series RC circuit

Switch K in position (a):

The equation of the mesh is written: $U = U_R + U_C$

We have, $i = C\frac{d\,U_C}{dt}$

$$U = i\,R + U_C = R\,C\frac{d\,U_C}{dt} + U_C \tag{2.65}$$

The equation (2.65) is a first order linear differential equation with constant coefficients and the second member. The solution to this equation is the sum of a general solution without a second member and a particular solution with a second member:

- The solution of the general equation without a second member $\left(R\,C\frac{d\,U_C}{dt} + U_C = 0\right)$ is:

  $U_C = A\,e^{-\frac{1}{\tau}t}$, with $\tau = RC$
- The solution of the particular equation with a second member when the current is constant ($U_C = cst \;\rightarrow \frac{d\,U_C}{dt} = 0$) is the next, $U_C = \;U$

Finally, the total solution is,

$$U_C = U + Ae^{-\frac{1}{\tau}t} \tag{2.66}$$

Taking into account the initial conditions, at $t = 0 \;\rightarrow U_C(0) = E + A = 0 \rightarrow A = -U$

$$U_C = U - Ue^{-\frac{1}{\tau}t} == U\left(1 - e^{-\frac{1}{\tau}t}\right) \tag{2.67}$$

The voltage across the resistor is given as follows:

$$U_R = R\,i = Ue^{-\frac{1}{\tau}t} \tag{2.68}$$

The current is given as follows:

$$i = C\,\frac{d\,i}{dt} = \frac{U}{R}e^{-\frac{1}{\tau}t} \tag{2.69}$$

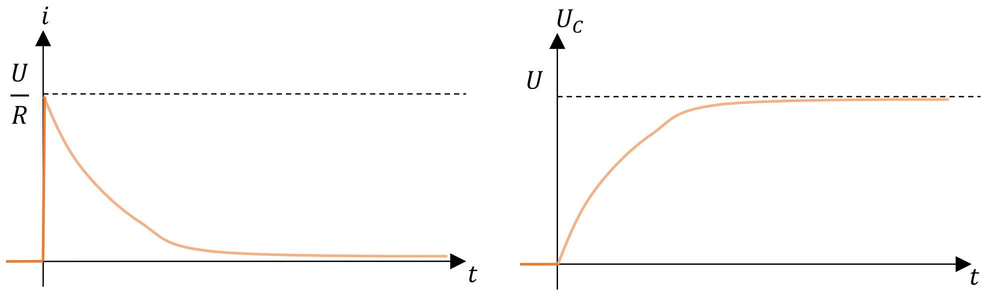


Figure 2.20. Transitional current i and voltage U_C in RC circuit with Switch K in position (a)

Switch K in position (b):

The equation of the mesh is written: $U_R + U_C = 0$

$$R\,C\frac{d\,U_C}{dt} + U_C = 0 \tag{2.70}$$

it is a first order linear differential equation with constant coefficients without second member. The solution to this equation is,

$$U_C = Ae^{-\frac{1}{\tau}t} \tag{2.71}$$

Taking into account the initial conditions, at $t = 0 \ \rightarrow U_C(0) = U \rightarrow i(0) = A$

So, $A = U$

Finally, the current is

$$U_C = Ue^{-\frac{1}{\tau}t} \tag{2.72}$$

The current is given as follows:

$$i = -\frac{U}{R}e^{-\frac{1}{\tau}t} \tag{2.73}$$

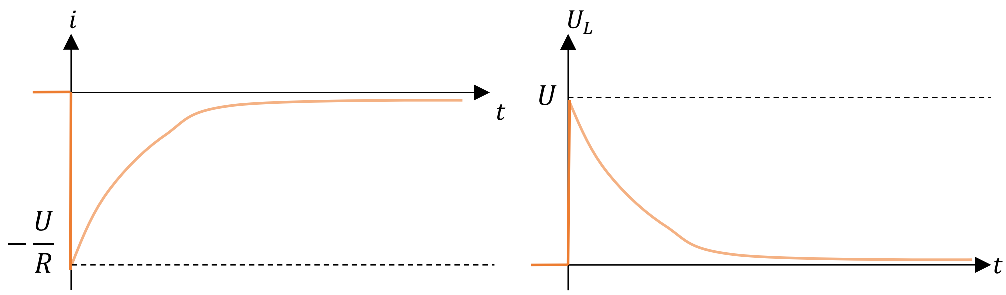


Figure 2.21. Transitional current i and voltage U_C in RC circuit with Switch K in position (b)

2.6.3. RLC circuit

The series RLC circuit corresponds to the case of a resistor, a capacitor and a coil placed in series. Everything is powered by a voltage source E. The figure below illustrates this case.

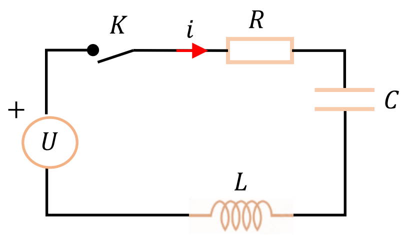


Figure 2.20. Series RLC circuit

We close the switch in the figure above (RLC circuit), the equation of the mesh is written:

$$U = R\,i + L\,\frac{d\,i}{dt} + \frac{1}{C}\int i\,dt \tag{2.74}$$

We note that the current is given by, $i = \frac{dq}{dt}$

$$U = R\,\frac{dq}{dt} + L\,\frac{d^2q}{dt^2} + \frac{q}{C} \tag{2.75}$$

The equation (2.75) is a second order linear differential equation with constant coefficients and the second member. The solution to this equation is the sum of a general solution without a second member and a particular solution with a second member.

Consider the differential equation without a second member:

$$R\,\frac{dq}{dt} + L\,\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \tag{2.76}$$

We pose:

$$\begin{cases} q = e^{rt} \\ \frac{dq}{dt} = re^{rt} \\ \frac{d^2q}{dt^2} = r^2e^{rt} \end{cases} \tag{2.77}$$

Now, we can write the differential equation without a second member as,

$$e^{rt}\left(r^2 + \frac{R}{L}r + \frac{1}{LC}\right) = 0 \tag{2.78}$$

The characteristic equation is a quadratic equation $r^2 + \frac{R}{L}r + \frac{1}{LC} = 0$

We calculate the discriminant,

$$\Delta = \frac{R}{L^2} - \frac{4}{LC} \tag{2.79}$$

We can find three solutions of this equation in function of discriminant Δ,

i) $\Delta > 0$

In this case, the characteristic equation admits two distinct real solutions:

$$\begin{cases} r_1 = \dfrac{\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} \\ r_2 = \dfrac{\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} \end{cases} \quad (2.80)$$

We find the next general solution of the equation without a second member,

$$q = A_1 e^{r_1 t} + A_2 e^{r_2 t} \quad (2.81)$$

ii) $\Delta = 0$

The characteristic equation admits a distinct real double solution:

$$r_1 = r_2 = -\frac{R}{2L} \quad (2.82)$$

We find the next general solution of the equation without a second member,

$$q = e^{rt}(A_1 + A_2) \quad (2.83)$$

iii) $\Delta < 0$

In this case, the characteristic equation admits two complex conjugate roots: $r_1 = \alpha - j\beta$ and $r_1 = \alpha + j\beta$

With

$$\begin{cases} \alpha = -\dfrac{R}{2L} \\ \beta = \dfrac{\sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2}}{2} \end{cases} \quad (2.84)$$

We find the next general solution of the equation without a second member,

$$q = e^{\alpha t}\left(A_1 cos(\beta\ t) + A_2 sin(\beta\ t)\right) \quad (2.85)$$

CHAPTER 3

Circuits and Electrical Power

Chapter 3: Circuits and Electrical Power

3.1. Objective

The objective of this chapter is to provide students with a detailed understanding of both single-phase and three-phase electrical power systems.

Students will learn how to analyze, calculate, and measure different types of powers, including active, reactive, and apparent, as well as understand the concept of power factor and its significance in electrical systems.

The chapter also aims to introduce students to three-phase alternating current systems, their configurations, and the methods to calculate power in such systems, preparing them for more advanced studies and practical applications in electrical engineering.

3.2. Prerequisites

For understanding the third chapter, students should be familiar with:

- Basic electrical concepts and laws, including Ohm's Law and Kirchhoff's Laws.
- Fundamental knowledge of AC circuits, including sinusoidal signals and phasor representation.
- Understanding of impedance and complex power in AC circuits.
- Previous exposure to electrical measurements and the use of measuring instruments.

3.3. Introduction

Understanding how to generate, distribute, and manage electrical power efficiently is crucial for electrical engineers. This chapter begins by exploring single-phase electrical power, focusing on the different types of power (active, reactive, and apparent) and their significance. The concept of power factor and its impact on electrical efficiency will also be discussed.

The chapter then progresses to three-phase alternating current systems, which are widely used in industrial and commercial applications. By understanding the principles and applications of both single-phase and three-phase power, apprentices will be equipped with the knowledge required to design and analyze the electrical power systems in various settings.

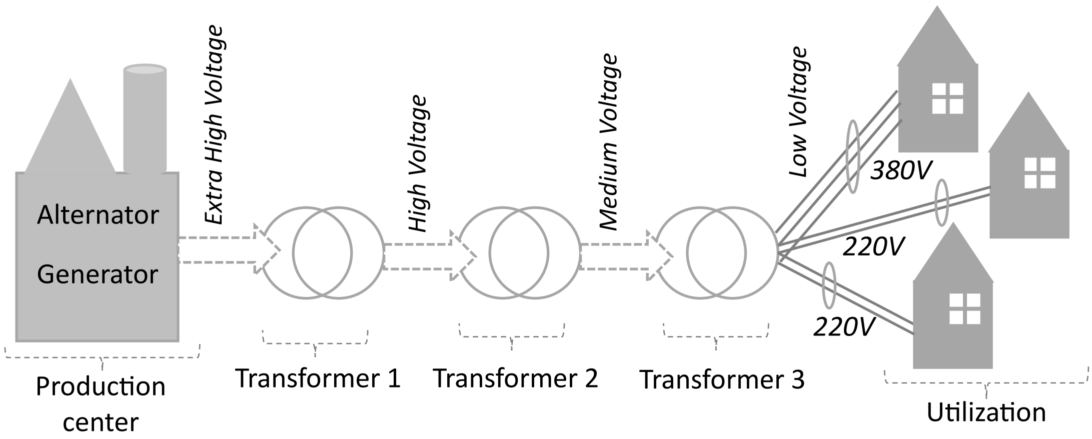


Figure 3.1. Voltage Transformation Process in Electrical Power Systems

3.4. Single-phase electrical powers

3.4.1. Basic linear components

The components of linear circuits, Resistor (R), Inductor (L), Capacitor (C) are fundamental in understanding AC circuit behavior and power calculations.

a) Relations of instantaneous values

The relationships between voltage and current at any given moment in time are,

$$\begin{cases} v(t) = R\, i(t) & for\ Risistor \\ v(t) = L\,\frac{d\, i(t)}{dt} & for\ Indictor \\ v(t) = \frac{1}{C}\int i(t)\, dt & for\ Capacitor \end{cases} \tag{3.1}$$

b) Relations of complex values

The complex relation involves resistor, inductor and capacitor are presented in next equations,

$$\begin{cases} V = Z_R\, I & for\ Risistor \\ V = Z_L\, I & for\ Indictor \\ V = Z_C\, I & for\ Capacitor \end{cases} \tag{3.2}$$

c) Impedance

The impedances of the three elements, the resistor, the inductor and the capacitor are presented respectively in the next,

$$\begin{cases} Z_R = R & Imaginary\ part\ is\ zero \\ Z_L = j\, L\omega & Real\ part\ is\ zero \\ Z_C = -j\,\frac{1}{C\omega} & Real\ part\ is\ zero \end{cases} \tag{3.3}$$

d) Modulus of impedance

The magnitudes of the three impedances are,

$$\begin{cases} |Z_R| = R \\ |Z_L| = L\omega \\ |Z_C| = \frac{1}{C\omega} \end{cases} \tag{3.4}$$

e) Argument of impedance

The phase angles between the voltage and the current in each component are,

$$\begin{cases} \varphi_R = 0^\circ & \varphi_R = 0\, radians & Z_R : real\ part\ is\ positive \\ \varphi_L = 90^\circ & \varphi_L = \frac{\pi}{2}\, radians & Z_L : imaginary\ part\ is\ positive \\ \varphi_C = -90^\circ & \varphi_C = -\frac{\pi}{2}\, radians & Z_C : imaginary\ part\ is\ negative \end{cases} \quad (3.5)$$

For the resistor, the current and the voltage are in phase.

For the indictor, the voltage is in advance to the current.

For the capacitor, the current is in advance to the voltage.

f) Admittance

The measure of how easily a circuit allows the flow of electric current, with $Y = 1/Z$.

$$\begin{cases} Y_R = \frac{1}{R} \\ Y_L = -j\frac{1}{L\omega} \\ Z_C = j\, C\omega \end{cases} \quad (3.6)$$

g) Modulus of admittance

The magnitudes of each admittance are,

$$\begin{cases} |Y_R| = \frac{1}{R} \\ |Y_L| = \frac{1}{L\omega} \\ |Y_C| = C\omega \end{cases} \quad (3.7)$$

h) Argument of admittance

The phase angles of three admittances are,

$$\begin{cases} \varphi_R = 0^\circ & \varphi_R = 0\, radians & Y_R : real\ part\ is\ positive \\ \varphi_L = -90^\circ & \varphi_L = -\frac{\pi}{2}\, radians & Y_L : imaginary\ part\ is\ negative \\ \varphi_C = 90^\circ & \varphi_C = \frac{\pi}{2}\, radians & Y_C : imaginary\ part\ is\ positive \end{cases} \quad (3.8)$$

3.4.2. Composed circuits of R, L and C

The application of the "Ohm and Kirchhoff" laws to circuits composed of impedances connected in series or in parallel allows us to state the following results:

The total impedance of a dipole made up of several impedances in series is equal to the complex sum of these:

$$Z = \sum Z_i \quad (3.9)$$

The total admittance of a dipole consisting of several admittances in series is equal to the complex sum of these:

$$Y = \frac{1}{Z} = \sum \frac{1}{Z_i} \quad (3.10)$$

So,

$$Y = \sum Y_i \tag{3.11}$$

Some impedances and their admittances corresponding to dipoles composed of R, L and C elements are defined in the next table, with the impedance is presented by $Z = R + j\,X$ and admittance by $Y = G + j\,B$.

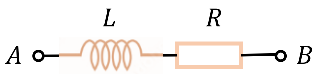
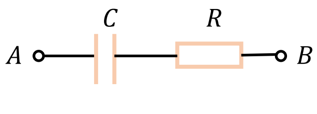
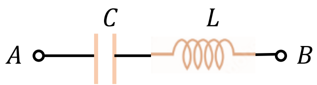
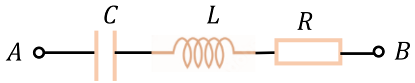
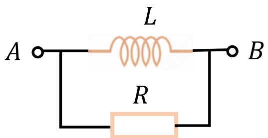
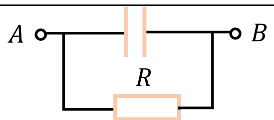
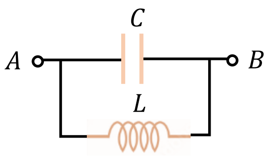
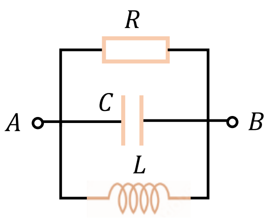
Composed circuits	Impedance	Admittance
	$R + j\,L\omega$	$\dfrac{R - j\,L\omega}{R^2 + (L\omega)^2}$
	$R - j\,\dfrac{1}{C\omega}$	$\dfrac{R + j\dfrac{1}{C\omega}}{R^2 + \left(\dfrac{1}{C\omega}\right)^2}$
	$j\left(L\omega - \dfrac{1}{C\omega}\right)$	$j\dfrac{C\omega}{1 - \omega^2 LC}$
	$R + j\left(L\omega - \dfrac{1}{C\omega}\right)$	$\dfrac{R + j\left(L\omega - \dfrac{1}{C\omega}\right)}{R^2 + \left(L\omega - \dfrac{1}{C\omega}\right)}$
	$\dfrac{R(L\omega)^2 + jR^2 L\omega}{(L\omega)^2 + R^2}$	$\dfrac{1}{R} - j\dfrac{1}{L\omega}$
	$\dfrac{R + j\omega C R^2}{1 + (\omega C R)^2}$	$\dfrac{1}{R} + jC\omega$
	$j\dfrac{L\omega}{1 - \omega^2 LC}$	$j\left(C\omega - \dfrac{1}{L\omega}\right)$
	$\dfrac{R - jR^2\left(C\omega - \dfrac{1}{L\omega}\right)}{1 + R^2\left(C\omega - \dfrac{1}{L\omega}\right)^2}$	$R + j\left(C\omega - \dfrac{1}{L\omega}\right)$

Tableau 3.1. Impedance and admittance of basic circuits used R, L and C

3.4.3. Measure active power P

Electrical equipment (motor, lamp, transformer, etc.) using a conventional power supply consumes three powers. An active power, a reactive power and another apparent power.

This is the energy that can be effectively utilized by the load, referred to as active power because it represents the genuinely useful power.

Active power is the average value of instantaneous power, measured in watts.

$$P = U\, I \cos(\varphi) \tag{3.12}$$

To measure power (P) using a wattmeter, make two connections: a parallel connection for voltage measurement and a series connection for current measurement (see Figure 3.2).

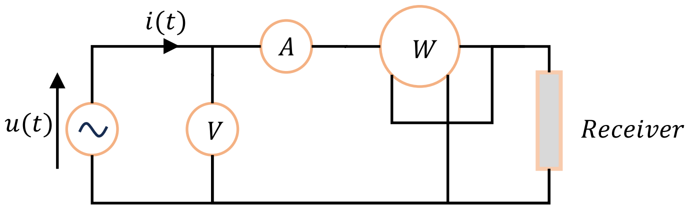


Figure 3.2. Measure active power P using wattmeter "*W*"

3.4.4. Measuring reactive power Q

Reactive power occurs when the installation includes inductive or capacitive components, measured in VAR.

$$Q = U\, I \sin(\varphi) \tag{3.13}$$

To measure reactive power (Q), connect an ammeter, a voltmeter, and a wattmeter as presented in last Figure (3.2), then calculate Q based on the type of receiver:

(i) for a resistive receiver $Q = 0$,

(ii) for an inductive receiver $Q > 0$ and

(iii) for a capacitive receiver $Q < 0$. As we note that,

$$\begin{cases} if\ \varphi > 0 & the\ receiver\ is\ inductive \\ if\ \varphi < 0 & the\ receiver\ is\ capacitive \end{cases} \tag{3.14}$$

3.4.5. Measuring apparent power S

The apparent power "S" is equal to the vector sum of the active and reactive powers, allowing the determination of the current absorbed by the load. It is expressed in VA.

$$S = U\, I \tag{3.15}$$

We can use other relation to calculate it using the Triangle of powers,

$$S = P + j\, Q \tag{3.16}$$

$$S^2 = P^2 + Q^2 \quad (3.17)$$

$$S = \sqrt{P^2 + Q^2} \quad (3.18)$$

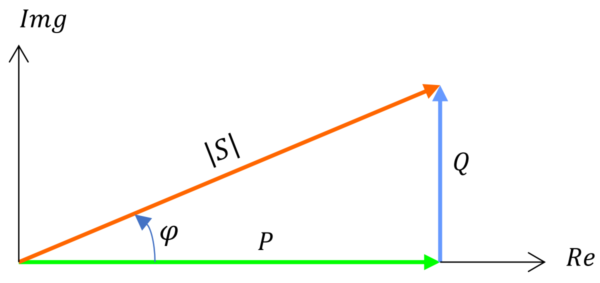


Figure 3.3. Complex power representation

for measuring this power, we use an ammeter and a voltmeter to determine the effective values respectively of current and voltage as presented in the next Figure.

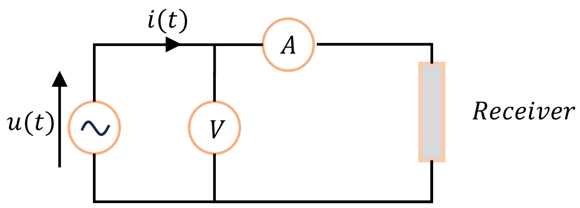


Figure 3.4. Measure apparent power "S" use an ammeter and a voltmeter

3.4.6. Power factor and Boucherot's Theorem

We recall here that, the total active power consumed by a system as presented in chapter 2 is the sum of the active powers consumed by each part of the global installation,

$$P_T = \sum_{i=1}^{N} P_i \quad (3.19)$$

The same method used to calculate the total reactive power,

$$Q_T = \sum_{i=1}^{N} Q_i \quad (3.20)$$

The apparent power consumed by a system is calculated from the next relation:

$$S_T = \sqrt{P_T^2 + Q_T^2} \quad (3.21)$$

Power factor is a factor measuring the active power production efficiency of the system. It calculated by,

$$\cos(\varphi) = \frac{P_T}{S_T} \quad (3.22)$$

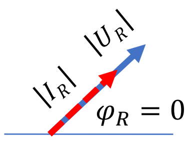
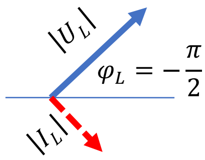
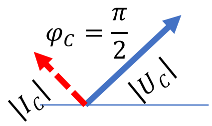
Components	Resistor	Inductor	Capacitor
Impedance	R	$j\,L\omega$	$-j\left(\frac{1}{C\omega}\right)$
Modulus of Z	R	$L\omega$	$\frac{1}{C\omega}$
Temporal voltage	$U_R(t) = R\,i_R(t)$	$u_L(t) = L\,\frac{di_L(t)}{dt}$	$u_C(t) = \frac{1}{C}\int i_C(t)dt$
Modulus of voltage	$U_R = R\,I_R$	$U_L = L\omega\,I_L$	$U_C = \frac{I_C}{C\omega}$
Phase angle	$\varphi_R = 0$	$\varphi_L = \frac{\pi}{2}$	$\varphi_C = -\frac{\pi}{2}$
Fresnel diagram			
Active power	$P = U_R\,I_R\cos(\varphi_R)$ $P = U_R\,I_R$ $P = R\,I_R^2$ $P = U_R^2/R$	0	0
Reactive power	0	$Q_L = U_L\,I_L\sin(\varphi_L)$ $Q_L = U_L\,I_L$ $Q_L = L\omega\,I_L^2$ $Q_L = U_L^2/L\omega$	$Q_C = U_C\,I_C\sin(\varphi_C)$ $Q_C = -U_C\,I_C$ $Q_L = -\,I_L^2/C\omega$ $Q_L = -U_L^2 C\omega$
Apparent power	$S = P$	$S = Q$	$S = Q$
Power factor	1	0	0

Tableau 3.2. Fresnel diagram and power of basic circuits used R, L and

3.5. Three-phase alternating current

Three-phase circuits are the foundation of the electricity distribution network. They are used to connect generators to industrial and residential networks. For example, a three-phase system transports electrical energy to a residential area, where it is then distributed as single-phase power. This network consists of three sinusoidal alternating voltages/currents of the same frequency and amplitude, each phase-shifted by $\frac{2\pi}{3}$.

3.5.1. Balanced three-phase system

A balanced three-phase voltage system is a configuration in which three sinusoidal voltages of the same frequency and amplitude are present, with each voltage phase-shifted by 120° relative to the others. This balance ensures that the sum of the voltages at any instant is zero, providing stable and efficient power distribution.

In the three-phase network, the distribution is made from four terminals:

(i) Three phase terminals marked 1, 2, 3 or A, B, C or R, S, T;

(ii) A neutral terminal *N*.

We note that the three-phase network is presented either with,

(i) Simple voltages ($v_1(t)$, $v_2(t)$ and $v_3(t)$), or

(ii) Compound voltages ($u_{12}(t)$, $u_{23}(t)$ and $u_{31}(t)$) as shown below.

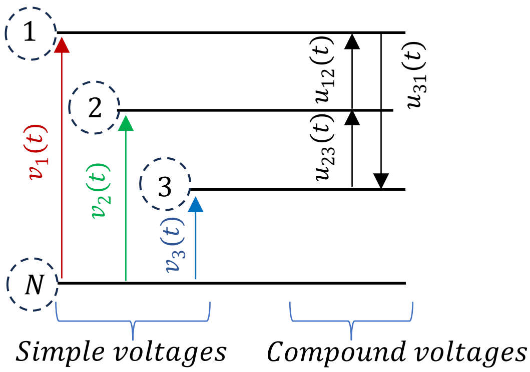


Figure 3.5. Representation of three-phase voltages (Simple and Compound)

A three-phase system is balanced when the three voltages have the same effective value and the phase between two voltages is $2\pi/3$.

3.5.2. Simple voltage

We define the simple voltage $v_1(t)$ as the observed voltage between the first phase and neutral terminals. All simple voltage $v_1(t)$, $v_2(t)$ and $v_3(t)$ have the same effective values.

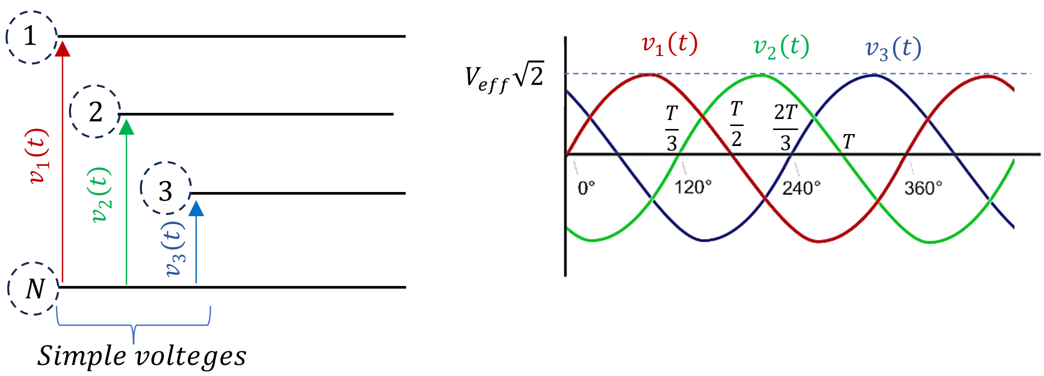


Figure 3.6. Temporal representation of three simple voltages

The expressions of the three simple voltages are given by,

$$v_1(t) = V_{eff}\sqrt{2}\sin(\omega t) \tag{3.23}$$

$$v_2(t) = V_{eff}\sqrt{2}\sin(\omega t - \tfrac{2\pi}{3}) \tag{3.24}$$

$$v_3(t) = V_{eff}\sqrt{2}\sin(\omega t - \tfrac{4\pi}{3}) \tag{3.25}$$

The Fresnel representation of these voltages is presented in the next figure.

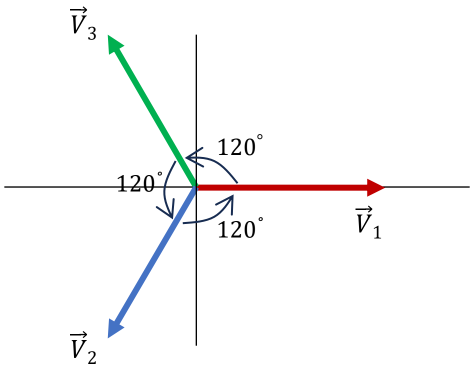


Figure 3.7. Complex representation of the three simple voltages

We can present these three vectors by,

$$\begin{cases} \vec{V}_1 = \begin{pmatrix} V \\ 0 \end{pmatrix}, \\ \vec{V}_2 = \begin{pmatrix} V \\ -\frac{2\pi}{3} \end{pmatrix}, \\ \vec{V}_3 = \begin{pmatrix} V \\ -\frac{4\pi}{3} \end{pmatrix}. \end{cases} \tag{3.26}$$

We note that the sum of the three simple voltage is zero, $\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = \vec{0}$, and $v_1(t) + v_2(t) + v_3(t) = 0$.

3.5.3. Composed voltage

The compound voltage $u_{12}(t)$ is the observed voltage between the first and second phase terminals. The compound voltages have the same frequency as simple voltages.

The compound voltages can be expressed as,

$$\begin{cases} u_{12}(t) = v_1(t) - v_2(t) \\ u_{23}(t) = v_2(t) - v_3(t) \\ u_{31}(t) = v_3(t) - v_1(t) \end{cases} \tag{3.27}$$

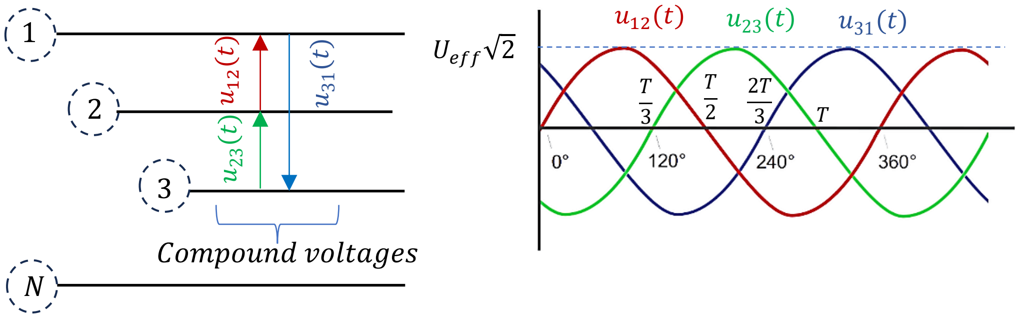


Figure 3.8. Temporal representation of three compound voltages

The Fresnel representation of these voltages is presented in the next figure.

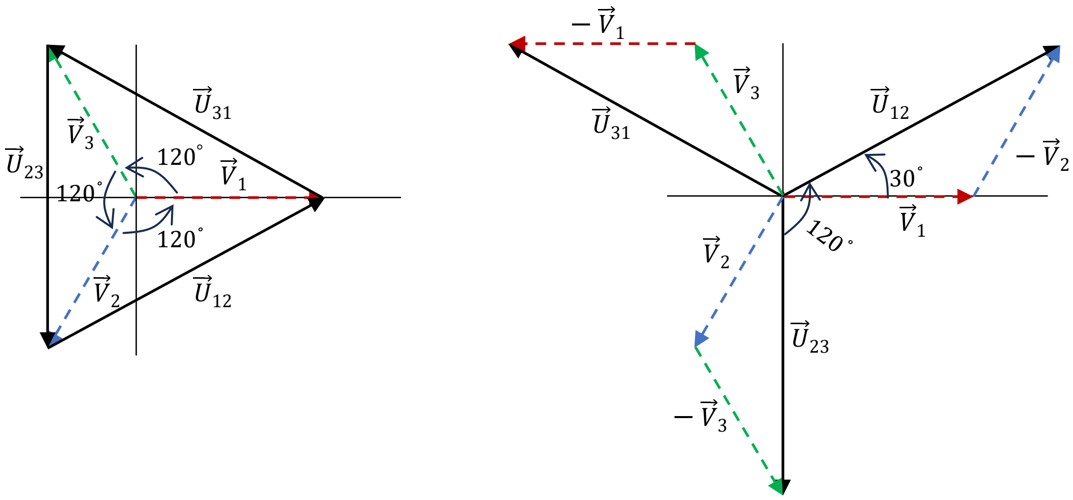


Figure 3.9. Complex representation of the three simple voltages

We can present the three compound voltage vectors by,

$$\begin{cases} \vec{U}_{12} = \begin{pmatrix} U \\ \frac{\pi}{6} \end{pmatrix}, \\ \vec{U}_{23} = \begin{pmatrix} U \\ -\frac{3\pi}{6} \end{pmatrix}, \\ \vec{U}_{31} = \begin{pmatrix} U \\ -\frac{7\pi}{6} \end{pmatrix}. \end{cases} \quad (3.28)$$

The temporal expressions of the three compound voltages are given by,

$$u_{12}(t) = U_{eff}\sqrt{2}\sin\left(\omega t + \frac{\pi}{6}\right) \quad (3.29)$$

$$u_{23}(t) = U_{eff}\sqrt{2}\sin\left(\omega t - \frac{3\pi}{6}\right) \quad (3.30)$$

$$u_{31}(t) = U_{eff}\sqrt{2}\sin\left(\omega t - \frac{7\pi}{6}\right) \quad (3.31)$$

We note that the sum of these voltages is zero, $\vec{U}_{12} + \vec{U}_{23} + \vec{U}_{31} = \vec{0}$, and $u_{12}(t) + u_{23}(t) + u_{31}(t) = 0$.

The relation between the simple and compound voltage is,

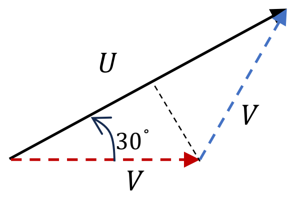


Figure 3.10. relation between the simple and compound voltages

$U = 2\,V\cos(30)$, we replace $\cos(30)$ by $\frac{\sqrt{3}}{2}$, we obtain $U = 2\,V\frac{\sqrt{3}}{2}$

$$U = V\sqrt{3} \tag{3.32}$$

3.5.4. Three-phase receivers

This type of receivers made up of three identical dipoles, of impedance Z. they are balanced because the three elements are identical. Generally, we can define two types of currents,

(i) Phase currents: these are the currents that flow through the Z elements of the three-phase receiver (symbolized by J).

(ii) Line currents: these are the currents that flow through the wires of the three-phase network (symbolized by I).

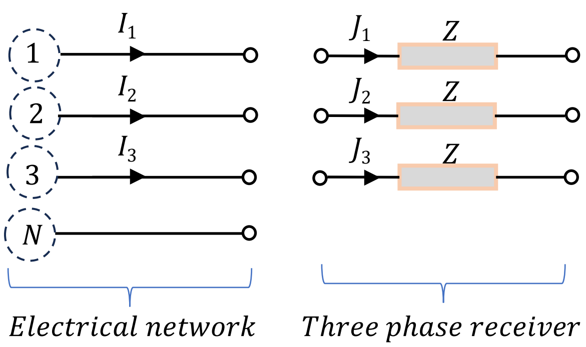


Figure 3.11. Electrical three-phase network and receiver

In practice, the three voltages in a three-phase system are generated by a three-phase alternator. Specifically, these voltages are produced by three stationary coils, each functioning as a voltage generator.

To utilize these voltages effectively, it is necessary to interconnect the three coils. There are two primary methods of coupling these coils: Star (⅄) and Triangle (Δ).

3.5.5. Star coupling (⅄)

The star coupling type is symbolized by (⅄). There are two types of voltages (i) The simple voltage between a phase and the neutral and (i) The compound voltage between two phases. The relationship between the simple voltage and the effective compound voltage is: $U = V\sqrt{3}$. The coupling of the star uses Neutral terminal as presented in Figure 3.12.

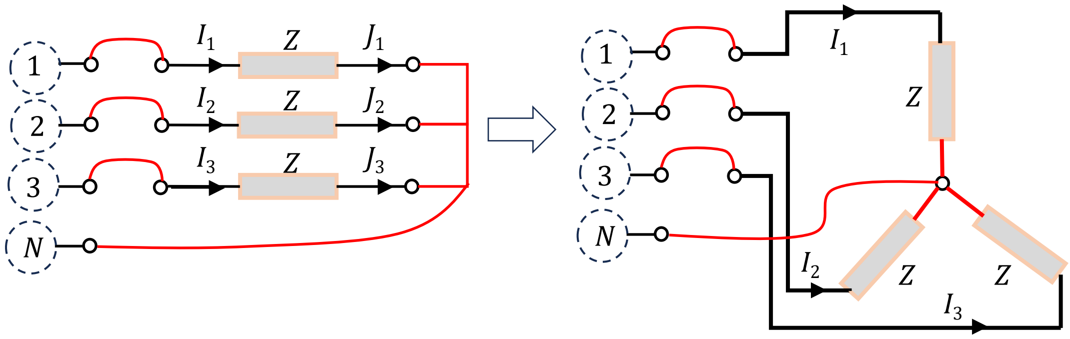


Figure 3.12. Receiver with star coupling type

In this case, the line current equal the phase current, $I_1 = J_1$, $I_2 = J_2$ and $I_3 = J_3$

3.5.6. Triangle coupling (Δ)

In this coupling, all three windings are connected end-to-end to form a closed loop, with each corner of the triangle representing a connection point to one of the phases. Unlike the star coupling, the triangle coupling does not have a neutral point.

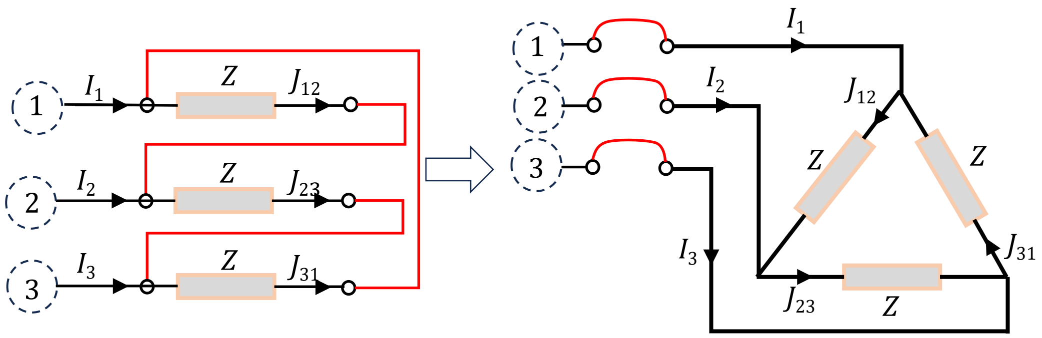


Figure 3.13. Receiver with triangle coupling type

The line voltage (voltage between any two lines) is equal to the phase voltage (voltage across each coil). There is no neutral point, so we don't distinguish between simple and compound voltages. The line current (current in each line) is related to the phase current (current in each winding) by a factor of $\sqrt{3}$.

In this case, the line currents are given by, $I_1 = J_{12}\sqrt{3}$, $I_2 = J_{23}\sqrt{3}$ and $I_3 = J_{31}\sqrt{3}$

The triangle coupling is commonly used in situations where a neutral point is not needed, and it is suitable for three-phase loads that are balanced.

3.5.7. Power in three-phase system

3.5.7.1. Power in star coupling

For a one receiver phase, the active and reactive powers are given respectively by,

$$\begin{cases} P_1 = V_1\, I_1 \cos(\varphi) \\ Q_1 = V_1\, I_1 \sin(\varphi) \end{cases} \quad (3.32)$$

with φ is the angle between $v_1(t)$ and $i_1(t)$

for the complete receiver, we have

$$\begin{cases} P = V_1\, I_1 \cos(\varphi) + V_2\, I_2 \cos(\varphi) + V_3\, I_3 \cos(\varphi) \\ P = 3\, V\, I \cos(\varphi) \end{cases} \quad (3.33)$$

$$\begin{cases} Q = V_1\, I_1 \sin(\varphi) + V_2\, I_2 \sin(\varphi) + V_3\, I_3 \sin(\varphi) \\ Q = 3\, V\, I \sin(\varphi) \end{cases} \quad (3.34)$$

with $V = U/\sqrt{3}$

The finale expressions of power are:

$$P = \sqrt{3}\, U\, I \cos(\varphi) \quad (3.35)$$

$$Q = \sqrt{3}\, U\, I \cos(\varphi) \quad (3.36)$$

The total apparent power is

$$S = \sqrt{3}\, U\, I \quad (3.37)$$

Power factor:

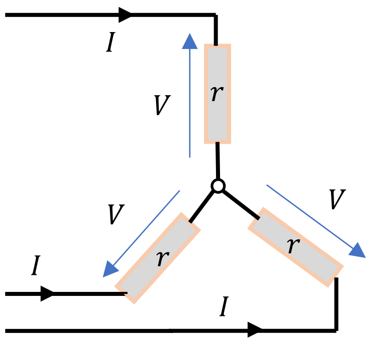
$$k = \cos(\varphi) = \frac{P}{S} \quad (3.38)$$

To see the Joule effect losses in the star coupling, we consider the resistive part of the receiver.

For one receiver phase, the Joule effect losses is defined as,

$$P_{j1} = r\, I^2$$

With r is the resistor between one phase and neutral. The resistor between two phase is $R = 2\, r$.



We define the Joule effect losses for the complete receiver by: $P_j = 3\, P_{j1} = 3\, r\, I^2$

Finally for the star coupling:

$$P_j = \frac{3}{2} R\, I^2 \quad (3.39)$$

3.5.7.2. Power in triangle coupling

In this case and for a one receiver phase, the active and reactive powers are given respectively by,

$$\begin{cases} P_1 = U_{12}\, J_{12} \cos(\varphi) \\ Q_1 = U_{12}\, J_{12} \sin(\varphi) \end{cases} \tag{3.40}$$

with φ is the angle between $u_{12}(t)$ and $j_{12}(t)$

for the complete receiver, we have

$$\begin{cases} P = U_{12}\, J_{12} \cos(\varphi) + U_{23}\, J_{23} \cos(\varphi) + U_{31}\, J_{31} \cos(\varphi) \\ P = 3\, U\, J \cos(\varphi) \end{cases} \tag{3.41}$$

$$\begin{cases} Q = U_{12}\, J_{12} \sin(\varphi) + U_{23}\, J_{23} \sin(\varphi) + U_{31}\, J_{31} \sin(\varphi) \\ Q = 3\, U\, J \sin(\varphi) \end{cases} \tag{3.42}$$

With $J = I/\sqrt{3}$

The finale expressions of power are:

$$P = \sqrt{3}\, U\, I \cos(\varphi) \tag{3.43}$$

$$Q = \sqrt{3}\, U\, I \cos(\varphi) \tag{3.44}$$

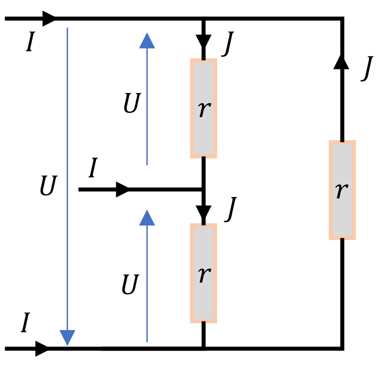
The total apparent power is

$$S = \sqrt{3}\, U\, I \tag{3.45}$$

Power factor:

$$k = \cos(\varphi) = \frac{P}{S} \tag{3.46}$$

To see the Joule effect losses in the triangle coupling, we consider the resistive part of the receiver.



For one receiver phase, the Joule effect losses is defined as,

$P_{j1} = r\, J^2$

The resistor between two phase is R defined as $2\,r$ in parallel with r.

$R = (2r)//(r) = \frac{2r\,.\,r}{2r+r}$, we have $R = \frac{2}{3}\, r$

We define the Joule effect losses for the complete receiver in case of triangle coupling by: $P_j = 3\, P_{j1} = 3\, r\, J^2 = 3\, \frac{3}{2}\, R\, \left(\frac{I}{\sqrt{3}}\right)^2$

Finally for the star coupling:

$$P_j = \frac{3}{2} R\, I^2 \tag{3.47}$$

3.5.8. Power measurement

3.5.8.1. Apparent power measurement

To measure the apparent power, we use a voltmeter and ammeter to determine the voltage and current in a balanced three-phase system, following the two configurations shown in the next Figure.

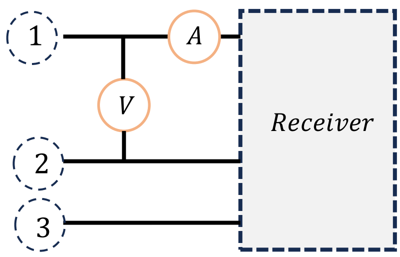
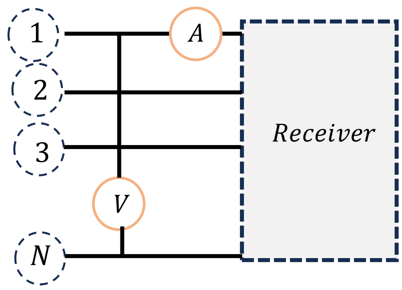
Electrical network without neutral	Electrical network with neutral
	
$S = \sqrt{3}\, V\, I$	$S = 3\, V\, I$

Figure 3.14. Method used to measure the apparent power

3.5.8.2. Active power measurement

A. Single wattmeter method with neutral

When the receiver is balanced, we can use one wattmeter to measure the active power absorbed as presented in the following figure.

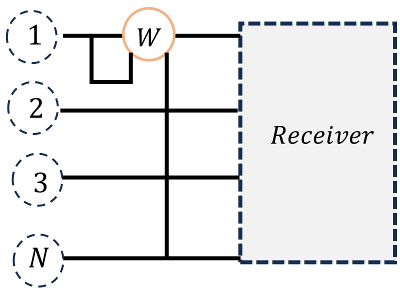


Figure 3.15. Single wattmeter method used to measure the active power

The power meter, as plugged in, measures power P_1:

$$P_1 = V\, I \cos(\varphi) \qquad (3.48)$$

The power absorbed by the balanced three-phase receiver is:

$$P = 3\, P_1 = 3\, V\, I \cos(\varphi) \qquad (3.49)$$

$$P = \sqrt{3}\, U\, I \cos(\varphi) \qquad (3.50)$$

We note that This method is used if the neutral terminal be accessible.

B. Two wattmeter method

For a general case (unbalanced or balanced systems) where the neutral is not accessible, we can measure the active power using two wattmeters as presented in the next Figure.

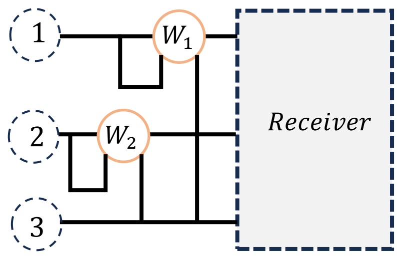


Figure 3.16. Single wattmeter method used to measure the active power

We note here that P_1 and P_2 are the two powers measured by the two wattmeters W_1 and W_2, respectively. We define the total active power of three-phase receiver by:

$$P = P_1 + P_2 \tag{3.51}$$

3.5.8.3. Reactive power measurement

A. Single wattmeter method with neutral

To measure reactive power using a single wattmeter, simply mount the voltage circuit between phase 2 and 3, as presented in the following figure.

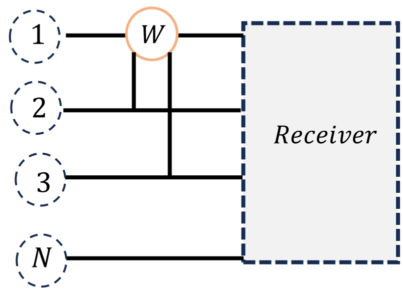


Figure 3.17. Single wattmeter method used to measure the reactive power

The reactive power is given by the following expression:

$$Q = \sqrt{3}\,P \tag{3.52}$$

In this case, P presents the power measured by the wattmeter W.

B. Two wattmeter method

For measuring the reactive power, we use the same configuration presented previously for measuring active power (see Figure 3.16), But we can use the following relation:

$$Q = \sqrt{3}\,(P_1 - P_2)$$

CHAPTER 4

Magnetic circuits

Chapter 4: Magnetic circuits

4.1. Objective

The objective of this chapter is to provide students with a comprehensive understanding of magnetic circuits and their importance in electrical machines. By the end of the chapter, students will be able to analyze magnetic circuits by calculating key parameters such as magnetic flux, reluctance, and magnetomotive force (MMF). They will also learn to compare magnetic circuits with electrical circuits, highlighting their similarities and differences. Furthermore, students will gain insights into the properties and applications of different magnetic materials and acquire the necessary skills to design and optimize simple magnetic circuits for efficient electromagnetic field management.

4.2. Prerequisites

For understanding this chapter, students should be familiar with:

- Analyze simple circuits using Ohm's Law
- Circuits and Electrical Power
- Concept of inductance, behavior of a coil carrying current, and magnetic field generation.
- Understanding of magnetic fields, flux, and electromagnetic induction (Faraday’s law, Ampère’s law basics).

4.3. Introduction

Magnetic circuits are essential for guiding and controlling magnetic fields in electrical machines. They play a crucial role in enhancing machine performance by efficiently directing magnetic flux through carefully designed paths. Built using materials such as ferromagnetic and ferrimagnetic substances, these circuits ensure optimal energy conversion and electromagnetic field management. Understanding the principles and design of magnetic circuits is fundamental to the study and development of efficient and reliable electrotechnical systems.

4.4. Magnetic field

A magnetic field is a region in space where magnetic forces can be observed, usually created by moving charges or changes in electric fields. In a magnetic medium subject to a magnetic excitation vector $\vec{H}$, the magnetic induction vector $\vec{B}$ can be defined. The relationship between these quantities depends on the medium properties. In a vacuum, the relationship between magnetic excitation and induction is given by:

$$\vec{B} = \mu_0 \vec{H} \tag{4.1}$$

where,

$\vec{B}$ is the magnetic induction vector, measured in Tesla [T].

$\vec{H}$ is the magnetic field vector, measured in Ampere per meter [A/m].

μ_0 is the permeability of free space [A/m].

In a magnetic material, the relationship becomes:

$$\vec{B} = \mu \vec{H} \tag{4.2}$$

Where μ is the magnetic permeability of the material, defined as $\mu = \mu_0 \mu_r$ with μ_r is the relative permeability of the material.

4.5. Ampère's theorem

Ampère's theorem is a fundamental principle that relates the magnetic field around a closed path to the current passing through the surface enclosed by that path. It states that the circulation of the magnetic field $\vec{H}$ along a closed contour C is proportional to the total current enclosed:

$$\oint_C \vec{H}\, d\vec{l} = \mu_0 \sum I \tag{4.3}$$

For a coil with N turns and current I flowing through each turn:

$$\oint_C \vec{H}\, d\vec{l} = \mu_0 N I \tag{4.4}$$

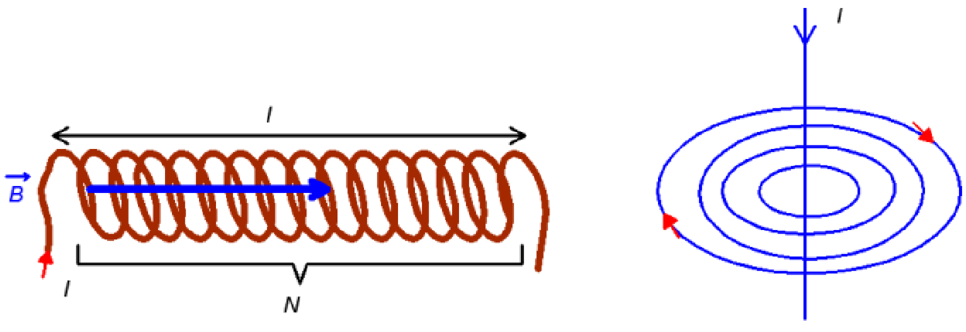


Figure 4.1. Wire-coil crossed by an electric current

4.6. Magnetic flux

Magnetic flux is a fundamental concept in the study of magnetic circuits and electromagnetic induction phenomena. Magnetic flux $\emptyset$ measures the amount of magnetic field passing through a given surface.

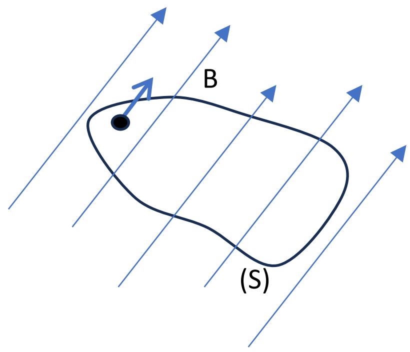


Figure 4.2. Presentation of Magnetic flux

It is defined using the magnetic induction vector $\vec{B}$ as follows:

$$\emptyset = \iint_{(S)} \vec{B}\,\vec{n}\,dS \tag{4.5}$$

where,

$\emptyset$ is the magnetic flux, measured in Weber (Wb).

S is the closed surface through which the flux is measured.

$\vec{n}$ is the unit vector normal to the surface.

The magnetic flux is maximum when the magnetic field $\vec{B}$ is perpendicular to the surface ($\vec{B} \| \vec{n}$) and zero when the field is parallel to the surface ($\vec{B} \perp \vec{n}$).

4.7. Homogeneous and linear magnetic circuits

Magnetic circuits are primarily constructed using ferromagnetic materials, as they provide high magnetic induction. In air or non-magnetic materials, magnetic field lines from a coil are not confined, leading to weak flux values. By using ferromagnetic materials, the magnetic flux can be efficiently channeled.

In the case of a homogeneous and linear magnetic circuit, such as the one shown in Figure 4.3, the following assumptions are made:

- The core material does not reach saturation (linear magnetic behavior).
- There is no hysteresis effect.

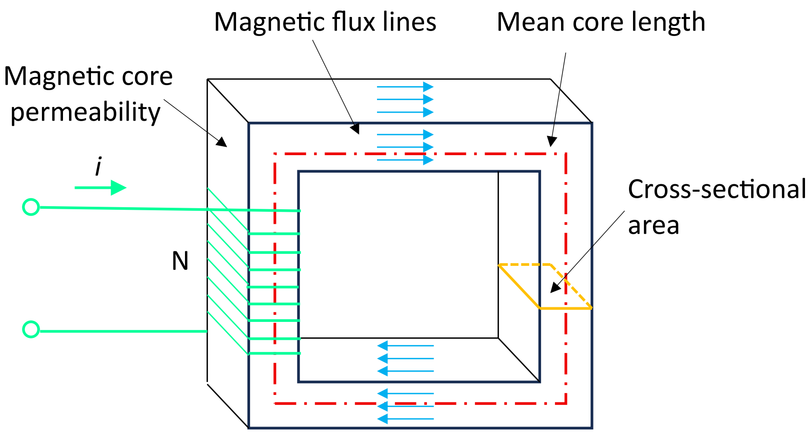


Figure 4.3. Magnetic core

Applying Ampère's theorem to this circuit gives:

$$\oint_C \vec{H}\, d\vec{l} = N\, I \tag{4.6}$$

Which becomes:

$$H\, L = N\, I \tag{4.7}$$

with,

H is the magnetic field strength.

L is the average length of the magnetic path.

N is the number of turns of the coil.

I is the current through the coil.

The magnetic induction BBB is related by:

$$B = \mu\, H = \frac{\mu\, N\, I}{L} \tag{4.8}$$

The magnetic flux Ø is expressed as:

$$\emptyset = B\, S = \frac{\mu\, S\, N\, I}{L} \tag{4.9}$$

4.7.1. Hopkinson formula

Hopkinson's formula generalizes the expression of the magnetomotive force (MMF) created in a magnetic circuit by the following relation:

$$F = N\, I = R\, \emptyset \tag{4.10}$$

R is the reluctance of the magnetic circuit, expressed in ampere-turns per weber (At/Wb).

N is the number of turns of the coil.

I is the current through the coil.

Ø is the magnetic flux in the circuit (Wb).

The reluctance RRR can also be expressed as a function of the length and cross-sectional area of the material:

$$R = \frac{L}{\mu\, S} \qquad (4.11)$$

4.7.2. Self-inductance

Self-inductance is defined as the proportional relationship between the current flowing through a coil III and the total magnetic flux intercepted by the coil. The fundamental relationship is given by:

$$\emptyset_t = N\,\emptyset = \frac{N^2\, I}{R} = L\, I \qquad (4.12)$$

where,

N: Number of turns,

R : Magnetic reluctance of the circuit,

$\emptyset$: Total flux,

L: Self-inductance.

The expression for self-inductance becomes:

$$L = \frac{N^2}{R} \qquad (4.13)$$

4.8. Heterogeneous and linear magnetic circuits

A magnetic circuit is said to be heterogeneous when it is composed of different materials or when its geometry has sections with varying properties. The figure below illustrates the Heterogeneous magnetic circuits case.

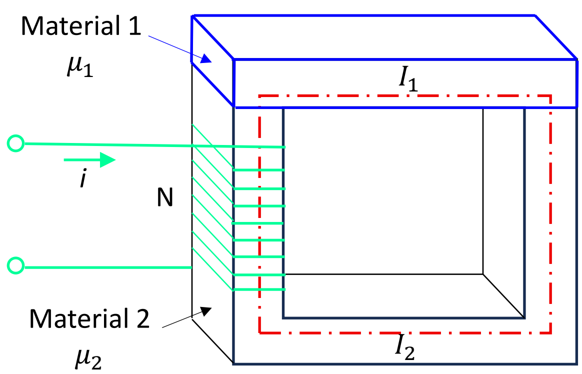


Figure 4.4. Heterogeneous magnetic circuit with two distinct regions

The figure 4.4 shows a heterogeneous magnetic circuit with two distinct regions:

- **Material 1** with permeability µ1
- **Material 2** with permeability µ2

The magnetic flux $\emptyset_t$ circulates through both materials, driven by a winding of N turns carrying a current I. The lengths of the magnetic paths in the two regions are denoted as l_1 and l_2, respectively.

The total reluctance of the circuit is the sum of the individual reluctances in each material:

$$R_{eq} = R_1 + R_2 \tag{4.14}$$

where:

$$R_1 = \frac{l_1}{\mu_1\, S} \tag{4.15}$$

$$R_2 = \frac{l_2}{\mu_2\, S} \tag{4.16}$$

We note that, S is the cross-sectional area assumed to be the same for both regions.

The total flux $\emptyset_t$ in the circuit is given by:

$$\emptyset_t = \frac{N\, I}{R_{eq}} = \frac{N\, I}{R_1 + R_2} \tag{4.17}$$

4.9. Magnetic circuits in sinusoidal alternating regime

The figure 4.5 illustrates a typical magnetic circuit with a coil wound around a magnetic core. The coil carries a current I that generates a magnetic flux $\emptyset(t)$ within the closed path of the magnetic material. The voltage source $V(t)$ drives the alternating current, producing a time-varying magnetic field in the circuit. We note that, a winding with N turns, an alternating voltage V(t) applied across the winding, Magnetic flux $\emptyset(t)$ circulating through the core. As we note that the flux is influenced by the cross-sectional area and magnetic properties of the core.

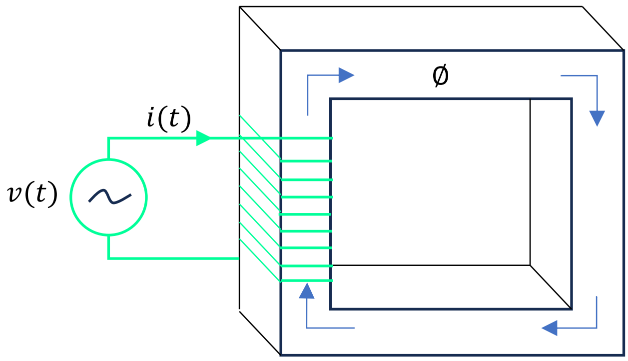


Figure 4.5. Magnetic circuit with a coil wound around a magnetic core

The relationship between the alternating voltage and magnetic flux is derived from Faraday's law of electromagnetic induction:

$$V(t) = \sqrt{2}\, V_{ef}\, \cos\,(\omega t) \tag{4.18}$$

$$V(t) = N\,\frac{d\emptyset(t)}{dt} \tag{4.19}$$

The flux is derived as:

$$\emptyset(t) = \frac{\sqrt{2}\, V_{ef}}{N\, \omega} \sin(\omega t) \quad (4.20)$$

The flux $\emptyset(t)$ can also be linked to the magnetic induction $B(t)$ over the core cross-sectional area S:

$$\emptyset(t) = S\, B(t) \quad (4.21)$$

The identification of amplitude values provides the following Boucherot relation:

$$B_{max} = \frac{\sqrt{2}\, V_{ef}}{S\, N\, \omega} \quad (4.22)$$

The V_{max} is given as

$$V_{max} = 2\pi f\ S\ N\ B_{max} \quad (4.23)$$

4.9.1. Ideal linear material

An ideal linear magnetic material is characterized by a linear relationship between the magnetic flux density B and the magnetic field strength H, expressed as:

$$B = \mu\, H \quad (424)$$

This linearity implies that the material does not exhibit saturation or hysteresis, making it ideal for theoretical and analytical studies.

From Faraday's law, the relationship between voltage and flux linkage is given by:

$$V(t) = \frac{d\emptyset(t)}{dt} \quad (4.25)$$

Given that the flux linkage $\emptyset(t)$ is proportional to the current through the inductance L:

$$\emptyset(t) = L\, i(t) \quad (4.26)$$

We obtain:

$$V(t) = L \frac{di(t)}{dt} \quad (4.27)$$

This equation defines a pure inductive response, implying that the coil behaves as an ideal inductor.

4.9.2. Nonlinear real material

Unlike ideal linear materials, nonlinear real materials exhibit a nonlinear relationship between the magnetic flux density B and the magnetic field strength H, described as $B = f(H)$. This nonlinearity is evident at low frequencies and results in the appearance of a hysteresis phenomenon.

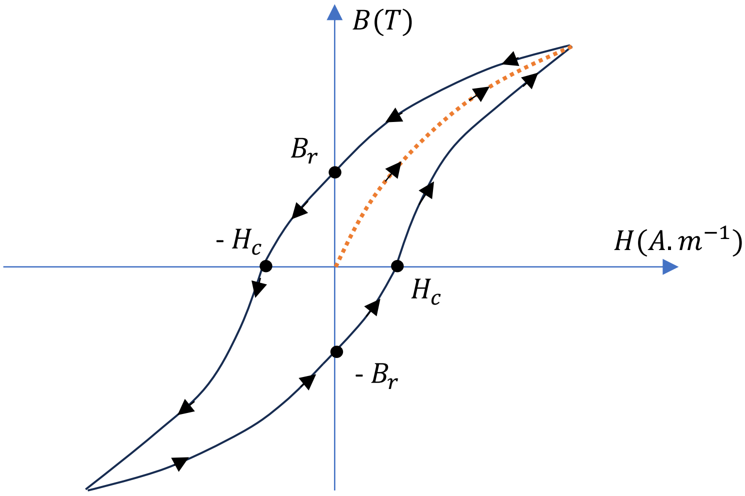


Figure 4.6. Concept of a magnetic hysteresis loop

In the design of electromagnetic devices, accurately assessing magnetic losses is crucial for evaluating the device's performance, particularly in terms of efficiency and heat management. Proper dimensioning of magnetic losses ensures improved thermal stability and optimized energy efficiency.

This phenomenon being non-linear, therefore, the inductance and permeability will no longer be constant. On the other hand, the real material is the source of losses in the metallic mass which we call "Iron Losses", they are made up of:

$$L = L_H + L_{EC} \tag{4.28}$$

With, L_H is the Hysteresis Losses and L_{EC} presents the Eddy Current Losses.

4.9.2.1. Hysteresis Losses

Hysteresis losses occur due to the repetitive magnetization and demagnetization of magnetic materials during each cycle of operation. The energy dissipated as heat is proportional to the area of the hysteresis loop. Hysteresis losses are given by:

$$L_H = \alpha_H \, f \, B_{max}^2 \tag{4.29}$$

α_H is Material constant (dimensionless or dependent on material).

4.9.2.2. Eddy Current Losses

Eddy currents are induced in the core due to variations in the magnetic field, causing Joule heating. These losses can be reduced by laminating the core. The formula for eddy current losses per cubic meter is:

$$L_{EC} = \alpha_{EC} \, f \, B_{max}^2 \tag{4.30}$$

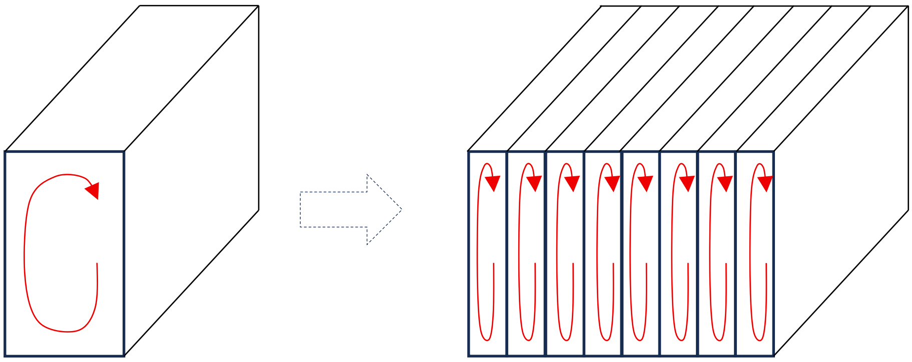


Figure 4.7. Eddy Current Losses in magnetic circuit

4.9.3. Inductance and Magnetic Flux in Electrical Circuits

4.9.3.1. Self-Inductance and Magnetic Flux

When a closed circuit carries a current $i(t)$, it generates a magnetic field $\vec{B}_p$ that induces a flux across the surface of the circuit.

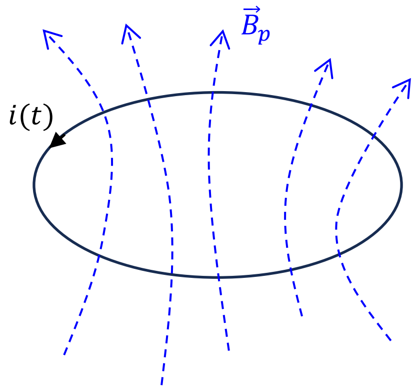


Figure 4.8. Flux across the surface of the circuit

The Self-Inductance Formula:

$$\emptyset_p = L\, i \tag{4.31}$$

where,

$\emptyset_p$ is the self-flux (Weber, Wb)

L is the self-inductance (Henry, H)

i is the current (Ampere, A)

A solenoid with N turns generates a uniform magnetic field when carrying current i.

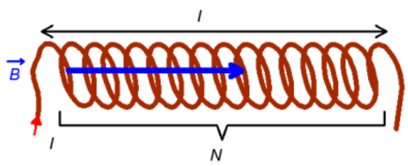


Figure 4.9. Self-Inductance

Magnetic Field:

$$B_p = \mu_0 \frac{N}{l} i \tag{4.32}$$

With l is the length of the solenoid and N is the number of turns.

Self-Inductance:

$$L = \frac{\mu_0 \, S \, N^2}{l} \tag{4.33}$$

And we can use other formula,

$$L = \frac{N^2}{R} \tag{4.34}$$

4.9.3.2. Mutual Inductance and Coupling Coefficient

Mutual inductance is the phenomenon by which a change in current in one circuit induces a voltage in a neighboring circuit due to the magnetic flux generated by the first circuit. This coupling occurs when two or more circuits are positioned such that the magnetic flux produced by one links with the other(s).

When a current i_1 flows through circuit 1, it produces a magnetic flux B_{p1} that can pass through circuit 2. Likewise, if a current i_2 flows through circuit 2, it produces a flux B_{p2} that links with circuit 1.

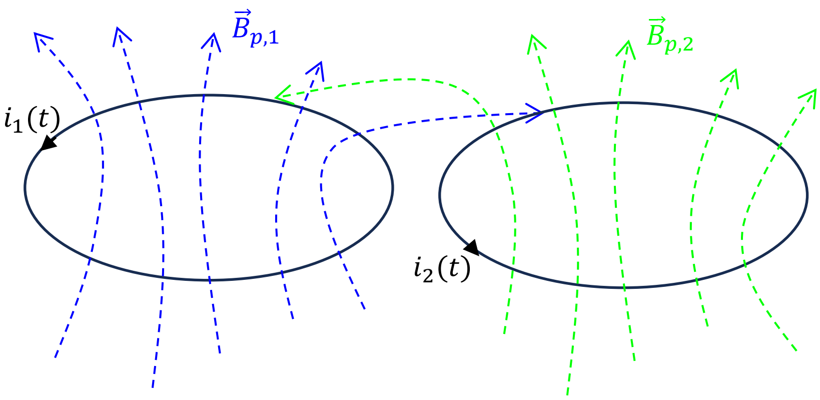


Figure 4.10. Mutual flux

The total flux linking a circuit due to both magnetic fields is:

$$B_{p,total} = B_{p,1} + B_{p,2} \tag{4.35}$$

The induced flux in each circuit can be described as:

$$\emptyset_1 = \emptyset_{11} + \emptyset_{21} \tag{4.36}$$

$$\emptyset_2 = \emptyset_{22} + \emptyset_{12} \tag{4.37}$$

We can write,

$$\emptyset_1 = L_1 \, I_1 + M_{21} \, I_2 \tag{4.38}$$

$$\emptyset_2 = L_2 \, I_2 + M_{12} \, I_1 \tag{4.39}$$

With,

L_1 and L_2 represents the self-inductances of Circuit 1 and Circuit 2, respectively.

M_{12} and M_{21} are the mutual inductances between the circuits.

Mutual Inductance (M):

The mutual inductance between two circuits is a measure of the flux in one circuit due to the current in the other:

$$M = M_{12} = M_{21} \tag{4.40}$$

Noting that, Mutual inductance depends on, (i) The geometric arrangement of the circuits and (ii) The permeability of the medium.

Coupling coefficient (k):

The coupling coefficient quantifies how well the two circuits are magnetically coupled. It is defined as:

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{4.41}$$

Where, $0 \leq k \leq 1$

If $k = 1$ indicates perfect coupling or all flux from one circuit links with the other.

If $k = 0$ means no magnetic coupling.

CHAPTER 5

Transformers

Chapter 5: Transformers

5.1. Objective

This chapter aims to provide students with a clear understanding of transformers, their types, and practical applications. Students will learn to define and illustrate various transformers, analyze their equivalent circuits, and evaluate their efficiency. Emphasis will be placed on modeling transformers, understanding power losses, and identifying factors that influence performance.

5.2. Prerequisites

Before studying this chapter, students should have:

- Concepts of magnetic flux and magnetomotive force (MMF).
- Analogy between magnetic and electrical circuits.
- Basic principle of mutual and self-induction.
- Sinusoidal waveforms, RMS values, phasors, and impedance.
- Power in AC circuits (real, reactive, and apparent power).

5.3. Introduction

The transformer is an essential electrical device designed to transfer energy in the form of alternating current (AC) from a source to a load while modifying the voltage level. This voltage transformation can either increase (step-up) or decrease (step-down) depending on the intended application. The key mechanism enabling this voltage change is the creation of a magnetic field within the transformer core.

5.4. Transformer symbols

The figure 5.1 illustrates standard symbols used to represent single-phase transformers.

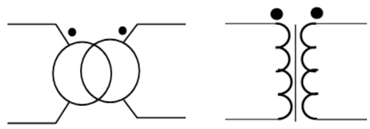


Figure 5.1. Standard symbols of single-phase transformers

5.5. Energy Transformation Stages

A single-phase transformer facilitates the transfer of energy from the primary side to the secondary side through different stages of transformation. The figure illustrates these steps, highlighting the associated power components and losses during the process.

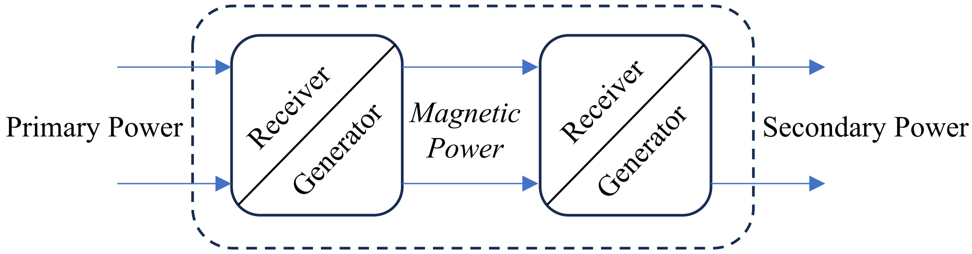


Figure 5.2. Powers presentation of single-phase transformer

The input power at the primary winding is given by the expression:

$$P_1 = U_1 \,.\, I_1 \cos(\varphi_1) \tag{5.1}$$

Part of this energy is dissipated as Joule losses in the primary winding.

The electrical energy is transformed into magnetic energy within the transformer core. However, some losses occur due to core phenomena such as hysteresis and eddy currents.

The magnetic energy induces a voltage in the secondary winding, converting back into electrical energy:

$$P_2 = U_2 \,.\, I_2 \cos(\varphi_2) \tag{5.2}$$

Secondary Joule losses are also present in this stage.

The efficiency of the transformer depends on minimizing losses at each stage. Key factors include optimized core design and high-quality materials to reduce hysteresis and eddy current losses.

5.6. Principle of Transformer

The operation of a transformer is based on the principle of electromagnetic induction. A variation of the magnetic flux through a coil induces an electromotive force e. Conversely, the presence of an EMF in a coil result in a variation of magnetic flux. This phenomenon is described by Faraday's law:

$$e = -\frac{d\emptyset}{dt} \tag{5.3}$$

For an ideal transformer, the primary winding with N_1 turns generate an EMF given by:

$$\begin{cases} e_1 = -N_1 \frac{d\emptyset}{dt} \\ u_1 = -e_1 \end{cases} \tag{5.4}$$

Assuming a sinusoidal voltage input $u_1 = U_1\sqrt{2}\,cos(\omega t)$, the rate of change of flux becomes:

$$\frac{d\emptyset}{dt} = \frac{U_1\sqrt{2}}{N_1}\,cos(\omega t) \tag{5.5}$$

This leads to the magnetic flux expression:

$$\emptyset = \frac{U_1\sqrt{2}}{\omega\,N_1}\,cos\left(\omega t - \frac{\pi}{2}\right) \tag{5.6}$$

The flux $\emptyset$ is defined as:

$$\emptyset = \int B\ dS \tag{5.7}$$

The magnetic field magnitude can be expressed as:

$$B = \frac{U_1\sqrt{2}}{\omega\,S\,N_1}\,cos\left(\omega t - \frac{\pi}{2}\right) \tag{5.8}$$

Thus, the peak magnetic field is:

$$B_{max} = \frac{U_1\sqrt{2}}{\omega\,S\,N_1} \tag{5.9}$$

Given that ω=2πf, the effective voltage becomes:

$$U_1 = 4.44\,N_1\,f\,B\,S \tag{5.10}$$

5.7. Ideal transformer

In an ideal transformer, all the magnetic flux generated by the primary winding passes through the secondary winding. This flux linkage ensures a proportional relationship between the voltages in the primary and secondary coils.

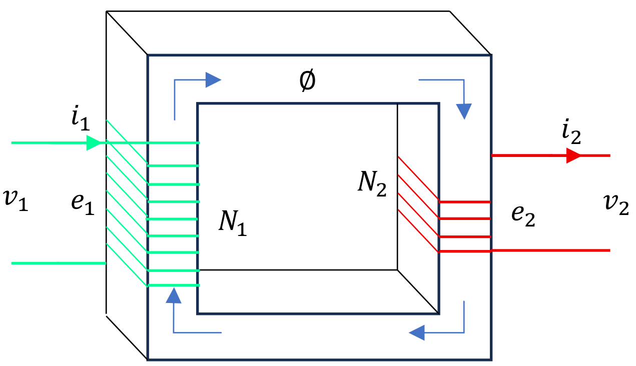


Figure 5.3. Input/Output of ideal transformer

The EMFs induced in the windings are defined as:

$$\begin{cases} v_1 = e_1 = -N_1\dfrac{d\emptyset}{dt} \\ v_2 = e_2 = -N_2\dfrac{d\emptyset}{dt} \end{cases} \tag{5.11}$$

From these expressions, the voltage ratio becomes:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \alpha \tag{5.12}$$

where α is the turns ratio.

The primary and secondary voltages u_1 and u_2 are in opposite phase due to the negative sign in the induced EMF equations. This phase opposition is inherent in the transformer's operation.

Now, let's connect a load by closing the switch in Figure 5.4, allowing a current i_2 to flow through the secondary winding. As a result, the secondary winding will generate a magnetomotive force (MMF) of $N_2 i_2$ in the core. This, in turn, will immediately induce a current i_1 in the primary winding, producing a counter-MMF $N_1 i_1$ to counteract $N_2 i_2$.

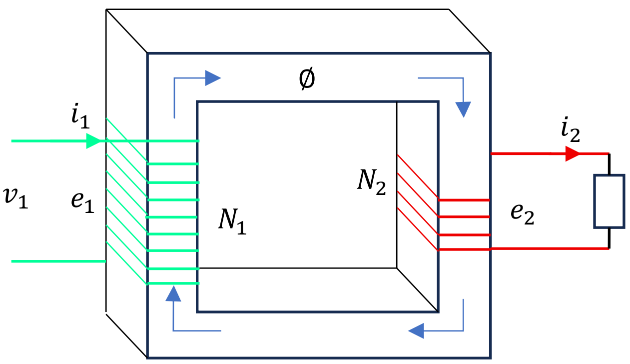


Figure 5.4. Transformer connect a load

$$N_1 i_1 = N_2 i_2 \tag{5.13}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{1}{\alpha} \tag{5.14}$$

$$V_1 i_1 = V_2 i_2 \tag{5.15}$$

5.8. Single-Phase Real Transformer

A real single-phase transformer differs from an ideal one due to inherent losses and non-ideal characteristics. In reality, the primary power P_1 is not equal to the secondary power P_2, and the efficiency η is always less than 1 due to various losses, including:

- **Joule losses** in the windings due to resistance.
- **Iron losses** in the magnetic core (hysteresis and eddy currents).
- **Mechanical losses** due to vibrations and noise.

Additionally, the magnetizing current is not zero even when the secondary is open-circuited, meaning the transformer requires reactive power to establish the magnetic field.

5.8.1. Input/Output Voltage Relationship

The voltage transformation ratio at no-load is given by:

$$m_v = \frac{V_{2,non-load}}{V_1} \approx \frac{N_2}{N_1} \quad (5.16)$$

Assuming an ideal transformer model for currents:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad (5.17)$$

5.8.2. Equivalent Circuit

A practical transformer can be represented using an equivalent circuit presented in Figure 5.5, where:

- R_1 and R_2 are the primary and secondary winding resistances.
- L_1 and $L_2$are the leakage inductances.
- The circuit can be further simplified by referring all parameters to the secondary side.

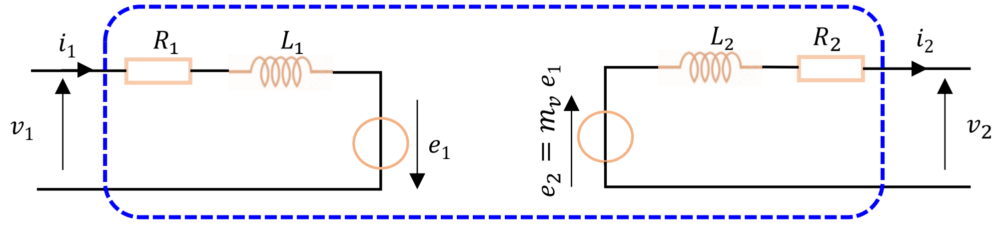


Figure 5.5. Equivalent circuit A real transformer

Using the transformed parameters:

$$R_s = R_2 + m_v^2\, R_1 \quad (5.18)$$

$$L_s = L_2 + m_v^2\, L_1 \quad (5.19)$$

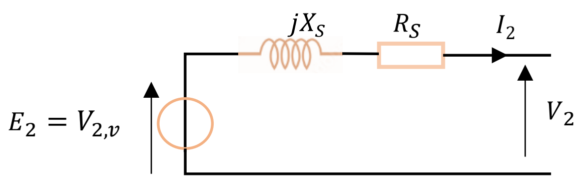


Figure 5.6. Equivalent circuit A real transformer with transformed parameters

By applying Kirchhoff's Voltage Law, the output voltage can be expressed as:

$$V_2 = V_{2,v} - (R_S + jX_S)I_2 \quad (5.20)$$

where X_S represents the leakage reactance.

This model helps in analyzing the voltage drop under load conditions and determining transformer efficiency.

5.9. Energy Balance and Efficiency

A real transformer undergoes several energy losses as it transfers electrical power from the primary to the secondary winding. The energy balance equation accounts for these losses and determines the efficiency of the transformer. The total electrical power absorbed by the transformer at the primary side is:

$$P_1 = U_1\, I_1\, cos(\varphi_1) \tag{5.21}$$

where,

U_1 and I_1 are the primary voltage and current, $cos(\varphi_1)$ is power factor at the primary side.

This absorbed power is distributed as follows:

Primary Copper Losses P_{j1}: Due to Joule heating in the primary winding resistance.

Iron Losses P_{ref}: Occur in the magnetic core and include:

- *Hysteresis losses:* Due to repeated magnetization and demagnetization of the core.
- *Eddy current losses:* Induced circulating currents in the core, proportional to B_{max}^2, U_1^2 and frequency f.

Secondary Copper Losses P_{j2}: Due to Joule heating in the secondary winding resistance.

Useful Output Power P_2: The actual power delivered to the load:

$$P_2 = U_2\, I_2\, cos(\varphi_2) \tag{5.22}$$

where,

- U_2 and I_2 are the secondary voltage and current,
- $cos(\varphi_2)$ is the power factor at the secondary side.

Considering all losses, the Power Balance Equation of the transformer is:

$$P_1 = P_{j1} + P_{fer} + P_{j2} + P_2 \tag{5.23}$$

The efficiency of a transformer η is the ratio of the useful power output to the total power input:

$$\eta = \frac{P_2}{P_1} \tag{5.24}$$

As we can use,

$$\eta = \frac{P_2}{P_1} \times 100\% \tag{5.25}$$

Since power losses reduce the efficiency, the efficiency equation can also be written as:

$$\eta = \frac{P_1 - (P_{j1} + P_{fer} + P_{j2})}{P_1} \tag{5.26}$$

Since transformers are designed to minimize losses, their efficiency is typically high, ranging from 95% to 99% in well-optimized designs.

5.10. Transformer types

5.10.1. Isolation Transformer

An isolation transformer is a special type of transformer where the turns ratio is equal to 1, meaning the primary and secondary voltages are identical. It is primarily used to provide galvanic isolation between circuits.

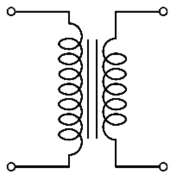


Figure 5.7. Example of isolation transformer

Applications:

- Prevents direct electrical connection between circuits, reducing risks of ground loops and electrical noise.
- Helps in adapting the neutral grounding scheme to match the installation requirements.
- Used in hospitals for patient safety and in industrial systems to isolate control circuits from high-voltage equipment.

5.10.2. Pulse Transformer

A pulse transformer is a type of transformer designed to transmit electrical pulses while providing galvanic isolation. It is primarily used for controlling thyristors and triacs in power electronics.

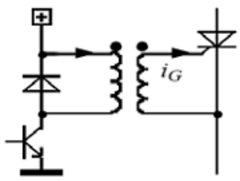


Figure 5.8. Example of pulse transformer

Applications:

- Used to provide isolated gate pulses to thyristors and triacs in power control circuits.
- Ensures that pulses retain their shape, amplitude, and timing while isolating the control and power circuits.
- Provides isolation to prevent damage from high voltages or electrical noise.

5.10.3. Autotransformer

An autotransformer is a type of transformer that has only a single winding, which acts as both the primary and secondary winding. Unlike conventional transformers, it does not provide galvanic isolation between the input and output.

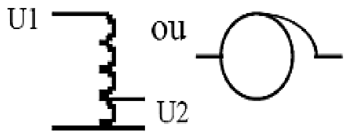


Figure 5.9. Example of autotransformer

Applications:

- Allows for fine tuning of the output voltage by shifting a sliding contact (cursor) along the winding.
- Since part of the winding is shared, it has a higher efficiency and smaller size compared to traditional transformers.

5.10.4. Mid-Point Transformer

A mid-point transformer is a type of transformer with a center-tapped secondary winding. This center tap provides a reference point that allows for dual polarity output, often used in rectifier circuits and balanced power distribution.

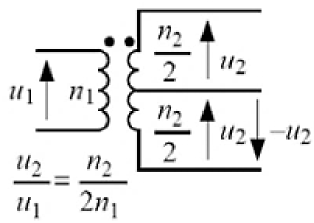


Figure 5.10. Example of mid-point transformer

Applications:

- Converts AC to DC with two diodes, providing a smoother DC output.
- Used in audio and communication systems for reduced noise.
- Provides ±V outputs for operational amplifiers and analog circuits.

5.10.5. Three-Phase Transformer

A three-phase transformer is an essential component in the generation, transmission, and distribution of electrical energy in three-phase alternating current (AC) systems. These transformers are widely used in power grids and industrial applications.

Magnetic Circuit and Construction

i. The magnetic circuit of a three-phase transformer is similar to that of a single-phase transformer but consists of three sets of primary and secondary windings.
ii. These windings are placed on a common magnetic core, often shaped like the "E" configuration.

Winding Configurations

A three-phase transformer can have different winding connections for both the primary and secondary sides: *Star (Y), Delta (D)* and *Zig-Zag (Z)*

The choice of configuration depends on the application, voltage levels, and load requirements. There are nine possible combinations of primary and secondary winding configurations: YY, YD, YZ, DD, DY, DZ, ZZ, ZY, ZD.

A capital letter represents the high-voltage side, while a lowercase letter represents the low-voltage side.

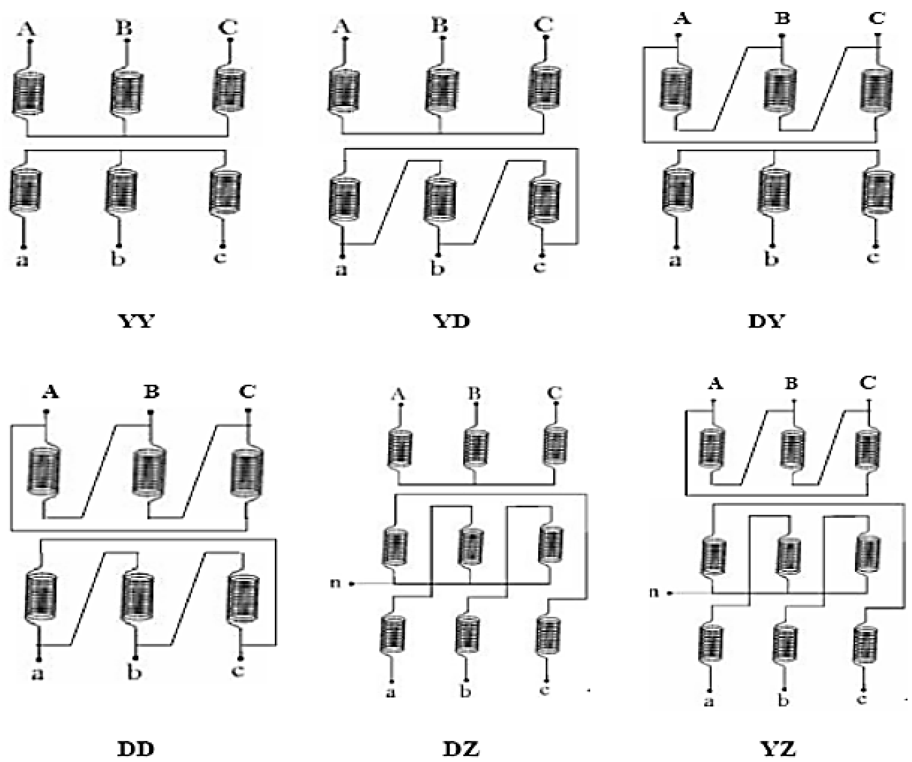


Figure 5.11. Example of three-phase transformer configurations

Applications:

- Power transmission and distribution networks.
- Industrial plants and factories with three-phase machinery.
- Electric motor drives in automation and manufacturing.

CHAPTER 6

Introduction to electrical machines

Chapter 6: Introduction to electrical machines

6.1. Objective

The objective of this chapter is to help students understand the general principles of electrical machines. By studying this chapter, students will learn the main parts of a machine and their functions, the basic operating principle in both motor and generator modes, and how to represent machines using equivalent circuit diagrams. They will also be able to analyze the power balance, identify the main sources of losses, and calculate the efficiency of a machine.

6.2. Prerequisites

Before starting this chapter, students should have acquired some basic knowledge from the previous five chapters, for example,

- Magnetic flux, flux density, reluctance, and the analogy with electrical circuits.
- Principle of electromagnetic induction (Faraday's law) and Laplace force.
- Active power, losses, and efficiency concepts.

6.3. Introduction

Electrical machines are fundamental devices in modern technology, ensuring the continuous exchange of energy between the electrical and mechanical domains. They are found everywhere, from industrial plants and transportation systems to household appliances and renewable energy sources.The essential role of electrical machines is to perform energy conversion: (i) As a generator, the machine converts mechanical energy into electrical energy, supplying power to external circuits. (ii) As a motor, it performs the reverse process, transforming electrical energy into mechanical energy, enabling the operation of countless mechanical systems.

Despite the diversity of machine types, all share a common structural principle composed of a magnetic field system, a rotating part, and a mechanism that ensures the interaction between current and magnetic flux. Understanding this structure and its operating principle is the first step toward analyzing their performance and evaluating their efficiency.

6.4. Machine Structure

An electrical machine, whether operating as a motor or a generator, is composed of essential parts that ensure the conversion of electrical energy into mechanical energy or vice versa.

The main components are the inductor (Stator), the armature (Rotor), and the commutation system (commutator). Each of these elements has a specific role in the functioning of the machine.

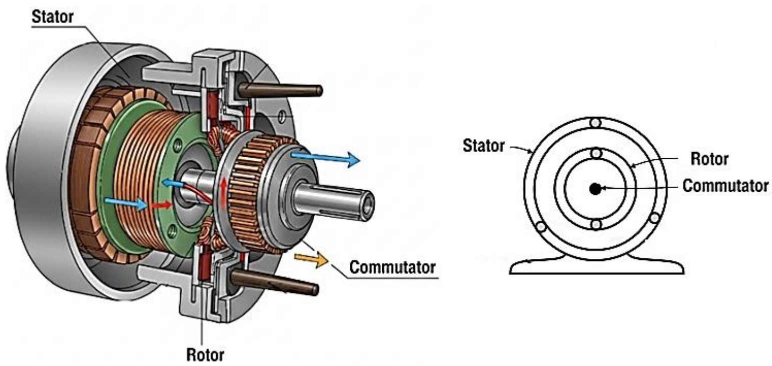


Figure 6.1. Parts of electrical machine

6.4.1. Inductor (Stator)

The inductor, also called the field system, is the part of an electrical machine that produces the magnetic field necessary for energy conversion.

Whether the machine operates as a motor or as a generator, the inductor plays the same fundamental role: it establishes the magnetic flux that interacts with the armature conductors. This field can be created by permanent magnets in small and simple machines, or more commonly by electromagnets consisting of coils supplied with current.

Figure 6.2. Examples of inductor

6.4.2. Armature (Rotor)

The armature is usually the rotating part, or rotor, of the machine. It is composed of a laminated magnetic core, designed to reduce eddy current losses, and conductors or windings placed in slots on its surface.

In operation, the role of the armature depends on whether the machine functions as a generator or as a motor.

(i) In a generator, the armature windings are cut by the magnetic flux, which induces an electromotive force.

(ii) In a motor, the armature windings carry current that interacts with the magnetic field, producing torque through the Laplace force.

The armature is mounted on a shaft, which serves as the mechanical interface, transferring power either to the external mechanical load in the case of a motor, or from the prime mover in the case of a generator.

Figure 6.3. Example of armature

6.4.3. Commutator

The commutator is a cylindrical structure composed of copper segments insulated from each other by mica. It is mounted on the shaft and rotates together with the armature.
Its main role is to ensure the mechanical rectification of current:

(i) In a generator, the commutator converts the alternating electromotive force (EMF) induced in the armature windings into a unidirectional (DC) voltage available at the output.

(ii) In a motor, it directs the current in the armature windings in such a way that the produced torque remains unidirectional.

The brushes, usually made of carbon or graphite, are stationary blocks that press against the surface of the commutator. Their function is to provide reliable electrical contact between the rotating commutator and the external circuit.

Important: Maintaining good contact between brushes and commutator is essential to minimize sparking and ensure proper current flow.

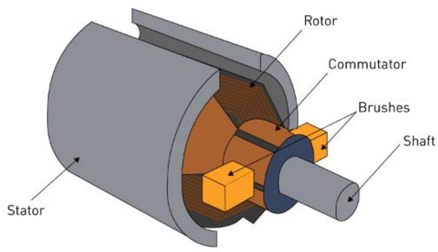


Figure 6.4. Important parts of commutator

6.5. Operating Principle

An electrical machine, whether functioning as a motor or a generator, is based on the principle of electromagnetic induction. According to Faraday's law, a changing magnetic field can induce a voltage in a conductor, while a conductor carrying current within a magnetic field is subjected to a force. All types of machines DC or AC, motors or generators depend on this interaction between magnetic fields and electric currents to achieve the conversion of energy from one form to another.

6.5.1. Motor

In motor operation, the machine transforms electrical energy into mechanical energy. Current is supplied to the rotor (armature) conductors, which are positioned within the magnetic field created by the stator.

The interaction between the current and the magnetic field generates forces on the conductors, as described by the Lorentz force law. These forces produce a torque that drives the rotor into rotation. The rotation is then transmitted to the shaft, where it can be used to power mechanical loads such as pumps, fans, or vehicles.

The direction of this force follows the right-hand rule, which links the orientation of the magnetic field, the current, and the resulting force.

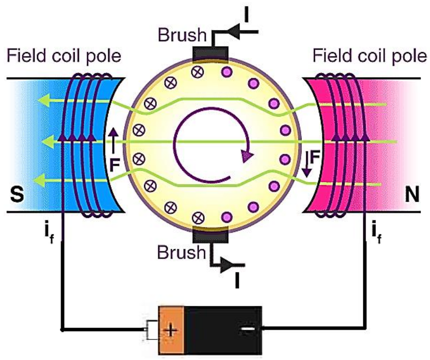


Figure 6.5. Operating principle of motor

6.5.2. Generator

In generator operation, the process is reversed: the machine converts mechanical energy into electrical energy.

A prime mover, such as a turbine or engine, rotates the rotor inside the stator's magnetic field. As the conductors on the rotor cut through the magnetic flux, a voltage or electromotive force is induced in them in accordance with Faraday's law. When the machine is connected to an external load, this voltage causes current to flow and supplies electrical power.

The direction of the induced current is given by the left-hand rule, while its magnitude depends on both the speed of rotation and the strength of the magnetic field.

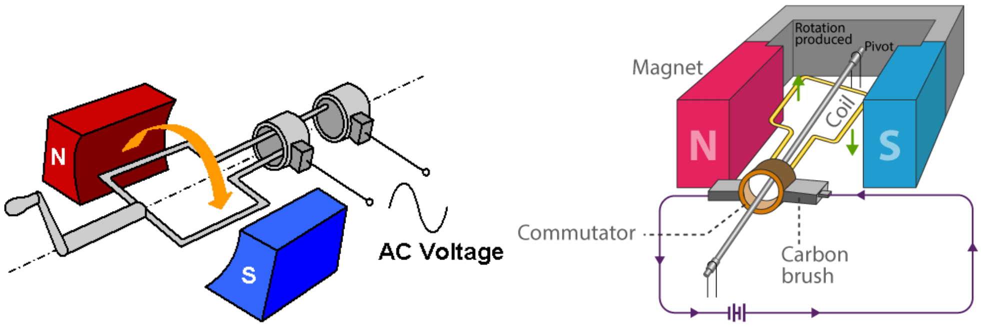


Figure 6.6. Operating principle of generator

6.6. Equivalent Circuit

When the stator and rotor windings are supplied with direct current (DC), there are no effects related to inductance or reactance, since these occur only with alternating currents. In this case, the only electrical component to be considered in the equivalent circuit is the ohmic resistance of the windings.

6.6.1. Equivalent Circuit of the Inductor

The equivalent circuit of the inductor, also known as the excitation winding, is a simplified model used to represent its electrical behavior.

The inductor is supplied with a direct voltage called the excitation voltage U_{ex}, which drives an excitation current J through the winding. This current is responsible for generating the magnetic flux required for the operation of the machine.

Since the winding is made of copper conductors, it has an electrical resistance known as the excitation resistance R_{ex}. Because the supply is direct current, the inductor does not exhibit inductive reactance, and its equivalent circuit reduces to a simple resistor. T

he relationship between the applied voltage, the resistance, and the current is expressed by Ohm's law as $U_{ex} = R_{ex}\, J$. Thus, the equivalent circuit of the inductor is represented by a resistance Rex across which the excitation voltage is applied, producing the excitation current that establishes the magnetic field of the machine.

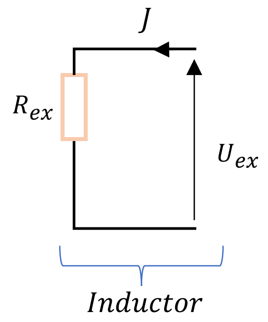


Figure 6.7. Equivalent Circuit of the Inductor

6.6.2. Equivalent Circuit of the Armature

The equivalent circuit of the armature represents its electrical behavior in both motor and generator operation. In either case, the armature winding carries an electric current called the armature current I. For a motor, this current enters the armature, while in a generator, it flows outward. The voltage across the terminals of the armature is denoted by U. Since the rotor always rotates within a magnetic field, the armature is the seat of an induced electromotive force E. This induced EMF opposes the applied voltage in the case of a motor (back EMF) and represents the generated voltage in the case of a generator. Therefore, the equivalent circuit of the armature consists of the applied terminal voltage U, the induced EMF E, and the resistance of the armature winding, through which the current I flows. This model allows us to analyze the energy conversion process by linking the electrical and magnetic phenomena occurring inside the machine.

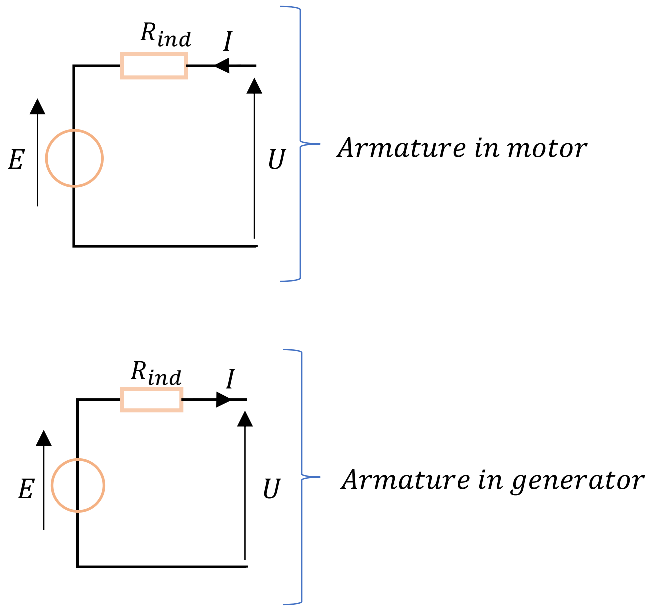


Figure 6.8. Equivalent Circuit of the armature in two cases, motor and generator

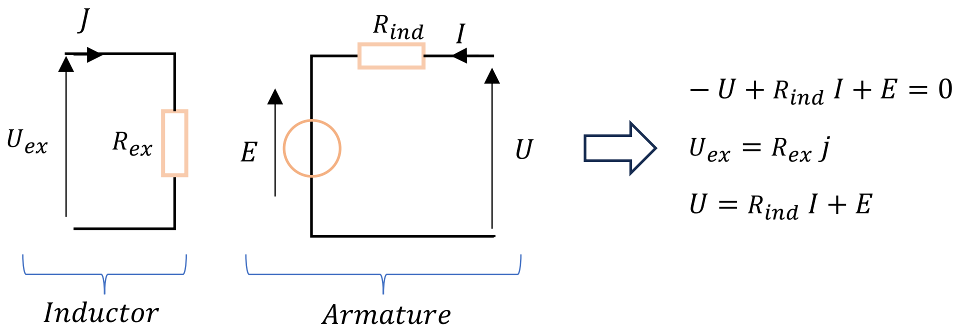


Figure 6.9. Global equivalent circuit of motor

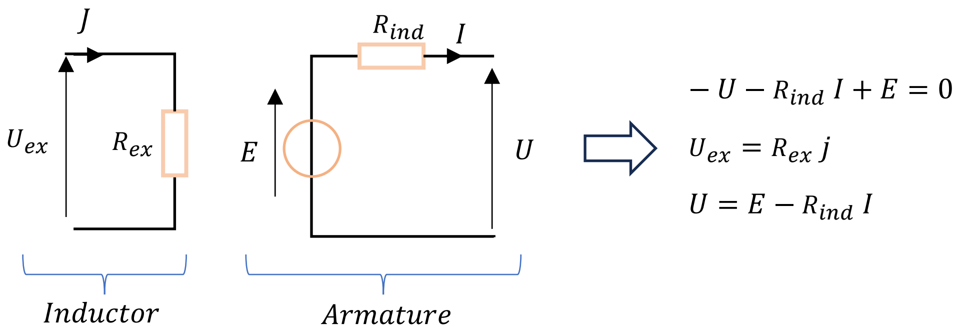


Figure 6.10. Global equivalent circuit of generator

6.7. Power Balance and Efficiency

6.7.1. Case of the Motor

An electric motor converts the absorbed electrical energy into mechanical energy to drive a mechanical load, such as a pump, a fan, or a propeller. However, during this conversion process, a portion of the absorbed energy is lost inside the machine in the form of heat, friction, and other parasitic effects.

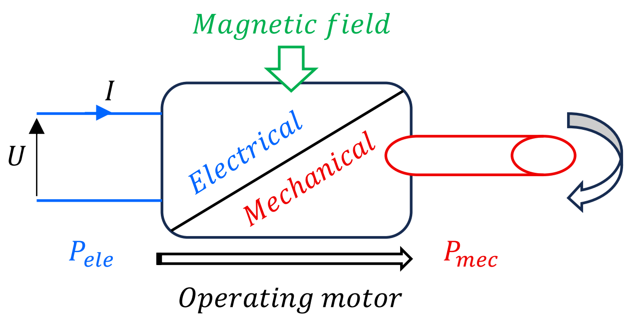


Figure 6.11. Input and output Powers in case of motor

The total electrical power (Absorbed Power) drawn from the supply, where U is the terminal voltage and I is the armature current.

$$P_{ele} = U\, I \tag{6.1}$$

The mechanical power (Useful Power) effectively delivered to the shaft, where T is the developed torque and Ω is the angular speed of the rotor.

$$P_{mec} = T\,\Omega \tag{6.2}$$

Losses in the Motor

a) Joule (copper) losses in the armature:

$$P_{J,Armature} = R_a\, I^2 \tag{6.3}$$

b) Joule (copper) losses in the inductor:

$$P_{J,Inductor} = R_f\, I_f^2 = U_f\, I_f \tag{6.4}$$

c) Collective or constant losses:

   a. Mechanical losses: due to friction in bearings, air resistance, vibrations, and ventilation.

   b. Iron (core) losses: resulting from hysteresis and eddy currents in the ferromagnetic material of the stator and rotor.

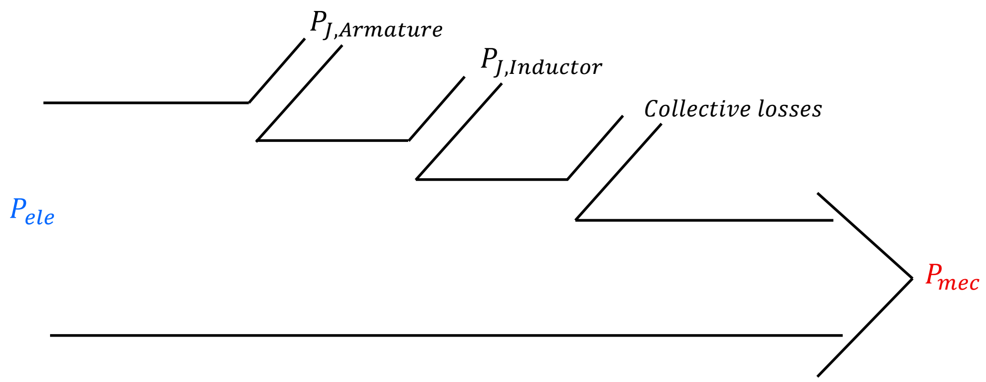


Figure 6.12. Powers Balance in case of motor

The efficiency expresses the ratio between the useful mechanical power and the absorbed electrical power:

$$\eta = \frac{P_{mec}}{P_{ele}} \tag{6.5}$$

6.7.2. Case of the Generator

A generator converts mechanical energy supplied by a prime mover (such as a turbine, an engine, or a motor) into electrical energy delivered to an external load. Similar to the motor, not all the absorbed mechanical power is transformed into useful electrical power, since part of it is dissipated inside the machine as losses.

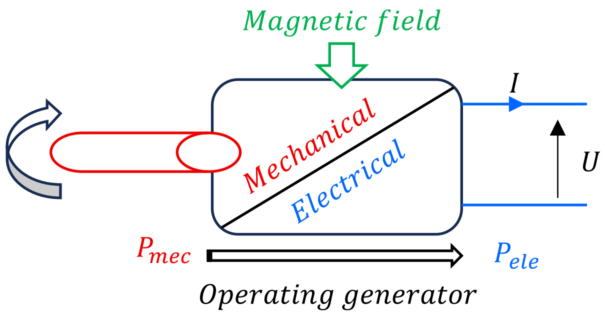


Figure 6.13. Input and output Powers in case of generator

The mechanical power provided to the generator shaft, where T is the torque applied by the prime mover and Ω is the angular speed.

$$P_{mec} = T\ \Omega \tag{6.6}$$

The electrical power delivered at the terminals, where U is the output voltage and I is the current supplied to the load.

$$P_{ele} = U\ I \tag{6.7}$$

Losses in the Generator:

a) Joule losses in the armature:

$$P_{J,Armature} = R_a\ I^2 \tag{6.8}$$

b) Joule losses in the inductor:

$$P_{J,Inductor} = R_f\ I_f^2 = U_f\ I_f \tag{6.9}$$

c) Collective losses:

   a. Mechanical losses: due to friction in bearings, vibrations, and ventilation.

   b. Iron losses: due to hysteresis and eddy currents in the ferromagnetic material.

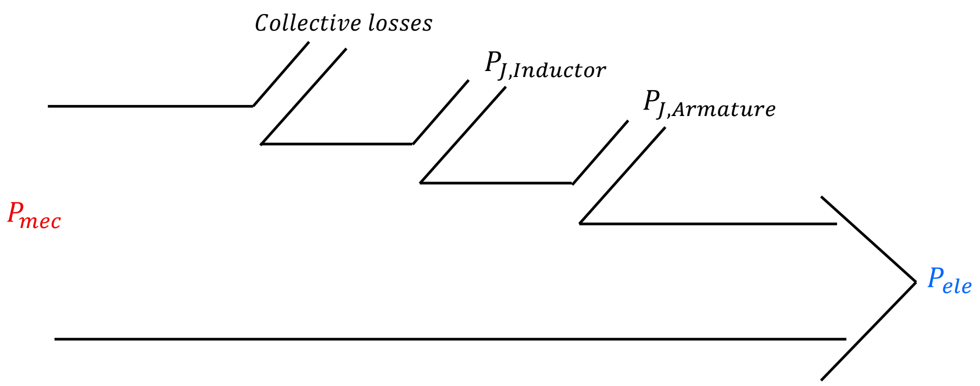


Figure 6.14. Powers Balance in case of generator

The efficiency expresses the ratio between the useful electrical power and the absorbed mechanical power:

$$\eta = \frac{P_{ele}}{P_{mec}}$$

Conclusion

This course has provided an overview of the fundamental principles of electricity and electrotechnics, progressively guiding students from the mathematical foundations to the practical aspects of machines and power systems.

Starting with the mathematical tools, such as complex numbers and their applications in AC circuit analysis, students developed the essential skills needed to understand electrical phenomena. The study of the basic laws of electricity and the behavior of resistive, capacitive, and inductive components built a strong foundation for analyzing circuits in both direct and alternating current regimes.

The course further expanded into single-phase and three-phase power systems, teaching students to evaluate active, reactive, and apparent powers, understand the power factor, and apply accurate measurement techniques. This knowledge was essential to bridge the gap between theoretical understanding and practical implementation.

The exploration of magnetic circuits and transformers introduced students to the principles of energy transfer and voltage transformation, which are at the heart of power generation and distribution systems.

Finally, the introduction to electrical machines offered a clear understanding of their structure, operation, and role in energy conversion, whether functioning as motors or generators. The analysis of power balance, losses, and efficiency equipped students with the tools to assess and optimize machine performance in real-world applications.

By completing this course, students have acquired a solid theoretical foundation and analyzing the electrical systems. This knowledge will serve as a cornerstone for further studies and professional growth in advanced areas such as power systems, automation, control, renewable energy, and electrical machinery.

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