People's Democratic Republic of Algeria

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

University of Blida1

Faculty of Technology

Department of Electrical Engineering

Electrical Engineering Fundamentals1

COURSE

Course Content

I. Mathematical Review of Complex Numbers

II. Electrical Circuits

III. Electrical Circuits and Power

IV. Magnetic Circuits

V. Transformers

VI. Introduction to Electrical Machines

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Academic Program

2nd Year undergraduate students in Electrical Engineering

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Foreword

The content of this course is fully aligned with the canvas of the Department of Electronics and is specifically designed for second-year undergraduate students (LMD system) specializing in Electrical Engineering.

This course begins with a comprehensive review of fundamental mathematical concepts, with particular emphasis on complex numbers and the basic principles of electricity. It then advances to the analysis of electrical circuits operating under direct current (DC), single-phase alternating current (AC), and three-phase AC systems.

Magnetic circuits and electrical transformers are presented in a clear, structured, and pedagogical manner, enabling students to develop a solid understanding of their operating principles and practical applications.

The course concludes with technological descriptions and practical insights into the different types of electrical machines, providing students with essential knowledge for both academic study and engineering practice.

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Introduction

Electrotechnics is one of the fundamental disciplines of electrical engineering, concerned with the study, analysis, and application of electrical and magnetic phenomena in physical systems. It forms the theoretical and practical backbone of numerous engineering fields, including power systems, electronics, telecommunications, and industrial automation. Mastering the principles of electrotechnics is therefore essential for any engineer or technician working in these domains.

This course, Fundamentals of Electrotechnics I, is designed for second-year undergraduate students specializing in Electronics and Biomedical Engineering. It provides a rigorous and structured introduction to the key concepts governing the behavior of electrical circuits, magnetic circuits, transformers, and electrical machines. The course builds upon the mathematical foundations of complex numbers and extends them to real-world electrical engineering applications.

Course Objectives

By the end of this course, students will be able to:

- Manipulate complex numbers in their various representations and apply them to sinusoidal electrical quantities.
- Analyze DC and AC electrical circuits using Kirchhoff's laws and fundamental theorems.
- Compute instantaneous, active, reactive, and apparent power in single-phase circuits.
- Understand and analyze magnetic circuits, electromagnetic induction, and inductance.
- Study the operating principles, equivalent circuits, and energy balance of single-phase transformers.
- Describe the structure, operating modes, and characteristics of direct current (DC) machines.
-

Course Structure

The course is organized into six chapters, each addressing a specific area of electrotechnics:

- Chapter I — Mathematical Review of Complex Numbers: algebraic, trigonometric, and exponential forms; arithmetic operations; application to sinusoidal signals and complex impedance.
- Chapter II — Electrical Circuits (DC & AC): dipoles, Kirchhoff's laws, series and parallel circuits, steady-state regime analysis.
- Chapter III — Electrical Circuits and Power: single-phase circuits, instantaneous, active, reactive, and apparent power, Boucherot's theorem.
- Chapter IV — Magnetic Circuits: magnetic quantities, Ampere's theorem, calculation of homogeneous and parallel magnetic circuits, inductance, and the magnetic-electrical analogy.
- Chapter V — Transformers: ideal and real transformer models, equivalent circuits, efficiency, voltage regulation, and practical types (isolation, pulse, autotransformer, three-phase).
- Chapter VI — Introduction to Electrical Machines: structure and operating principles of DC machines, equivalent circuits, EMF, torque expressions, and power balance inmotor and generator modes.

Each chapter is complemented by worked examples and exercises designed to reinforce understanding and develop problem-solving skills. Students are encouraged to actively engage with these exercises, as they reflect the type of problems encountered both in examinations and in professional practice.

Chapter I

Mathematical Review

Of

Complex Numbers (CN)

Chapter I

Mathematical Review of Complex Numbers (CN)

I.1 Definition of a Complex Number

Complex numbers extend the set of real numbers. The equation $x^2+1=0$ has no solution within the real domain R, since it requires the square root of a negative number. This necessity naturally leads to the introduction of complex numbers. In the complex domain C, this equation admits two roots: j and −j.

A complex number is composed of a real part and an imaginary part, and can be represented as a point with two coordinates in a plane known as the *complex plane*.

The applications of complex numbers are numerous, particularly in electromagnetism and electronics, where their trigonometric form greatly simplifies the mathematical treatment of wave-related phenomena.

I.2 Different Forms of a Complex Number

I.2.1 Cartesian Representation

A complex number z can be expressed in its algebraic form as*:*

$$Z = a + jb \ (a, b \in R, j^2 = -1) \qquad (1.1)$$

Here, the first term *Re(Z)= a* represents the *real part*, while the second term *Im(Z)=b* represents the imaginary part.

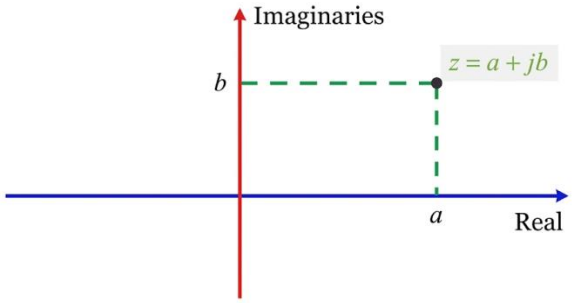


Figure.I.1 Cartesian Representation of a Complex Number

I.2.2 Geometric Representation

In a plane equipped with an orthonormal coordinate system $(O, \vec{u}, \vec{v})$, every complex number $z = a + jb$ is associated with a point $P(a, b)$. Conversely, to every point in the plane one can associate a unique complex number.

The point $P(a, b)$ is the image of the complex number $z = a + jb$, and z is the affix of the vector $\overrightarrow{OP} = a\vec{u} + b\vec{v}$

I.2.3 Trigonometric Representation

For the point *P*, which is the image of the complex number $z = a + jb$ in the plane withrespect to the orthonormal frame $(O, \vec{u}, \vec{v})$:

- **Themodulus** of z is the positive real number $r = |z| = \sqrt{a^2 + b^2}$
- **Theargument** of $z(z \neq 0)$ is the angle θ, defined up to an additive multiple of $2k\pi(k \epsilon z)$, such that:

Thus, the **trigonometric form** of a complex number z is expressed as: $z = r(\cos\theta + j\sin\theta)$

Geometrically, θ is, up to an additive multiple of 2kπ, the measure of the angle between the vector $\vec{u}$ and the vector $\overrightarrow{OP}$. Therefore, the trigonometric form of the complex number *z* is given by:

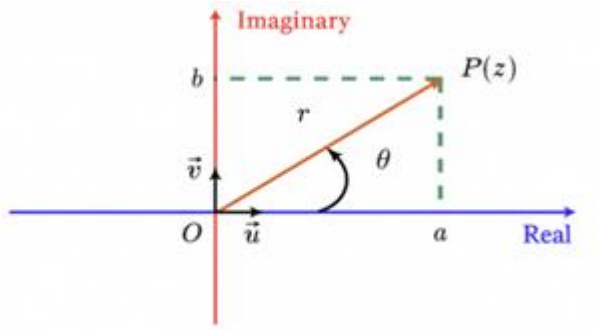


Figure.I.2 Trigonometric Representation of a Complex Number

Moivre's Theorem

The n-th power of a complex number z is given by De Moivre's formula:

$$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$$

Example:

Consider a complex number given in algebraic form: z = 3 + j4. Its geometric representation is illustrated in the figure below.

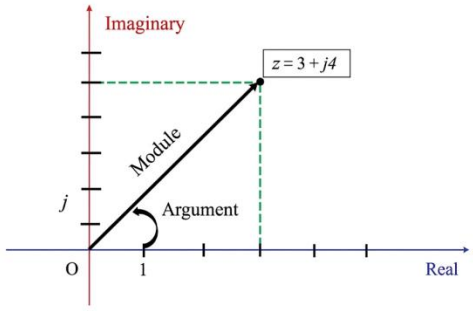


Figure.I.3 Geometric Representation of CN: z=3+j4

The modulus of z is a positive real quantity: r = $\sqrt{3^2+4^2}$ = 5

The argument of z is the angle given by: arg (z) = $(3 + j4)$ = $tan^{-1}\,(4/3) \approx 0.92\ rad \approx 53.13°$

I.2.4 Exponential Representation

Every complex number can also be expressed in the form: z =[r, Θ] known as the *polar form*.

Euler's formula establishes the fundamental relation between the complex exponential and the trigonometric functions in the complex plane:

$\forall\theta\epsilon$ R; $e^{j\theta} = cos\theta + jsin\theta$

Euler's Theorem

For any real number θ, Euler's identities are expressed as:

$$\begin{cases} \cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j} \end{cases}$$

Thus, a complex number may be written in exponential form as: $z = re^{j\theta}$

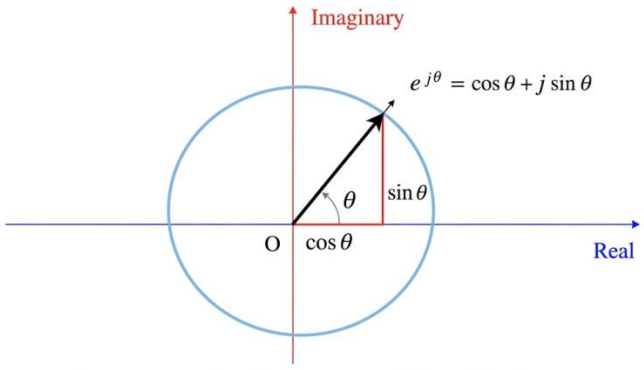


Figure.I.4 Exponential Representation of a Complex Number

Example :

Linearizing a Trigonometric Polynomial Using the Binomial Theorem

We are given: $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$$\cos\Theta^4 x = \left(\frac{e^{jx} + e^{-jx}}{2}\right)^4 = \frac{(e^{jx} + e^{-jx})^4}{16}$$

$$\cos\Theta^4 x = \frac{e^{4jx} + 4e^{3jx}e^{-jx} + 6e^{2jx}e^{-2jx} + 4e^{jx}e^{-3jx} + e^{-4jx}}{16} = \frac{e^{4jx} + e^{-4jx}}{16} + \frac{e^{2jx} + e^{-2jx}}{4} + \frac{3}{8}$$

$$= \frac{1}{8}\left(\frac{e^{4ix} + e^{-4ix}}{2}\right) + \frac{1}{2}\left(\frac{e^{2ix} + e^{-2ix}}{2}\right) + \frac{3}{8}$$

$$\cos\Theta^4 x = \frac{1}{8}\cos 4x + \frac{1}{2}\cos 2x + \frac{3}{8}$$

I.3 Arithmetic Operations on Complex Numbers

Let us consider two complex numbers:

$$Z = a + jb \text{ and } Z' = a' + jb' \ (a \epsilon R\,;\ a' \epsilon R\,;\ b \epsilon R\,;\ b' \epsilon R).$$

I.3.1 The Sum

The real part of the sum of two complex numbers is equal to the sum of their real parts, andthe imaginary part is equal to the sum of their imaginary parts:

$$z+z' = (a+a') + j(b+b')$$

I.3.2 The Product

The product of two complex numbers is given by:

$$z.z' = (a.a' - b.b') + j(a.b' + a'.b)$$

The modulus of the product is equal to the product of the moduli:

$$|z.z'| = |z| \times |z'|$$

The argument of the product is equal to the sum of the arguments:

$$arg(z \times z') = arg\ z + arg\ z'.$$

Thus, in trigonometric form, the product is expressed as:

$$z.z' = r.r'(\cos(\Theta + \Theta') + j\sin(\Theta + \Theta')) .$$

I.3.3 The Quotient

To obtain the algebraic form of the quotient of two complex numbers, the numerator anddenominator are multiplied by the complexconjugate of the denominator.

$$\frac{z}{z'} = \frac{a+jb}{a'+jb'} = \frac{(a+jb)+(a'-jb')}{(a'+jb')(a'-jb')} = \frac{(aa'+bb')+j(a'b-ab')}{a^2+b^2}$$

The modulusofthequotient is equal to the quotient of the moduli:

$$\left|\frac{z}{z'}\right| = \frac{|z|}{|z'|}$$

The argumentofthequotient is equal to the difference of the arguments:

$$arg\frac{z}{z'} = argz - argz'(z \neq 0\ etz' \neq 0).$$

$$\frac{Z}{Z'} = \frac{r}{r'}(\cos(\theta-\theta') + j\sin(\theta-\theta'))$$

I.4 The Conjugate of a Complex Number

The complexconjugate of $z = a + jb$ denoted by $\bar{z}$. It represents the same number with theimaginary part having the opposite sign: $\bar{z} = a - jb$

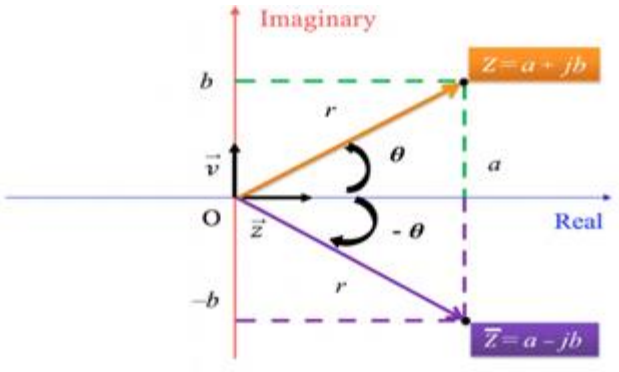


Figure.I.5 Conjugate Complex Number

- The product of a complex number and its conjugate is a real number equal to the square of their common modulus: $z.\bar{z} = a^2 + b^2 = |z|^2 = |\bar{z}|^2$
- The conjugate of a sum is equal to the sum of the conjugates:

  if$z = a + jb$ and $z' = a' + jb'$ then$\overline{z + z'} = \bar{z} + \bar{z'} = (a + a') - j(b + b')$

- The conjugate of a product is equal to the product of the conjugates :

$$\overline{z.z'} = \bar{z}.\bar{z'} = (a.a' - b.b') - j(a'b + ab')$$

I.5 Square Root of a Complex Number

Definition: Let Z be a given complex number. A *complex square root* of Z is any complex number z such that: $z^2=Z$

The simplest way to determine the square roots of a complex number Z written in algebraic form a+jb is to set z=x+jy and then solve the resulting system of two equations with two unknowns.

$$z^2 = Z <=> \begin{cases} z^2 = Z \\ |z|^2 = |Z| \end{cases} <=> \begin{cases} (x+iy)^2 = a + bi \\ x^2 + y^2 = \sqrt{a^2 + b^2} \end{cases}$$

$$<=> \begin{cases} x^2 - y^2 + 2xyi = a + bi \\ x^2 + y^2 = \sqrt{a^2 + b^2} \end{cases} <=> \begin{cases} x^2 - y^2 = a \\ 2xy = b \\ x^2 + y^2 = \sqrt{a^2 + b^2} \end{cases}$$

Example:

We wish to determine the square roots of 3+4i

$$z^2 = 3 + 4i <=> \begin{cases} x^2 - y^2 = 3 \\ 2xy \\ x^2 + y^2 = \sqrt{3^2 + 4^2} \end{cases} <=> \begin{cases} x^2 - y^2 = 3 \\ 2xy = 4 \\ x^2 + y^2 = 5 \end{cases}$$

$$<=> \begin{cases} x^2 = 4 \\ 2xy = 4 \\ x^2 + y^2 = 5 \end{cases} <=> \begin{cases} x = 2 \\ y = 1 \end{cases} ou \begin{cases} x = -2 \\ y = -1 \end{cases}$$

It follows that there are two square roots of 3+4i : - 2 - i et 2 + i

I.6 Application of Complex Numbers to Electricity

I.6.1 Sinusoidal Quantities (Voltages and Currents)

For each sinusoidal quantity expressed in the general form:

$$u(t) = U_{max}\sin(\omega t + \varphi) = \sqrt{2}Usin(\omega t + \varphi)$$

One associates a Fresnel vector. Thus, the corresponding complex representation of u(t) is:

$$u(t) = [U, \varphi] = U(cos\varphi + jsin\varphi)$$

Similarly, for the electric current : $i(t) = I\sqrt{2}Usin(\omega t + \varphi)$

$$i(t) = [U, \varphi] = i(cos\varphi + jsin\varphi)$$

$U = \frac{U_{MAX}}{\sqrt{2}}$ and $I = \frac{I_{MAX}}{\sqrt{2}}$ represent the effective (RMS) values of the voltage and current,respectively.

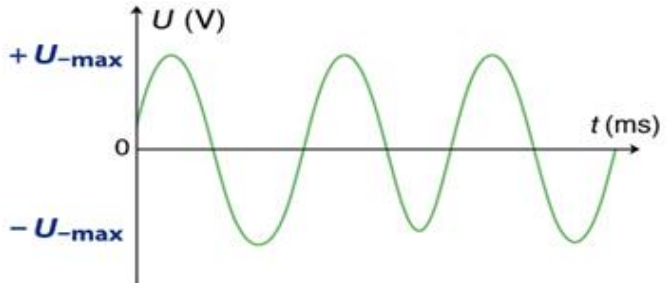


Figure.I.6 Representation of Sinusoidal Quantities

$$T : Period$$

$$f = \frac{1}{T} \qquad \omega = 2\,(frequency)$$

$$\omega = 2\pi f \qquad \omega = 2\,\pi f(angular\ frequency)$$

I.6.2 Complex Impedance of a Dipole

Consider a dipole of impedance Z, subjected to a sinusoidal alternating voltage u(t) and carrying a current of intensity i(t)

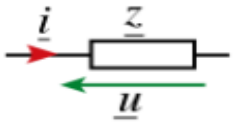


Figure.I.7 Complex Impedance of a Dipole

The **complex impedance** of the dipole is defined as the complex number:

$\underline{Z}=\underline{U}\underline{I}$, where $\underline{U}$ and $\underline{I}$ are the complex quantities associated with the voltage u(t) and the current i(t) respectively.

Exercises

Exercise 01

Let the complex number z with modulus r, argument θ, and complex conjugate $\underline{z}$.

1. Calculate the inverse of z=3+j4.
2. Find the solution of $2z+\underline{z}=9+j$.
3. What is the modulus of $z = \frac{-\sqrt{2}}{1+j} e^{i\frac{\pi}{3}}$?
4. What is the argument of $\frac{-1+j\sqrt{3}}{Z}$?

Exercise 02

Given: $z1 = 1 + j\sqrt{3}$ and $z2 = \sqrt{3} - j$

1. Write each of the complex numbers z_1 and z_2 in trigonometric form.
2. Write z_1^{21} in trigonometric form, then calculate its value.
3. For $z=z_1+z_2$, deduce the modulus and argument of z.

Exercise 03

1. Use Euler's formulas to transform the following expression into a sum: f(x)=sin(2x)sin(3x)
2. Linearize : $g(x)=\sin^3(2x)$

Exercise 04

Let the complex number be z=1−j. By applying Moivre's Theorem, calculate $(1-j)^{20}$.

Exercise 05

A current $i(t) = 2\sqrt{2}\sin(314t + \frac{\pi}{6})$ flows through a dipole when a voltage

$u(t) = 220\sin(314t + \frac{\pi}{6})$ is applied to it. Determine the impedance of this dipole.

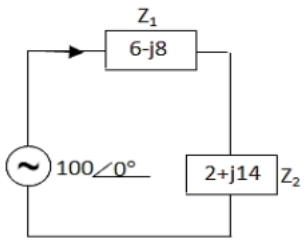
Exercise 06

A sinusoidal voltage U(t)=1.3 sin(ωt+0.3) is applied to an electrical impedance

Z=49.36-j31.19

- Write the expression for the impedance in polar form.
- Give the expression for the current I(t) that flows through this impedance.

Exercise 07



Consider the circuit in the figure beside

- Calculate the equivalent impedance $Z_{eq} = Z_1 + Z_2$
- Calculate the current I that flows through the circuit.

Exercise 08

A resistor and a coil are connected in parallel.

- Give the expression for the equivalent impedance. Calculate the expression for the impedance in Ohms (the modulus of the impedance).
- Calculate the phase shift between the voltage and the current (the argument of the impedance).

Exercise 09

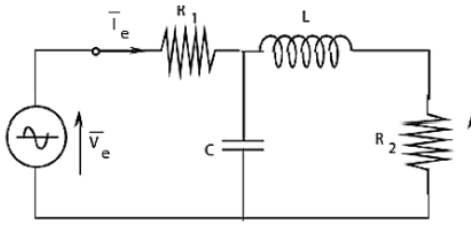
A resistor, an inductor, and a capacitor are connected in series.

- Give the expression for the real part of the equivalent impedance Zeq. The reactance X corresponds to the imaginary part of the complex impedance Z.
- Give the expression for the reactance X. The impedance of the series connection (in ohms) is the modulus of the complex impedance.
- Give the expression for the modulus of the impedance. The phase shift of the current relative to the voltage is given by the argument of the impedance. Give the expression for the phase shift.

Exercise 10

Consider the RLC circuit shown in the figure.

Given the impedance values at 1 kHz: R=20 Ω, R=50 Ω, Lω=25 Ω, 1/Cω=40 Ω.



Chapter II

Review of the Fundamental Laws of Electricity

Chapter II

Review of the Fundamental Laws of Electricity

II.1 Electrical Circuits

An electrical circuit is made up of a set of components (elements or linear dipoles) interconnected. In general, the circuit contains at least one voltage or current source, passive components, and possibly one or more active components.

II.2 Steady-State Regime

In the steady-state or direct current (DC) regime, current and voltage quantities are constant or time-independent. In this specific regime, inductors act as short circuits, and capacitors act as open circuits.

II.3 Electrical Dipole

An electrical dipole is a single component or a set of components connected to two terminals.

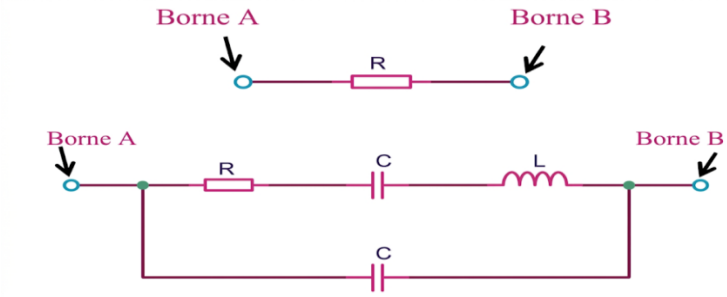


Figure.II.1 Electrical Dipoles

II.3.1 Passive Dipole

A passive dipole is defined as a two-terminal element that consumes electrical energy but does not provide any energy source. Typical examples include resistors, inductors, and lamps.



Figure.II.2 Passive Dipoles

II.3.2 Active dipole:

It is a dipole that contains an energy source, such as a battery or a direct current motor.



Figure.II.3 Active Dipoles

II.4 Kirchhoff's Laws

II.4.1 Node Law (Kirchhoff's Current Law – KCL):

The sum of the current intensities entering a node is equal to the sum of the current intensities leaving the node.

$$I1 + I2 + I3 + I4 = 0$$

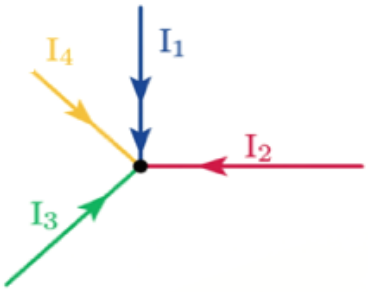


Figure.II.4 Node Law (Kirchhoff's Current Law)

II.4.2 Loop Law (Kirchhoff's Voltage Law – KVL):

The sum of the voltages oriented along a closed loop of an electrical circuit is equal to zero.

$$V1 - V2 - V3 + V4 = 0$$

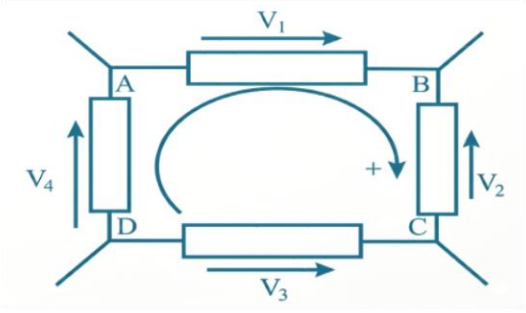
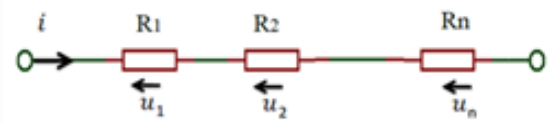


Figure II.5 Loop Law (Kirchhoff's Voltage Law – KVL)

II.5 Association of Elementary Dipoles R, L, and C

II.5.1 Series Connection:

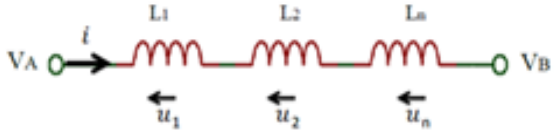
Dipoles are connected in series when they are arranged one after another. In this case, the current i *(t)* is common to all dipoles, and the voltage *u (t)* is the sum of the voltages across each dipole.



$$u(t) = R.i(t)$$

$$u = u_1 + u_2 + u_3 + \cdots + u_n$$

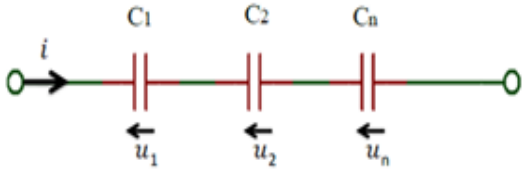
$$= (R_1 + R_2 + R_3 + \cdots + R_n)i = R_{eq}i$$



$$u(t) = L.di(t)/dt$$

$$u = u_1 + u_2 + u_3 + \cdots + u_n =$$

$$(L_1 + L_2 + L_3 + \cdots + L_n)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$



$$u(t) = 1/C.\int i(t)\,dt$$

$$u = u_1 + u_2 + u_3 + \cdots + u_n$$

$$= \frac{1}{c_1}\int i.dt + \frac{1}{c_2}\int i.dt + \frac{1}{c_3}\int i.dt + \cdots$$

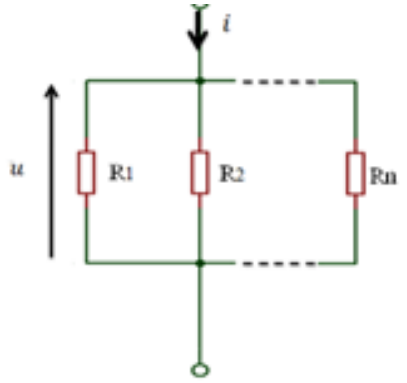
$$+ \frac{1}{c_n}\int i.dt$$

$$= (\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \cdots + \frac{1}{c_n})\int i.dt$$

Figure.II.6 Series Connection of R, L, C Dipoles

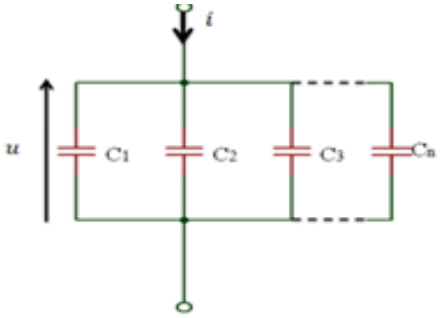
II.5.2 Parallel Connection:

The voltage *u(t)* is common to all dipoles, and therefore the total current *i(t)* is the sum of the currents through each dipole.



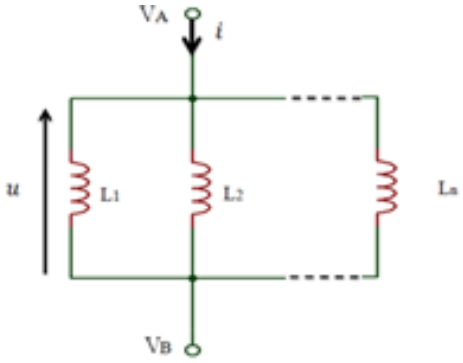
$$= \frac{u}{R_1} + \frac{u}{R_2} + \frac{u}{R_3} + \cdots + \frac{u}{R_n}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right) u$$



$$i = i_1 + i_2 + i_3 + \cdots + i_n$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} = \sum_1^n \frac{1}{L_n}$$



$$= c_1 \frac{du}{dt} + c_2 \frac{du}{dt} + c_3 \frac{du}{dt} + \cdots + c_n \frac{du}{dt}$$

$$= (c_1 + c_2 + c_3 + \cdots + c_n) \frac{du}{dt}$$

Figure.II.7 Parallel Connection of R, L, C Dipoles

II.6 Harmonic (Sinusoidal) Regime

Today, all power networks operate with alternating currents and voltages of sinusoidal form. Sinusoidal quantities are particular periodic quantities, whose study is important in electronics and electrical engineering. The objective of studying the behavior of dipoles in the harmonic regime is to determine the nature of the associated impedances.

Classically, two types of variable regimes are distinguished, i.e., those in which electrical quantities depend on time: transient regimes and periodic regimes.

II.6.1 Representation of Sinusoidal Quantities

In electrical engineering, electrical loads are almost always connected to the terminals of the same source supplying a sinusoidal voltage *u(t)*, which is characterized by its root mean square (RMS) value. The objective is to express the sinusoidal quantity in its analytical time-domain form.

The real representation of sinusoidal quantities (current and voltage) is given by:

$$u(t) = U_M \,.\, sin(\omega t + \varphi u)$$

$$i(t) = I_M \,.sin(\omega t + \varphi i)$$

where**:**

- U_M and I_M: Maximum values of u(t) and i(t), respectively
- ωt+φ: Instantaneous phase
- ω: Angular frequency in rad/s, with ω=2πf f: Frequency in Hz
- T: Period in seconds
- φ: Initial phases of u(t) and i(t), respectively
- φ=φu−φi: Phase difference between u(t) and i(t)

II.6.2 Average and RMS Values

- **Average Value:**

A periodic function u(t) of period T has an average value U_{avg} given by:

$$U_{avr} = \frac{1}{T}\int_0^T u(t)dt$$

The average value of a sinusoidal signal is zero.

- **RMS Value:**

In the electrical domain, the RMS (root mean square) value of an alternating voltage is equal to the value of a DC voltage that would produce the same heating effect in the same resistance.

For a periodic function u(t) of period T, the RMS value is given by:

$$U_{RMS} = \frac{1}{T}\int_0^T u(t)^2\,dt = \frac{U_m}{\sqrt{2}}$$

II.6.3 Vector Representation of a Sinusoidal Quantity

A sinusoidal quantity (current or voltage) can also be represented by a rotating vector in the Oxy plane, rotating at an angular velocity ω\omegaω in the counterclockwise direction. This is known as the Fresnel vector associated with the sinusoidal quantity.
Depending on the type of circuit considered (resistive, capacitive, or inductive), currents and voltages exhibit different phase shifts with respect to a given reference.
To simplify the representation of Fresnel vectors, they are generally drawn at t=0, which does not alter the final result.

The magnitude of the Fresnel vector of the quantity u(t)) is equal to its RMS value.

$$u(t) = \sqrt{2}U_{RMS}.\sin(\omega t + \varphi)$$

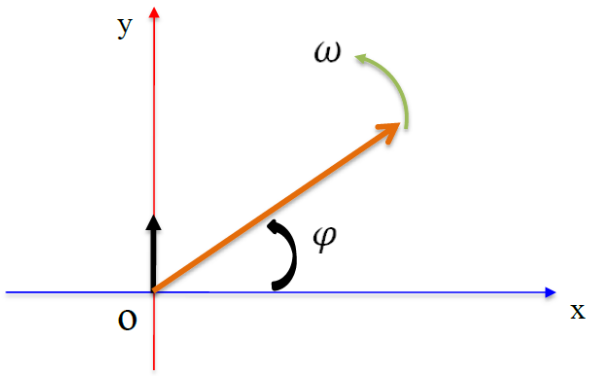


Figure.II.8 Fresnel Vector Representation

Example:

Let two vectors be: $u(t) = 5\sqrt{2}.\sin \omega t\ i(t) = 2\sqrt{2}.\sin(\omega t - \frac{\pi}{6})$

Give the Fresnel vector representation for the two quantities u(t)u(t)u(t) and i(t)i(t)i(t).

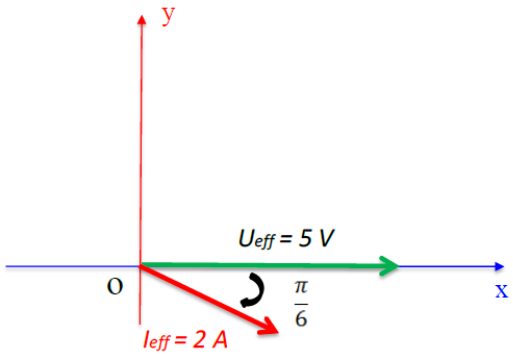


Figure.II.9 Fresnel Vector Representation (Example)

II.6.4 Complex Impedances

Electrical impedance measures the opposition of an electrical circuit (or dipoles) to the flow of an electric current.

In the sinusoidal regime, the loop equations become differential equations, whose resolution becomes complicated for circuits containing more than one or two receivers. Therefore, it is necessary to use a specific notation and methodology.

The complex impedance $\underline{Z}$ of a dipole in sinusoidal steady-state operation is the ratio of the complex voltage $u(t) = U_{max}\, e^{j(wt+\varphi u)}$ to the complex current $i(t) = I_{max} e^{j(wt+\varphi i)}$

We note:

- The complex impedance of a dipole: $\underline{Z} = \underline{U} / \underline{I}$

The impedance magnitude is Z, expressed in ohms (Ω).

- The complex admittance: $Y = \frac{I}{U}$

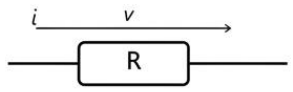
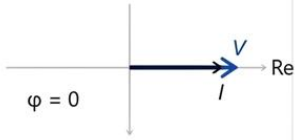
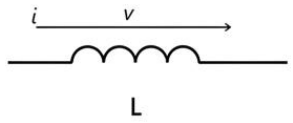
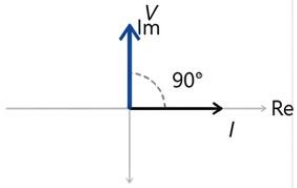
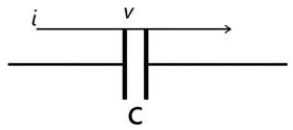
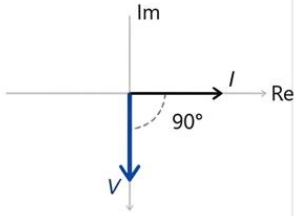
The admittance magnitude is Y, expressed in Siemens (S).

- Complex impedances are complex numbers. Classically if $Z = R + jX$

Then R represents the series resistance of the impedance and X its series reactance.

- Complex impedances follow the same combination rules as resistances.

II.6.5 Application to Linear Dipoles

Component	Phasor diagram	Impedance	Phase angle
		Z = R purely real	φ = 0 V and I in phase
		Z = jLω purely imaginary	φ = +π/2 V leads I by 90°
		Z = 1 / jCω purely imaginary	φ = −π/2 V lags I by 90°

Circuit Diagram — Fresnel Vectors — Complex Impedance — Phase Shift

II.7 Electric Power

The concept of electric power is a fundamental tool in electrotechnics. It not only makes it possible to obtain an overall view of the energy behavior of a system, but also to solve certain problems more easily using the power balance method.

The power consumed by the dipole is expressed as: $P = k.U_{eff}.I_{eff}$

(k: power factor, representing the phase shift between voltage and current, ranging between 0 and 1).

k=1 corresponds to the ideal case (voltage and current in phase, purely resistive circuit).

Receiversign convention :

- If P>0, the dipole receives energy: it is then in receiver mode (case of a motor, a resistor, etc.).
- If P<0, the dipole supplies energy: it is then in generator mode (case of an alternator, a voltage source, etc.).

Several other types of electric power can be distinguished, corresponding to concepts related to the technological aspects of electric power distribution.

II.7.1 Instantaneous Power

It is the product of current and voltage at any instant: $P(t) = u(t).\, i(t)$

$$\text{Let}\begin{cases} u(t) = u_{eff}\sqrt{2}\sin(\omega t + \varphi_u) \\ i(t) = I_{eff}\sqrt{2}\sin(\omega t + \varphi_i) \end{cases}$$

$$P(t) = 2U_{eff}I_{eff}\sin(\omega t + \varphi_u)\,\sin(\omega t + \varphi_i)$$

$$P(t) = U_{eff}I_{eff}(\cos(\varphi_u - \varphi_i) - \cos(2\omega t + \varphi_u + \varphi_i))$$

Considering that

$$sina.sinb = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\varphi_u - \varphi_i = \varphi$$

$$P(t) = U_{eff}I_{eff}cos\varphi - U_{eff}I_{eff}\cos(2\omega t + \varphi_u + \varphi_i)$$

The instantaneous power therefore appears as the sum of a constant term — representing the active power P, which is the average value consumed by the dipole and a sinusoidal function with a variable term.

This latter term is a sinusoidal function of double frequency and with zero mean value, called the oscillating (or fluctuating) power.

II.7.2 Fluctuating Power

This is the variable part of the instantaneous power:

$$p(t) = U_{eff}I_{eff}\cos(2\omega t + \varphi_u + \varphi_i)$$

II.7.3 Active Power

It represents the average value of the instantaneous power:

$$p = U_{eff}.I_{eff}.cos\varphi$$

This is the power that corresponds to effective physical work. Its unit is the watt (W)**.**

The power consumed by the dipole is expressed as:

$$P = k.U_{eff}.I_{eff}$$

where k is the power factor, ranging between 0 and 1.

II.7.4 Reactive Power

Reactive power is the power without physical effect in terms of work, which corresponds to the reactive part of the current:

$$Q = U_{eff}.I_{eff}.sin\varphi$$

It is expressed in volt-ampere reactive (VAR)**.**

II.7.5 Apparent Power (for sizing)

Apparent power is the product of the effective values:$S = U_{eff}.I_{eff}$It is expressed in volt-ampere (VA)**,** This allows the introduction of the power factor**:**$k = P/S = cos\varphi$

II.8 Boucherot's Theorem

In a network at constant frequency, there is conservation of active power on the one hand and reactive power on the other hand.

Remark: Boucherot's theorem is not valid for apparent power.

Statement:

The active power of a system is equal to the sum of the active powers of the elements that compose it; likewise for reactive power. However, this is not true for apparent power.

II.9 Transient response:

These are the particular variations of electrical quantities that occur during sudden changes in the characteristics of an electrical circuit. In general, they do not occur repeatedly; otherwise, the system is referred to as operating in a sustained periodic regime.

II.9.1 RL Circuit:

The RL circuit consists of a generator connected in series with an inductor.

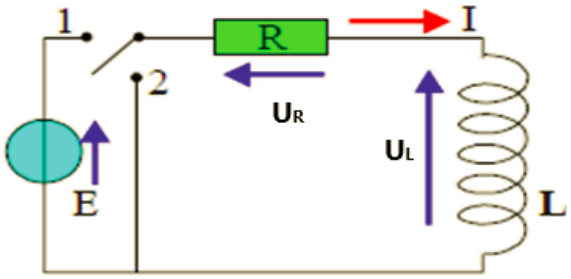


Figure II.10 RL Circuit — Transient Regime

Depending on the position of the changeover switch, the circuit operates either in the free (natural) regime *(position 2)* or in the forced (driven) regime *(position 2)*.

Forced regime:$E = U_R + U_L = RI + L\frac{dI}{dt}$

The differential equation is $\frac{dI}{dt} + \frac{I}{\tau} = \frac{E}{L}$ avec $\tau = \frac{L}{R}$which is a first-order differential equation. Its solution is the sum of two components: the homogeneous (zero-input) solution and a particular (forced) solution.

- **General (homogeneous) solution (SSM — *sans second membre*):**

$$\frac{dI}{dt} + \frac{I}{\tau} = 0 \rightarrow \frac{dI}{I} + \frac{I}{\tau}dt = 0 \rightarrow \int \frac{dI}{I} = -\frac{1}{\tau}\int dt \rightarrow I = Ae^{-\frac{t}{\tau}}$$

- **Particular solution PS :**

Particular solution. Assuming I = Constsubstitute into$\frac{\mathrm{dI}}{\mathrm{dt}} + \frac{\mathrm{I}}{\tau} = \frac{\mathrm{E}}{\mathrm{L}}$to obtain $I = \frac{E}{R}$

General solution of the differential equation:$I_L(t) = Ae^{-\frac{t}{\tau}} + \frac{E}{R}$

Initial condition.

if $t = 0$, $I_L = 0$ then $A = -\frac{E}{R}$

Therefore, $I_L(t) = \frac{E}{R}(1 - e^{-\frac{t}{\tau}})$

$$U_L = L\frac{dI}{dt} = -\frac{L}{\tau}I_0 e^{-\frac{t}{\tau}} = -RI_0 e^{-\frac{t}{\tau}}$$

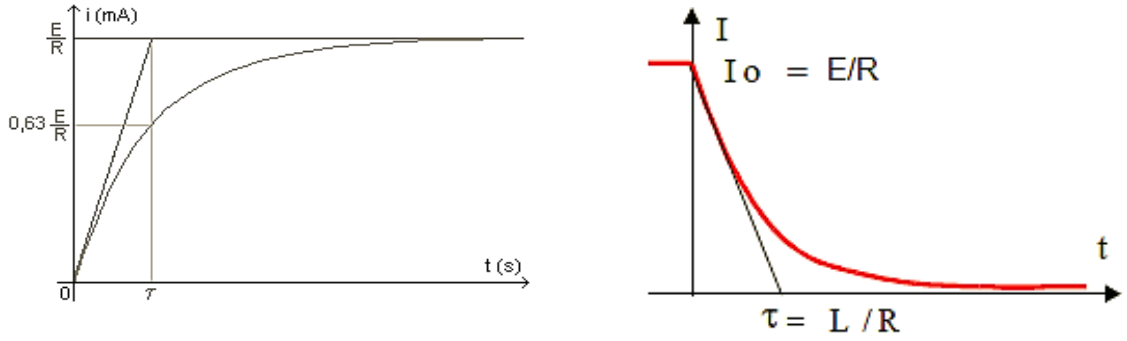


Figure II.11RL Circuit — Current and Voltage Response

II.9.2 RC Circuit – Charging and Discharging of a Capacitor

The RC circuit is composed of a generator, a resistor, and a capacitor. In their series configuration, RC circuits can be used to implement electronic filters, such as **low-pass** or **high-pass filters**. The time constant τtauτ of an RC circuit is given by the product of the values of its two components: τ=R·C.

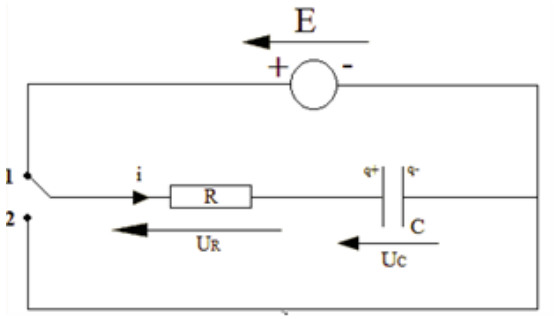


Figure II.12RC Circuit — Charging and Discharging of a Capacitor

- **Charging of the Capacitor**

When the switch is moved to position 1, a voltage source E is applied across the terminals of the capacitor.

$$E = RI + U_C \quad , I = C\frac{dU_c}{dt}$$

$$E = RC\frac{dU_c}{dt} + U_C \quad \rightarrow \quad \frac{dU_c}{dt} + \frac{1}{RC}U_C = \frac{E}{RC}$$

The solution of this equation is: $U_C(t) = Ae^{-\frac{t}{\tau}} + E$

If $t = 0 \quad U_c = 0 \ Then \ A = -E \ and \ U_C(t) = E(1 - e^{-\frac{t}{\tau}})$

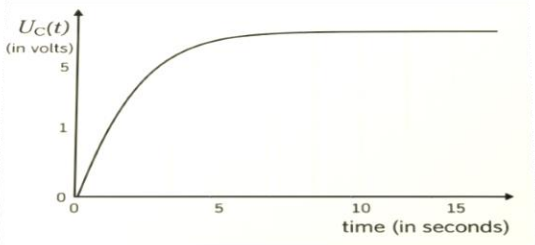


Figure II.13RC Circuit — Capacitor Charging

- **Discharging of the Capacitor**

When the switch is moved to position 2, the voltage source E is applied across the terminals of the capacitor, initiating the discharging process.

$$RI - U_C = 0 \ , \qquad I = c\frac{dU_c}{dt}\tau\frac{dU_c}{dt} - U_c = 0$$

The solution of this equation is:$U_C = Ee^{-\frac{t}{\tau}}$

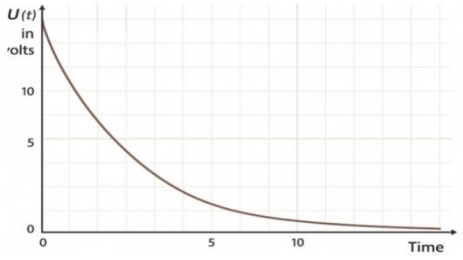


Figure II.14RC Circuit — Capacitor Discharging

II.9.3 RLC Circuit — Transient Response (Three Regimes)

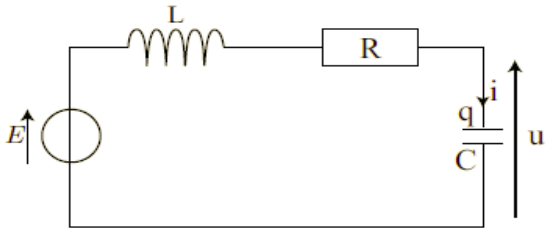


Figure II.15 RLC Circuit

According to Kirchhoff's Voltage Law, we have:

$$-E + RI + L\frac{dI}{dt} + \frac{1}{C}\int Idt = 0$$

Let us derive the differential equation.

Consider the voltage and current across the capacitor:

$$I = \frac{dq}{dt}, \qquad q = CU_c \; alors \; I = C\frac{dU_C}{dt}$$

$$E = RC\frac{dU_c}{dt} + LC\frac{d^2U_c}{dt^2} + U_c \;\; \rightarrow \ddot{U}_C + \frac{R}{L}\dot{U}_C + \frac{1}{LC}U_C = \frac{E}{LC}$$

We set$\omega^2 = \frac{1}{LC}$which represents the naturalangularfrequency, i.e., the frequency at which the system oscillates in free evolution.

$2\lambda = \frac{\omega_0}{Q} = \frac{R}{L} \rightarrow Q = \sqrt{\frac{L}{C}}X\frac{1}{R}$The qualityfactorQ is a dimensionless measure of the damping rate of an oscillator.

We also define: $\Omega = \sqrt{\lambda^2 - \omega^2}$

- If$\lambda < \omega_0$underdampedregime (pseudo-periodic response, curve 1).
- If$\lambda > \omega_0$overdampedregime (aperiodic response, curve 2).
- If$\lambda = \omega_0$criticallydampedregime (curve 3).

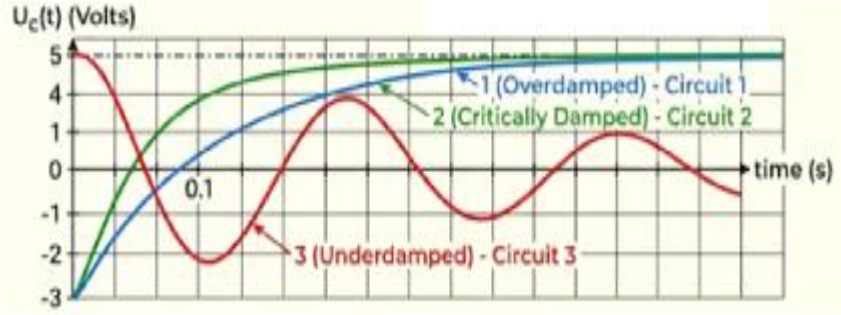


Figure II.16 RLC Circuit — Transient Response (Three Regimes)

Exercises

Exercise 01

We consider an RLC circuit supplied by a sinusoidal alternating voltage of frequency 50Hz.

Given: R=30 Ω L=0.2 H C = 100μF.

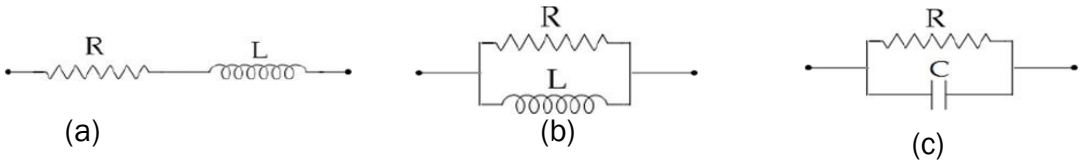
1. Calculate the angular frequency ω.
2. Calculate the complex impedance of the resistor.
3. Calculate the complex impedance of the inductor.
4. Calculate the complex impedance of the capacitor.
5. Deduce the complex impedance of the RLC series circuit.

Exercise 02

For the circuits shown below, the following values are given:

R=20ΩL=50$mH$$C$=100$\mu F$et f=50Hz

Calculate the complex impedance, impedances, the phase shifts corresponding to each case.

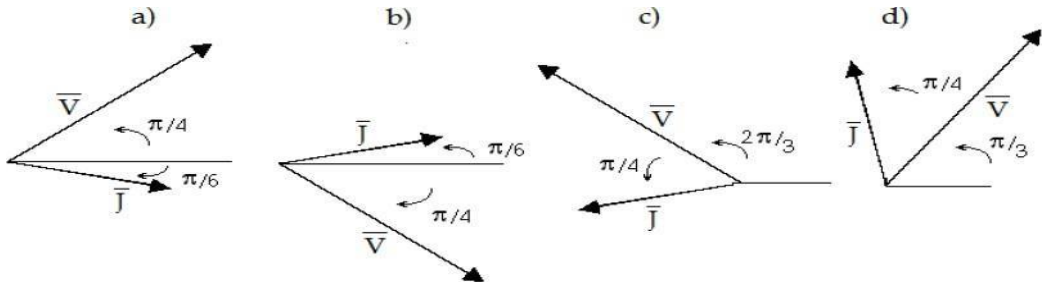


Exercise 03

For the following phasor diagrams, determine:

1. The analytical (time-domain) and complex (phasor) representations of V and J.
2. The values of the resistance and the total reactance corresponding to each diagram, assuming it represents a series RLC circuit.

Given: V=380$V$and =19A



Exercise 04

A measurement with a multimeter (in AC ~ mode) gives: I_{Reff} = 5,00 Ma I_{Leff} = 3,98 mA

Calculate the rms value of the total current i(t). Determine the phase shift of the voltage u(t) with respect to the current i(t), denoted $\varphi_{u/i}$.

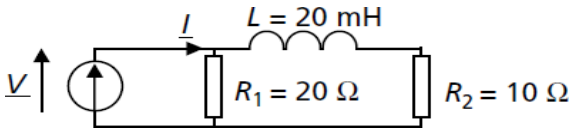
Exercise 05

A single-phase motor of a washing machine draws a current of 5 A under a supply voltage of 230 V – 50 Hz. The power factor is given as cosφ = 0.75**.**

1. Calculate the apparent power of the motor.
2. Calculate the active power absorbed by the motor.
3. Calculate the reactive power absorbed by the motor.
4. Calculate the electrical energy consumed for a continuous operation of 2 hours**.**
5. Given that the price of1 kWh = 1.779 DA (Sonelgaz tariff), calculate the cost of this operation.

Exercise 06

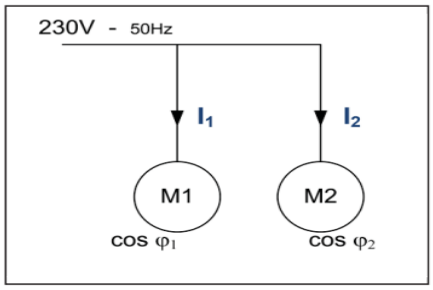
We consider the single-phase load shown below, supplied by a sinusoidal voltage with an rms value of V=230 V and a frequency of 50 Hz.



1. Calculate the rms value $I_1$of the current flowing through the resistor R_1.
2. Calculate the rms value I_2 of the current flowing through the resistor R_2.
3. Calculate the rms value I of the total current absorbed by the whole circuit.
4. Calculate the values of the active power P, reactive power Q, and apparent power S of this circuit.
5. Deduce the value of the power factor of this load.

Exercise 07

Consider the group of motors, shown in the figure opposite, supplied by an rms voltage of 230V.



The group consists of two dipoles: D1 is a motor such that Il = 5 A ; cos1 = 0,8 et D2 is another motor such that I2 = 10 A ; cosφ_2 = 0,7.

1. Calculate the different powers (active, reactive, and apparent) of each motor, as well as those of the group.
2. Calculate the phase shift between the supply voltage and the total supply current.

Chapter III

Electrical Circuits and Power

Chapter III

Electrical Circuits and Power

III.1 Introduction to Single-Phase and Three-Phase Systems

In the autumn of 1882, the first electrical networks appeared simultaneously in New York and Bellegarde, in France. They were very local and used direct current.

However, they experienced relative failures because they did not allow the transmission of industrial power (in 1882, Deprez succeeded in transmitting 400 W over a distance of 57 km, but with an overall efficiency of only 30%).

In 1884, engineers Lucien Gaulard and John Gibbs, on the other hand, worked on alternating current. They developed a high-power transformer using three-phase current.

Electricity production has been carried out since the end of the 19th century from various primary energy sources. The first power plants operated on wood.

Today, production can be carried out from fossil energy (coal, natural gas, or oil), nuclear energy, hydroelectric energy, solar energy, wind energy, and biomass.

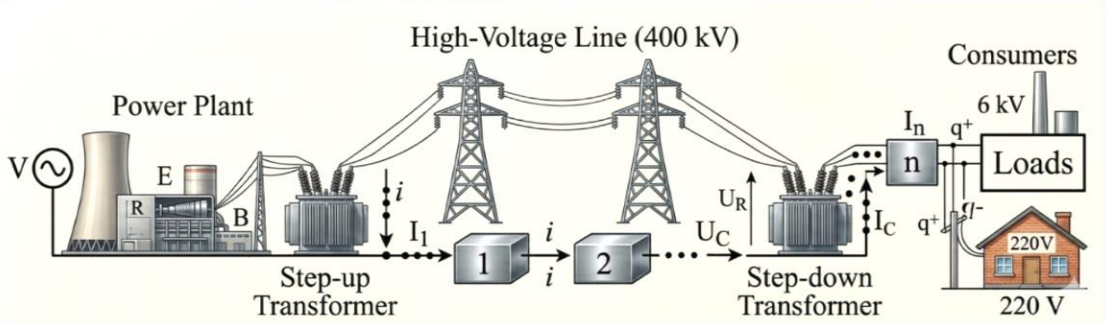


Figure III.1 Electricity Production and Distribution System

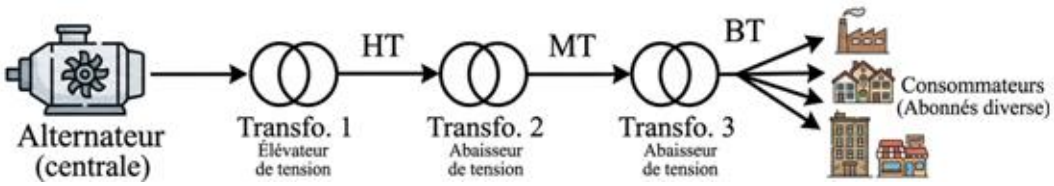


Figure III.2 Three-Phase Alternator and Turbine

The most convenient solution for industrial production of electrical energy is driving an alternator with a turbine, both rotating around an axis. Naturally, such installations produce sinusoidal voltages.

III.2 Single-Phase Circuits and Electrical Power

Single-phase current is the most widely used for the general public. It uses two conductors: the phase and the neutral (generally connected to the ground at the last transformer, just like the neutral in a three-phase system).

Usually, a third conductor called ground is used to ensure safety and to discharge current in case of an electrical fault. It is necessary for any device with a metal casing; however, a 30 mA residual-current circuit breaker ensures personal safety when a fault occurs in the electrical installation.

Single-phase distribution systems are generally used in rural areas, where the cost of a three-phase network is too high and where the electrical loads do not require such infrastructure.

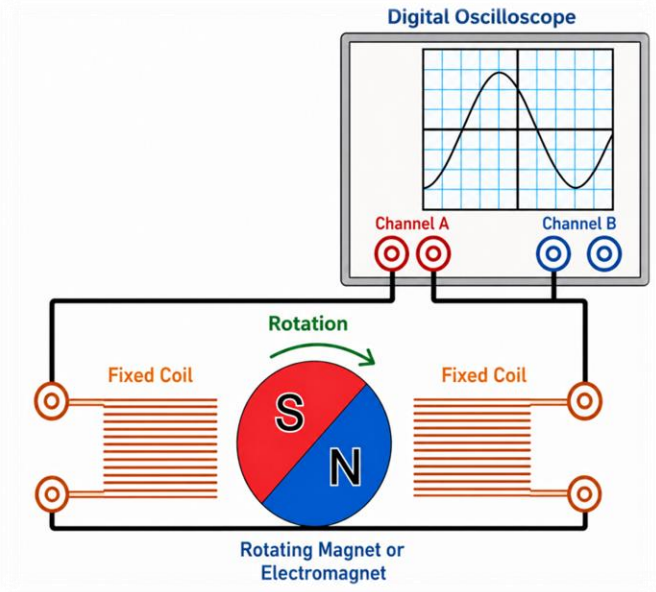


Figure III.3 Single-Phase Distribution Network

III.2.1 Power of Ideal RLC Receivers

Impedance	*Oms's Law*	*Phase shift*	*Power factor*	*Active Power*	*Reactive Power*	*Apparent Power*
$Z=R$	$U=Z\,I$	$\varphi=0$ $\varphi=0$ rads	$\cos\varphi=1$ $\sin\varphi$ $\sin\varphi=0$	$P=U\,I$ $P=R\,I^2$ $P=U^2/R$	$Q=0$ VAR	$S=P$ $S=U\,I$
$Z=L\,\omega$	$U=L\,\omega I$	$\varphi=90$ $\varphi=\pi/2$ rads	$\cos\varphi=0$ $\sin\varphi$ $\sin\varphi=1$	$P=0$ W	$Q=UI\sin\varphi$ $Q=U\,I$ $Q=L\,\omega\,I^2$	$S=Q$ $S=L\,\omega\,I^2$
$Z=1/C\omega$	$U=I/C\,\omega$	$\varphi=-90$ $\varphi=-\pi/2$ rads	$\cos\varphi=0$ $\sin\varphi$ $\sin\varphi=-1$	$P=0$ W	$Q=U\,I\sin\varphi$ $Q=-U\,I$ $Q=-U^2C\,\omega$	$S=Q$ $S=-U^2C\,\omega$

III.2.2 Measurement of Active Power in Single-Phase Current

An electrical device (motor, lamp, transformer, etc.) supplied by a standard electrical network provided by Sonelgaz consumes three types of power: active power, reactive power, and apparent power. As its name indicates, active power is fully converted into mechanical a series connection power (work) and heat (losses). It is denoted by P and expressed in Watts(W).

The formula used to calculate active power is: $P = U\,I\cos\varphi$ (Watts)

To measure P, a wattmeter must be connected according to the downstream connection diagram. It has at least four terminals: two terminals for measuring voltage and two terminals for measuring current. Therefore, two connections must be made:

- a parallel connection, like a voltmeter, to measure voltage, and like an ammeter, to measure current.

The wattmeter also takes into account the phase shift.

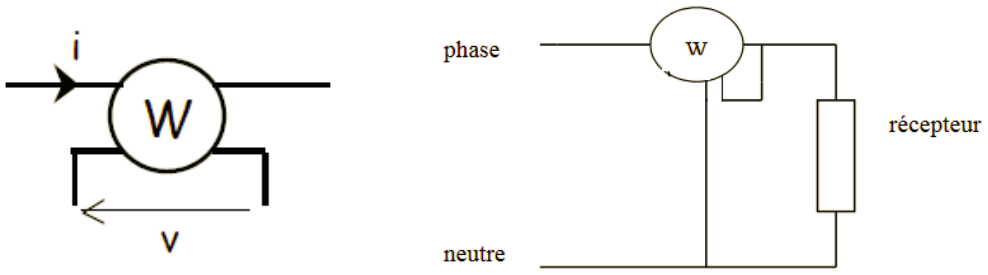


Figure III.4 Wattmeter Connection — Upstream and Downstream

III.2.3. Power Factor Problem and Reactive Power Compensation

The presence of a power factor that is much smaller than 1 in an electrical installation has a very negative consequence:

The current required to produce the same amount of power becomes higher compared to the case where the power factor is equal to 1.

Energy billing takes into account only the active power consumed.

For this reason, electric power companies penalize users whose $\cos\varphi$ is < 0.8, in order to compensate for the oversizing of the network caused by the need to supply a larger current.

When an installation or an electrical network exhibits a $\cos\varphi < 0.8$, it becomes necessary to modify the installation to increase this factor.

Since the vast majority of installations are inductive—meaning that the low power factor is mainly due to the presence of inductances in the circuits—the simplest way to raise the power factor is to install a bank of capacitors at the head of the installation.

This process is called reactive power compensation.

Determining the required capacitance to increase the power factor from $\cos\varphi$ to $\cos\varphi'$.

This method of compensating reactive power is known as "static compensation."

a. Initial State and Modification:

We begin with an active power P and an initial reactive power Q. Adding a capacitor (which provides reactive power Qc) modifies the total power balance:

- **Active Power:** $P' = P$ (remains unchanged).
- **New Reactive Power:** $Q' = Q + Q_c$

Note: In practical applications, since a capacitor provides negative reactive power relative to an inductive load, this addition effectively reduces the total reactive power Q.

b. Trigonometric and Electrical Relation ships

The derivation uses relation ships derived from the power triangle:

- $Q = P \tan(\varphi)$ (initial state)
- $Q' = P \tan(\varphi)$ (final state after compensation)
- Reactive Power of the Capacitor: $Q_c = Q' - Q$
- Electrical Formula for Q_c : $Q_c = C\ \omega\ U^2$

c. Expression for Capacitance C

By isolating C and substituting ω with $2\pi f$, and replacing Q' and Q with their trigonometric expressions, we arrive at the final formula :

$$c = \frac{P(tg\varphi' - tg\varphi)}{2\pi f U^2}$$

III.3 Balanced Three-Phase Systems

Three-phase machines have power ratings that are more than 50% higher than those of single-phase machines of the same mass, and therefore their cost is lower (since the price is directly proportional to the machine's mass).

When electrical energy is transmitted, losses are lower in a three-phase system. For this reason, electricity distribution (from the power plant to the consumer areas) is carried out using three phases.

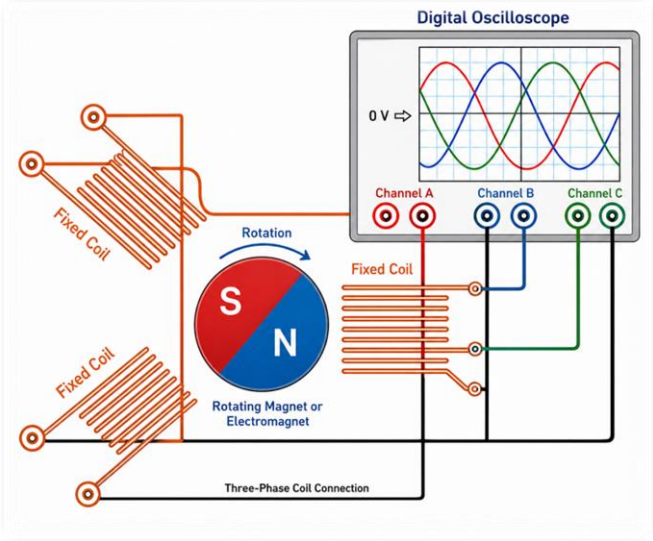


Figure III.5Three-Phase Balanced System — Three Coils Connection

In the case of the local distribution network, the Sonelgaz company supplies us, at the point of use, with a three-phase voltage system through four terminals.

- Three phase terminals labeled 1, 2, 3 or A, B, C or R, S, T;

- One neutral terminal N.

The three-phase system is balanced. In fact, the three voltages have the same RMS value and are phase-shifted by $2\pi/3$ of a period with respect to one another.

In a three-phase voltage system, we find two types of voltages:

Phase (or star) voltages between each phase and the neutral, noted v_1, v_2, v_3

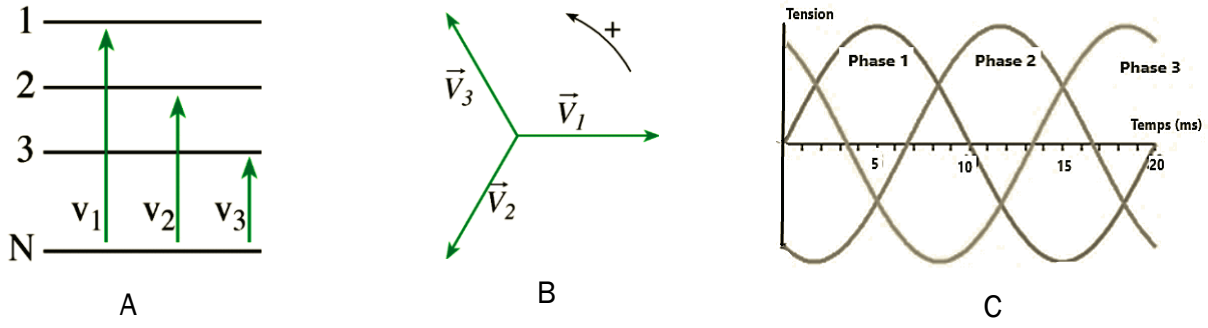


Figure III.6:a.Neutral Point and Phase Voltages in a Three-Phase System
b.Vector Diagram of Three-Phase Voltages
c.Three-Phase Sinusoidal Voltage Waveforms

$$v_1(t) = V\sqrt{2}\sin(\omega t)\; v_2(t) = V\sqrt{2}\sin\left(\omega t - \frac{2\pi}{3}\right) v_3(t) = V\sqrt{2}\sin\left(\omega t - \frac{4\pi}{3}\right)$$

✓ Composed voltages between the phases u_{12}, u_{23}, u_{31}



Figure III.7 a Line Voltages in a Three-Phase Electrical System -**b** Phasor Diagram of Line and Phase Voltages in a Three-Phase System

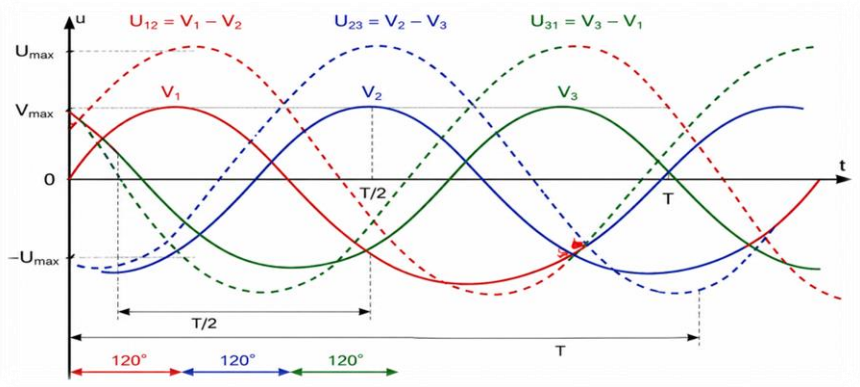


Figure III.8 Phase and Line Voltage Waveforms in a Three-Phase System

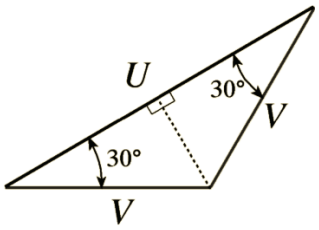
$$\bar{U}_{12} = \bar{V}_1 - \bar{V}_2 \bar{U}_{23} = \bar{V}_2 - \bar{V}_3 \bar{U}_{31} = \bar{V}_3 - \bar{V}_1$$

$$u_{12}(t) = U\sqrt{2}\sin\left(\omega t + \frac{\pi}{6}\right) u_{23}(t) = U\sqrt{2}\sin\left(\omega t - \frac{\pi}{2}\right) u_{31}(t) = U\sqrt{2}\sin\left(\omega t - \frac{7\pi}{3}\right)$$

III.3.1 Relationship between U and V

$U = 2Vcos30 \quad soitU = 2V\frac{\sqrt{3}}{2}$

Finally$U = \sqrt{3}V$



III.3.2 Balanced Three-Phase Loads

III.3.2.1 Definitions

- **Three-phase loads:** These are loads composed of three identical branches, each with impedance Z.
- **Balanced:** Because the three components are identical.
- **Phase currents:** These are the currents flowing through each impedance Z of the three-phase load. **Symbol:** J
- **Line currents:** These are the currents flowing through the conductors of the three-phase network. **Symbol:** I

The network and the load can be connected in two different ways: **star (Y)** or **delta (Δ)**.

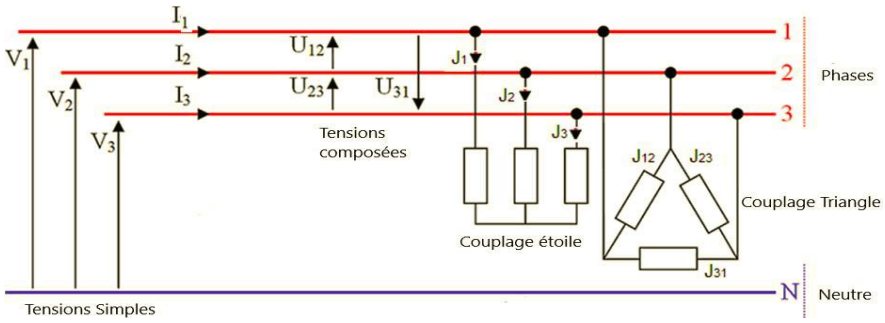


Figure III.9 Delta-Connected Three-Phase Load

III.3.2.2 Star Connection

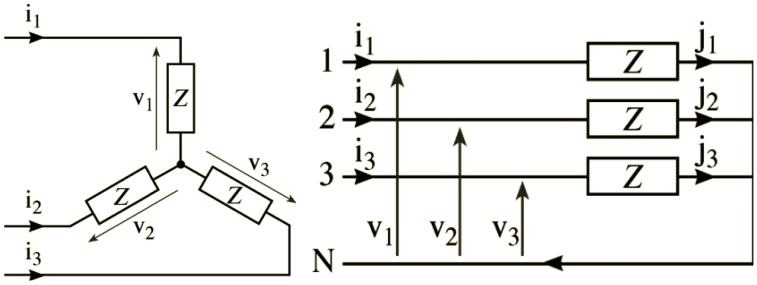


Figure III.10 Three-phase star-connected load

Since they are the same impedances, therefore *i1 + i2 + i3 = 0*, thus i_n=0. The current in the neutral wire is zero. The neutral wire is therefore not necessary.

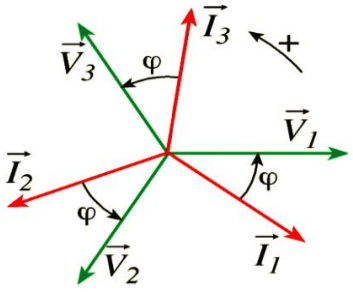
For a balanced three-phase system, the neutral wire is useless.

The line currents are equal to the phase currents.

$$i_1 = j_1\ ,\ \ i_2 = j_2\ \ ,i_3 = j_3$$

Moreover, the load and the network are balanced, therefore

$$I_1 = I_2 = I_3 = J_3$$



- **Powers**
  - For one phase of the load: $P_1 = VIcos\varphi avec\varphi(\vec{I},\vec{V})$
  - For the complete load:$P = 3VIcos\varphi deplusV = \frac{U}{\sqrt{3}}$
  - Finally, for the star connection:$P = \sqrt{3}UIcos\varphi$

$$Q = \sqrt{3}UIsins\varphi$$

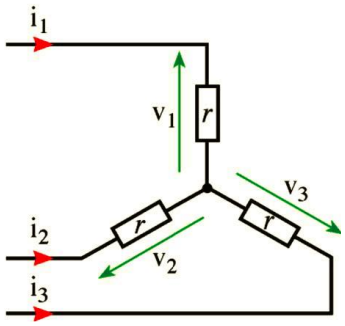
$$S = UI$$

  - Power factor:$K = cos\varphi$

- **Joule losses**

Let us consider that the resistive part of the load

  - For one phase of the load:$P_{j1} = rI^2$
  - Pour le récepteur complet : $P = 3P_{j1} = 3\,rI^2$
  - Resistance seen between two terminals:$R = 2\,r$
  - Finally, for the star connection:$P = \frac{3}{2}RI^2$



III.3.2.3 Delta connection

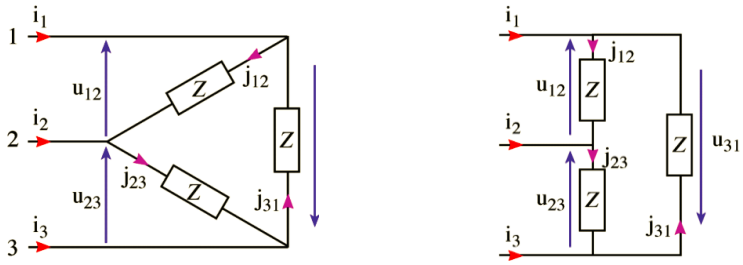


Figure III.11 Three-phase delta-connected load

Since they are the same impedances, therefore $i_1 + i_2 + i_3 = 0$ $and J_{12} + J_{23} + J_{31} = 0$

Here, in no case is the neutral wire necessary.

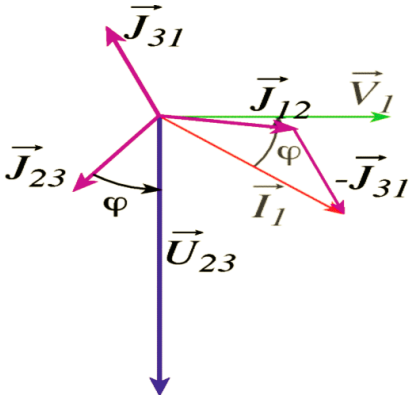
Relations between the currents

According to the diagram in paragraph 3.2.3.

$$i_1 = J_{12} - J_{31} \quad \rightarrow \overrightarrow{I_1} = \overrightarrow{J_{12}} - \overrightarrow{J_{31}}$$

$$i_2 = J_{23} - J_{12} \quad \rightarrow \overrightarrow{I_2} = \overrightarrow{J_{23}} - \overrightarrow{J_{12}}$$

$$i_3 = J_{31} - J_{23} \quad \rightarrow \overrightarrow{I_3} = \overrightarrow{J_{31}} - \overrightarrow{J_{23}}$$



The three-phase system is balanced:$I_1 = I_2 = I_3 = I\ et\quad J_{12} = J_{23} = J_{31} = J$

For the delta connection, the relationship between I and J is the same as the relationship between V and U.

For the delta connection: $J = \frac{I}{\sqrt{3}}$

- **Powers**
  - For one phase of the load: $P_1 = UJcos\varphi$
  - For the complete load: $P = 3\,P_1 = UJcos\varphi$; $J = \frac{I}{\sqrt{3}}$
  - Finally, for the star connection: $P = \sqrt{3}UIcos\varphi$

    In the same way: $Q = \sqrt{3}UIsins\varphi$ and $S = UI$
  - Power factor: $K = cos\varphi$

- **Joule losses**

Let us consider that the resistive part of the load.

- For one phase of the load: $P_{j1} = rJ^2$
- For the complete load: $P = 3P_{j1} = 3\,rJ^2$
- Resistance seen between two terminals: $R = \frac{2rr}{2r+r} = \frac{2}{3}r$
- Finally, for the star connection: $P = \frac{3}{2}RI^2$

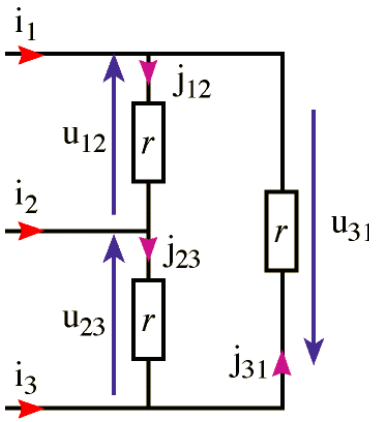


Figure III.12 Balanced delta connection with resistors

	Star connection	Delta connection
Relationship between U and V	$U = V\sqrt{3}$	$U = V\sqrt{3}$
Relationship between I and J	$I = J$	$I = J\sqrt{3}$
Phase shift	$\varphi(\vec{I},\vec{V})$	$\varphi(\vec{J},\vec{U})$
Active power	$P = 3.P_1 = 3VIcos\varphi$ $P = \sqrt{3}UIcos\varphi$	$P = 3.P_1 = 3UJcos\varphi$ $P = \sqrt{3}UIcos\varphi$
Joule losses	$P = 3rI^2$ $P = \frac{3}{2}RI^2$	$P = 3rJ^2$ $P = \frac{3}{2}RI^2$
Equivalent resistance	$R = 2r$	$R = \frac{2}{3}r$
Reactive power	$\varrho = \sqrt{3}UIsin\varphi$	$\varrho = \sqrt{3}UIsin\varphi$
Apparent power	$S = \sqrt{3}UI$	$S = \sqrt{3}UI$
Power factor	$k = cos\varphi$	$k = cos\varphi$

III.4 Measurement of Electrical Power in a Three-Phase System

In a three-phase system, the measurement of active power is essential for evaluating the electrical energy actually consumed by the loads. Several methods can be used depending on the network configuration and the type of load. In the most general case, especially when the system is unbalanced, the three-wattmeter method provides an accurate measurement of the total active power by using one wattmeter for each phase.

III.4.1 Measurement of Active Power in the Most General Case (Three-Wattmeter Method)

In this case, three wattmeters are required to measure the active power transmitted by this line.

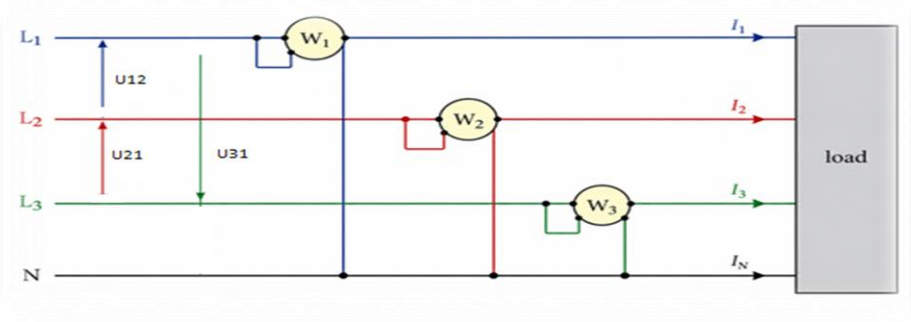


Figure III. 14 Three-Wattmeter Method for Active Power Measurement
P = reading of W1 + reading of W2 + reading of W3

III.4.2 Measurement of Active Power in the Specific Case Where the Neutral is Not Connected (Two-Wattmeter Method))

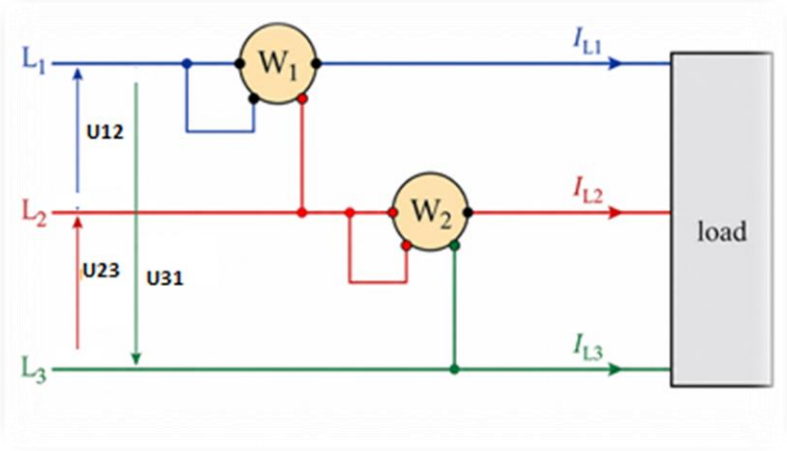


Figure III.15 Two-Wattmeter Method (No Neutral)

If the neutral is not connected or if the current flowing through it is zero: i1 + i2 + i3 = 0

In this case, it is possible to measure the active power transmitted by a three-phase line (balanced or not, with sinusoidal or non-sinusoidal signals) using only two wattmeters.

P=reading of W1+reading of W2

III.4.3 Measurement of Active Power in the Specific Case Where Voltages and Currents Are Balanced (but not necessarily sinusoidal AC)

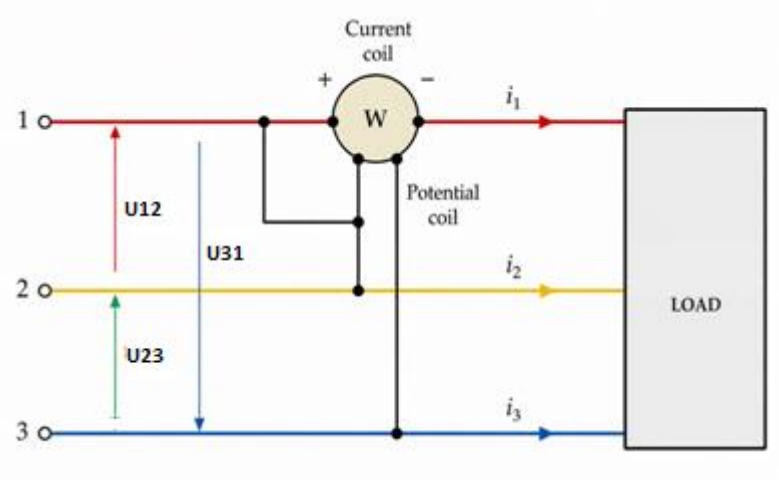


Figure III.16 One-Wattmeter Method (Balanced Loads)

The active power transmitted by the three-phase line can in this case be measured with a single wattmeter.

P=3×reading of W

III.5 Improvement of Power Factor in Three-Phase Systems

III.5.1 Capacitor Connection in Delta

Voltage across a capacitor: U

Reactive power absorbed by one capacitor: $Q = C\omega U^2$

Reactive power absorbed by the three capacitors: $Q_C = 3Q_{C1} = -3C\omega U^2$

	Active power	Reactive power	Power factor
Load only	P	$Q = P\ tg\varphi$	We have $cos\varphi$
The three capacitors only	0	Q_C=-3C$\frac{\omega}{U^2}$	
Load + capacitors	P	$Q'=Q+Q_C=P\ tg\varphi'$	We want $cos\varphi'$

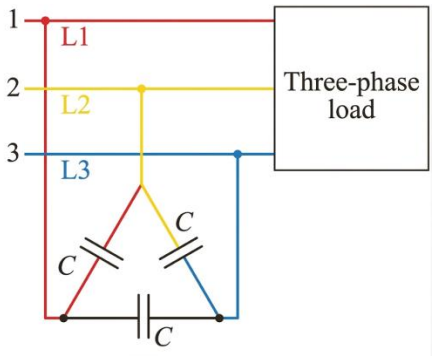


Figure III.17 Star Connection of Capacitors for Power Factor Correction

We deduce the capacitance of the capacitor as follows:

$$Q_C = 3Q_{C1} = -3C\omega U^2 = Q' - Q$$

$$-3C\omega U^2 = Ptg\varphi' - Ptg\varphi$$

Finally:

$$C = \frac{P(tg\varphi - tg\varphi')}{3\omega U^2}$$

III.5.2 Star Connection of Capacitors

Using the same reasoning as above, we show that the capacitance of the capacitor is given by the following relation:

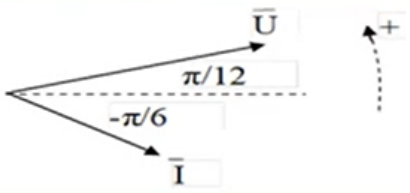
$$C = \frac{P(tg\varphi - tg\varphi')}{\omega U^2}$$

The star connection is therefore less interesting since the required capacitor capacitance is three times greater than for the delta connection. The larger the capacitance, the bulkier and more expensive the capacitor becomes.

Exercises

Exercise 01

Consider the vector (phasor) representation of the voltage *u* and the current *i* of an R–X circuit**:**



Given: U=380V, I=10 A, f=50 Hz

a) Give the analytical and complex representations of *u* and *i*.

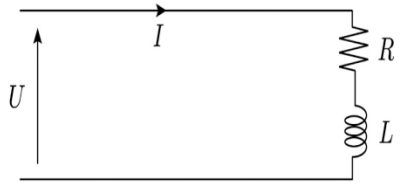
b) Determine *R* and *X*.

c) State whether the circuit is inductive or capacitive.

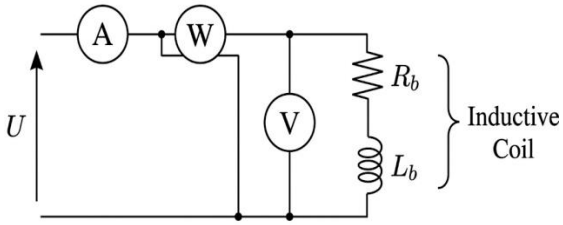
Exercise 02

Given the following R–L circuit:

We are given: R=10 Ω, L=20mH, f=50Hz, U=120V.



1. Calculate the impedance of the circuit.
2. Calculate the RMS value I of the current and its phase shift with respect to the voltage.
3. Calculate the active and reactive components of the current: Ia and Ir.
4. Calculate the active, reactive and apparent powers: P, Q and S



Exercise 03

In the following series RLC circuit, we have:

R=35.2Ω, L=0.16H, C=133.4μF.

The RMS value of the current in the circuit is 5 A, and its frequency is 50 Hz.

1. Calculate the reactances $X_L=L\omega$ and $X_C=1/C\omega$.
2. Calculate the impedance of the circuit, specifying its modulus and its argument.
3. Calculate the RMS value of the source voltage U.
4. Determine the RMS values of the voltages across each element: U_R, U_L and U_C.

5. Taking the current I as the phase reference (φi=0), draw the phasor (Fresnel) diagram representing all the electrical quantities of the circuit.
6. From this geometric construction, recover the value of the voltage U.

Exercise 04

A single-phase motor draws a current of 16 A from a 240 V, 60 Hz. A wattmeter connected to the line indicates 2765 W.

1. Calculate the power factor of the motor and the reactive power absorbed.

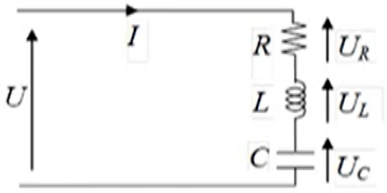
If a capacitor having a reactance of 30 Ω is connected in parallel with the motor, calculate:

2. The active power indicated by the wattmeter.
3. The total reactive power absorbed by the capacitor and the motor.
4. The apparent power drawn from the supply line.
5. The line current.
6. The power factor of the motor–capacitor combination.

Exercise05

A single-phase 13.2 kV, 60 Hz line connects a substation to an industrial load. The line has a resistance of 2.4Ω and a reactance of 12Ω.

The metering equipment at the substation indicates that the line voltage is 12.5 kV and that the line draws 3 MW of active power and 2MVAR of reactive power.



Calculate :

1. The current flowing in the line.
2. The active and reactive power consumed by the line.
3. The active, reactive and apparent power absorbed by the load.
4. The voltage across the load.

Exercise 06

A single-phase heating device absorbs 4 kW from a 240 V supply line. A capacitor connected in parallel with the resistor supplies 3 kVAR to the line.

1. Calculate the value of the line current.
2. If the capacitor is removed, calculate the new line current.

Exercise 07

We define a low-voltage network of 230 V / 400 V at a frequency of 50 Hz.

1. What do 230 V and 400 V mean?
2. Write the instantaneous expressions of the three phase (line-to-neutral) voltages.
3. Write the instantaneous expressions of the three line-to-line voltages.

Exercise 08

A three-phase motor draws a power of 2.8 kW from the supply at 3 × 400 V, with a frequency of 50 Hz and a power factor cosφ = 0.85. Its windings are Y-connected (star connection). Calculate:

1. The voltage across each winding;
2. The line current;
3. The current in each winding;
4. The reactive power.

Exercise 09

A balanced three-phase load is made of three resistive elements R = 100 Ω.This load is supplied by a 220 V / 380 V, 50 Hz three-phase system.

1. Star (Y) connection: Compute the RMS line current I_line and the active power P.
2. Delta (Δ) connection: Compute the RMS line current I_line and the active power P.
3. What can we conclude**?**

Exercise 10 Study of a hoist driven by a three-phase AC motor

1. The motor is supplied by a 220/380 V – 50 Hz three-phase network.

The input power is measured using the two-wattmeter method, giving: Wattmeter 1: P_1 = 4800 W ; Wattmeter 2: P_2 = 1500 W

1.1. Calculate the active power and reactive power. Deduce the line current and the power factor of the motor.

1.2. Draw (or provide) the diagram used to measure: the line current, the line-to-line voltage, and the power using the two-wattmeter method. Specify the ranges (calibers) of the measuring instruments.

1.3. Propose another method of measuring the active power.

2. Study of the motor nameplate

Frequency	Speed	Voltage	Output power	Power factor	Current
50 Hz	1450 tr/min	220/380 V	5.00 Kw	0.74	13A/22.5A

2.1. What is the nominal voltage across one winding of the motor? Deduce the correct connection type (star or delta) for operation on the balanced 220/380 V three-phase system.

2.2. What is the nominal current in one phase of the motor? What does the second current value correspond to?

2.3. Calculate the active input power of the motor and deduce its efficiency.

Chapter IV

Magnetic Circuits

IV.1 Introduction

Inductors, transformers, alternators, induction machines, and similar devices are all based on the use of magnetic circuits, meaning masses of materials known as *magnetic materials*, whose purpose is to guide a magnetic induction. Rather than speaking only of induction, we often refer to the magnetic flux associated with this induction. The figure below provides a summary of the quantities involved in linear magnetic circuits.

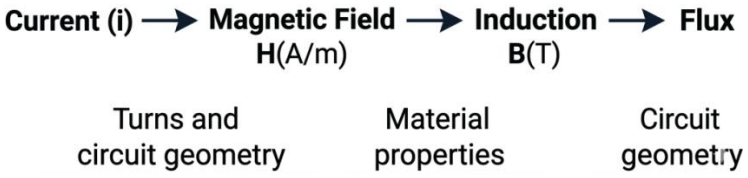


Figure IV.1 Summary of Magnetic Circuit Quantities

IV.2 Notions on Magnetic Quantities

IV.2.1 Fundamental Laws in Electromagnetism

All phenomena involved in electrical engineering and electrical machines are based on two basic laws, namely:

- Biot–Savart Law or Ampere's Theorem (used in magnetic circuits and transformers)
- Laplace Force or Lorentz Force Expression (used in electrical machines

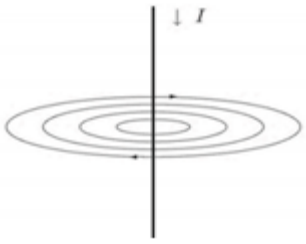


Figure IV.2 Fundamental Laws in Electromagnetism

IV.2.2 Production of a Magnetic Field

If we consider a straight cylindrical conductor through which a current **I** flows, this current creates a magnetic field around the conductor.

The intensity of this magnetic field is given by Ampere's Law $\int Hdl = I$(Ampere's Theorem).

To create a uniform magnetic field, a coil is used to concentrate the magnetic field lines in the same region.

Inside the coil, the magnetic fields produced by each turn **add together**, generating a stronger and more uniform magnetic field.

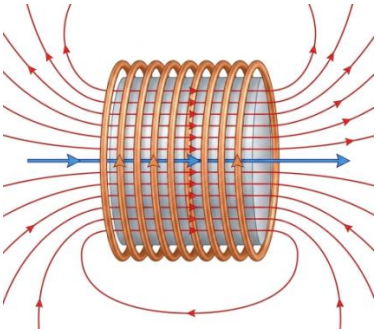


Figure IV.3 Magnetic Field Produced by a Straight Conductor

IV.2.3 Origin of the Magnetic Field

When a material is present along the path of the magnetic field, the properties of this material will modify the value of the magnetic field.

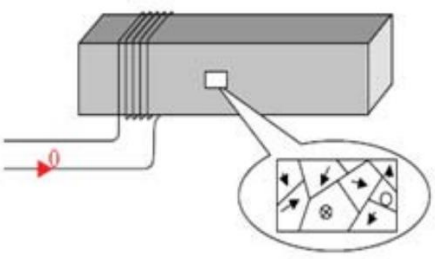
The field becomes channeled and altered by the presence of the material. In this case, we speak of magnetic induction (inside the material).

$$\boldsymbol{B = \mu H}$$

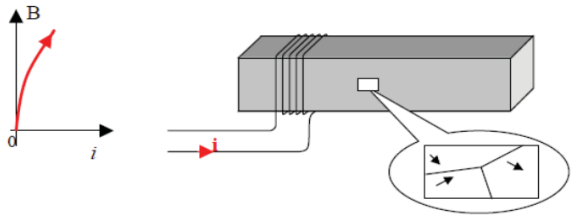
μ:magnetic permeability

- When the material is not magnetized (H = 0), the effects of the small magnetic domains (Weiss domains) cancel each other out.

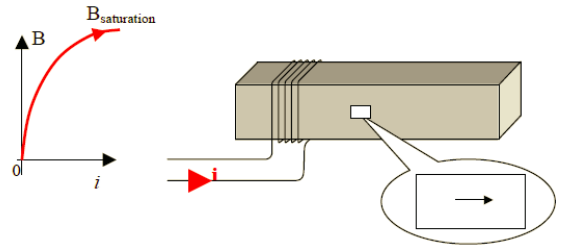
  As a result, there is no magnetic induction, and therefore: B_0=0.



- If we increase the magnetic field H = H_1 (that is, we increase the current flowing through the coil), we obtain a magnetic induction $B_1 \neq 0$.



- If we further increase the magnetic field to H = H_2 (with H_1 greater than H_2), we obtain an induction B_2, which is greater than B_1.

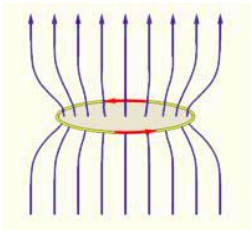


IV.2.4 Magnetic Flux

The magnetic flux passing through a surface S corresponds to the total number of magnetic field lines that cross this surface.

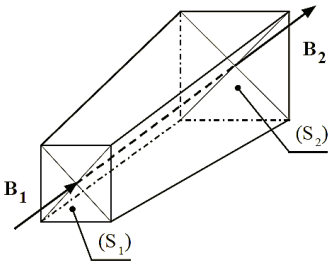
- The magnetic flux is conserved within a flux tube.

$$\Phi = \int_S B.ds = Bs$$



- The same magnetic flux that crosses surface **S_1** also crosses surface **S_2**.

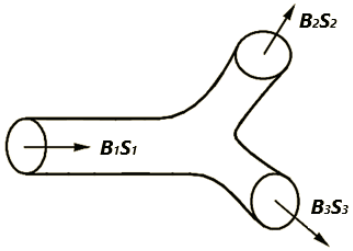
$$\emptyset = B_1S_1 = B_2S_2$$



- In the case of a branching or a node (where the flux encounters two different paths), the magnetic flux splits between the available paths.

$$\Phi_{Total} = \Phi_1 + \Phi_2$$

$$B_1S_1 = B_2S_2 + B_3S_3$$



IV.2.5 Ampere's Theorem

Ampere's theorem makes it possible to determine the value of the magnetic field from the electric currents that produce it. Consider, for example, a coil through which a current **I** flows. The coil then creates a magneto motive force **(**MMF**)** that drives a magnetic flux through the medium. This phenomenon is similar to what occurs in electrical circuits: an electromotive force **(**EMF**)** moves electrons, causing an electric current to flow through the medium. Ampere's theorem states that the circulation of the magnetic field vector H along any closed curve **(**C**)** is equal to the algebraic sum of the electric currents crossing the surface bounded by this contour.

$$FMM = \oint_c H\, dl = \sum_k \mp I_k$$

• The current is taken as positive if it flows in the direction of the normal to the surface (according to the right-hand corkscrew rule relative to the direction in which the contour C is traversed).

• The current is taken as negative if it flows opposite to the direction of the normal to the surface (according to the right-hand corkscrew rule relative to the direction in which the contour **C** is traversed).

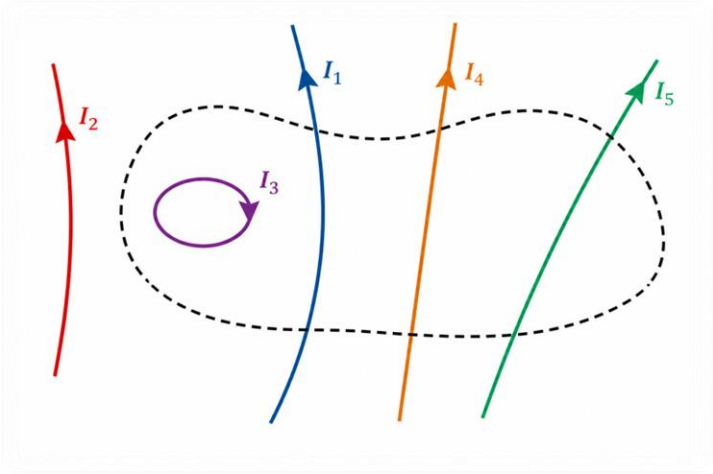


Figure IV. Ampere's Theorem — Coil and Magnetomotive Force

The current I_2 does not appear in the calculation. Applying Ampere's theorem gives:

$$\oint Hdl = I_1 - I_3 + I_4 - I_5$$

IV.3 Calculation of the Magnetic Circuit

Magnetic circuits are based on the use of ferromagnetic materials, with the goal of producing a magnetic induction B in a specific region (the air gap).

To achieve this, an excitation magnetic field H is generated using a coil, and then directed (or "channeled") toward the functional region — the air gap.

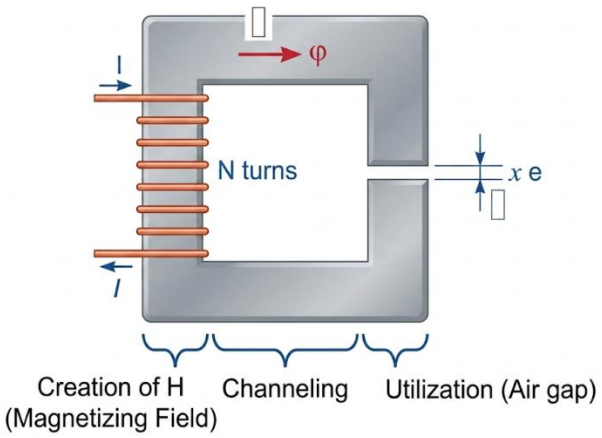


Figure IV. Heterogeneous Magnetic Circuit with Air Gap

We identify three main elements**:**

1. The winding, which generates the excitation and therefore the magnetic field.
2. The yoke, which directs the field H toward the useful area. The yoke forces the magnetic field to follow a specific path because its magnetic permeability is much higher than that of air. The material that forms the *yoke* behaves like a flux tube.
3. The air gap, where the magnetic field is intended to be used. The air gap is the region where interaction with the external environment takes place.

The mathematical formulation is based on three fundamental laws**:**

- Flux Conservation
- Ampere's Theorem
- Material Law (constitutive relation)

IV.3.1 Case of a Homogeneous Series Magnetic Circuit

The circuit is made of a single material forming a closed magnetic loop. In this case, the magnetic circuit is equivalent to a flux tube of length L. All the magnetic flux is guided through the circuit. Furthermore, the circuit behaves linearly atevery point.

$$B = \mu H \qquad (\mu = \mu_0 \mu_r)$$

μ_0 is the permeability of free space (vacuum permeability). $\mu_0 = 4\pi \times 10^{-7}$

μ_r is the relative permeability of the material.

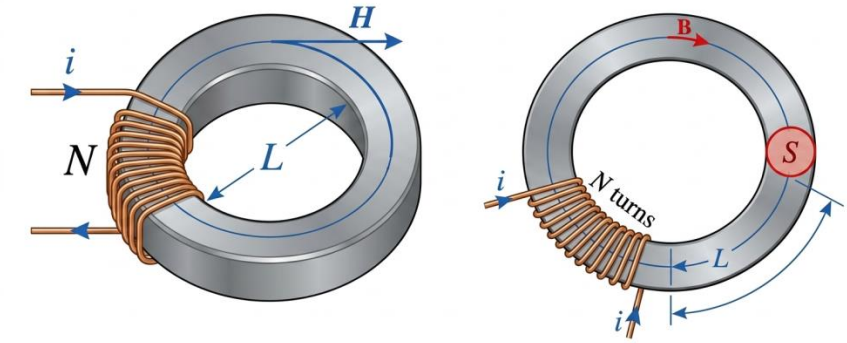


Figure IV. Homogeneous Series Magnetic Circuit

According to Ampere's theorem$\sum_i^n H_i \, L_i$, in our case:$B = \frac{\mu_0 \mu_r NI}{L}$

L: average length of the magnetic field lines [m]

N: number of turns of the coil

I: current flowing through the coil [A]

H: magnetic field strength (magnetic excitation) [A/m]

IV.3.2 Case of a Heterogeneous Series Magnetic Circuit (magnetic circuit with air gap)

In this case, the magnetic circuit is the same as the one described above, except that we have introduced an airgap. Therefore, the circuit contains two different materials:

- a ferromagnetic material with permeability μ
- air, with permeability μ_0

$$H(L - \delta) + H_0 \delta = NI$$

IV.3.3 Parallel Magnetic Circuit (magnetic circuit with branches)

This magnetic circuit is composed of three segments of lengths l, l′, l′′, l′′, with cross-sectional are as S, S′, S′′, and permeabilities μ, μ′, μ′′,each one being crossed respectively by the magnetic fluxesΦ, Φ′, Φ′′.

We can apply Ampere's theorem to three different closed magnetic paths, using the average flux paths of lengths: land l′, or l′ and l′′, or l' and l′′.

$$Ni = Hl + H^{'}l^{'}, \qquad Ni = Hl + H^{''}l^{''} \qquad 0 = H^{'}l^{'} - H^{''}l^{''}$$

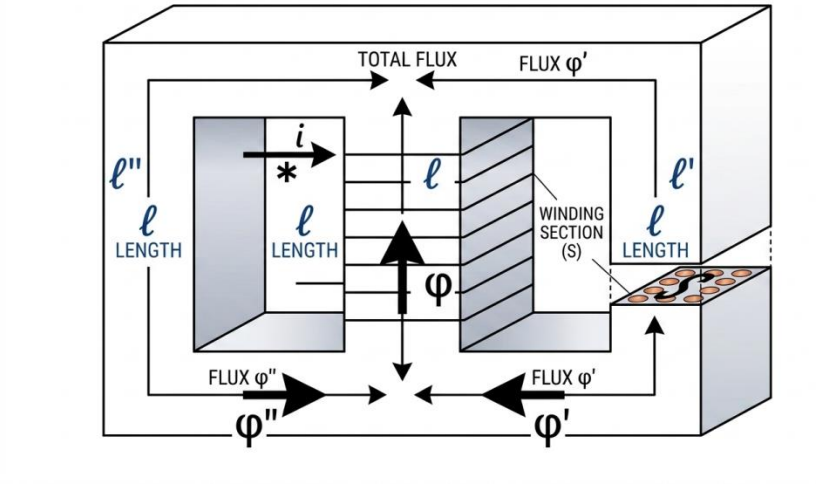


Figure IV. Parallel Magnetic Circuit — Flux Distribution

By applying the corresponding relations$H = \frac{B}{\mu} = \frac{\varphi}{\mu S}$ to each of the three sections of the magnetic circuit, the previous equations become:

$$Ni = \frac{\emptyset . l}{\mu S} + \frac{\emptyset' . l'}{\mu' S'} \ , \quad Ni = \frac{\emptyset . l}{\mu S} + \frac{\emptyset'' . l''}{\mu'' S''} \ , \quad 0 = \frac{\emptyset' . l'}{\mu' S'} - \frac{\emptyset'' . l''}{\mu'' S''}$$

These three equations are dependent, meaning that any one of them can be deduced from the other two.

Therefore, if we want to perform calculations on this magnetic circuit, only two of the three equations should be kept, because the third one does not provide any additional information.

The problem then reduces to solving a system of three equations with three unknowns:

$$Ni = \frac{\emptyset . l}{\mu S} + \frac{\emptyset' . l'}{\mu' S'} \ , \quad Ni = \frac{\emptyset . l}{\mu S} + \frac{\emptyset'' . l''}{\mu'' S''} \ , \quad \emptyset = \emptyset' + \emptyset''$$

If we denote the$\Re$reluctance by$\frac{l}{\mu S}$, $\Re'$reluctance by$\frac{l'}{\mu' S'}$, $\Re''$reluctance by$\frac{l''}{\mu'' S''}$; these equations become:

$$Ni = \Re\emptyset + \Re'\emptyset', \quad Ni = \Re\emptyset + \Re''\emptyset'' \ , \ \emptyset = \emptyset' + \emptyset''$$

IV.4 Inductance of a Coil

Consider a coil with N turns carrying a current I. The coil is placed in a magnetically linear medium (such as air). The magnetic flux produced by the coil is Ø. The total magnetic flux linked to the coil is:$\lambda = N\emptyset$

Magnetic Quantities	Electrical Quantities
Magnetomotive Force (MMF) = NI	Electromotive Force (EMF) = U (V)
Magnetic Flux Φ (Wb)	Electric Current I (A)
Reluctance$\Re\Re = \frac{1}{\mu_0 \mu_r}\frac{L}{S}$	Resistance R $R = \rho\frac{L}{S}$
Magnetic voltage (magnetic drop)MMF=$R\,\emptyset$	Electrical voltage (electric drop) U = R I
Magnetic loop (magnetic circuit loop) $\sum_{maille} \boldsymbol{F_{MM}}$	Electrical loop (Kirchhoff's voltage law) $\sum_{maille} \boldsymbol{U}$
Magnetic node $\sum_{Noeud} \boldsymbol{\emptyset}$	Electrical node (Kirchhoff's current law) $\sum_{Noeud} \boldsymbol{I}$

The inductance of the coil is defined as:$L = \frac{\lambda}{I} = \frac{N\emptyset}{I} = \frac{N^2}{\Re}$

IV.5 Magnetic–Electrical Analogy

To every magnetic circuit, we can associate an equivalent electrical representation, allowing us to study the behavior of the magnetic circuit using electrical relationship

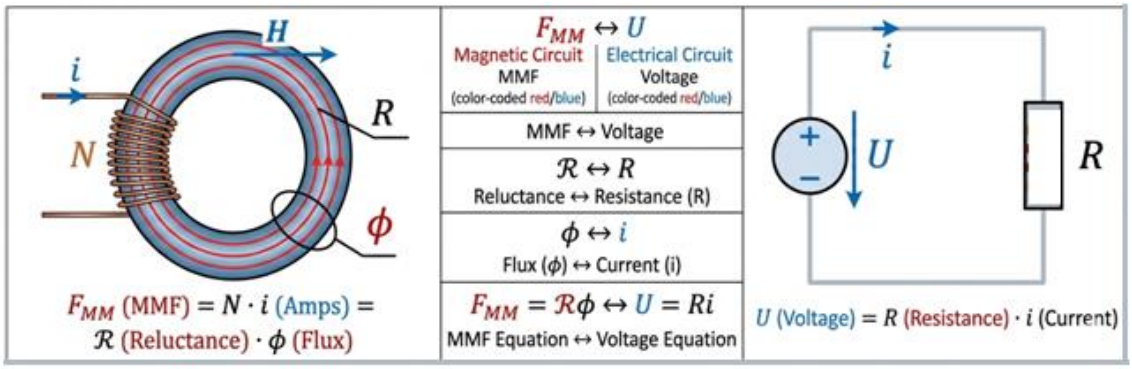


Figure IV. Magnetic–Electrical Analogy

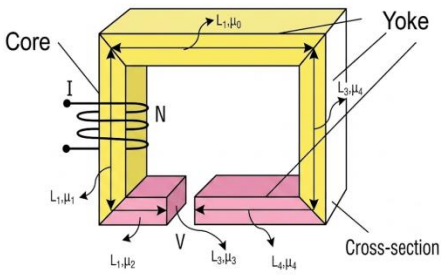
Exercises

Exercise 01

Consider the magnetic circuit shown opposite. The magnetic core is made of a single material with relative permeability μr, an average length L=0.7m and a cross-sectional area S=0.003m^2.
Calculate the current required to obtain a flux Ø=36×10^{-4}Wb.
The number of turns is N=200N .
From the magnetization curve B=f(H), we are given that if B=1.2T, then H=800A.turn/m.

Exercise 02

Consider the magnetic circuit (shown in the figure) composed of:



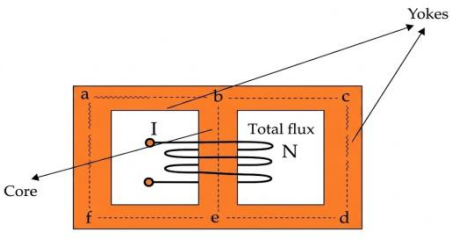
- **Sections 1, 2, and 3** made of the same ferromagnetic material with : L_1=0.15m, L_2=0.2m, L_3=0.15m, $S_1=S_2=S_3$=0.0003m^2
- **Sections 4 and 5** made of the same ferromagnetic material with: L_4=0.12m, L_5=0.075m, $S_4=S_5$=0.004 m^2
- An **air gap** with: L_0=0.005 m, S_0=0.004 m^2

Calculate the current III required to obtain a fluxΦ = 36.10^{-4}Wb
The number of turns is N=200.
From the magnetization curves *B=f(H)*we are given(*si B=1.2T, H=800 Art/m*), (*si B=0.9 T, H=500Atr/m)*

Exercise 03

Consider the magnetic circuit shown opposite.

The magnetic core is made of a single material with relative permeability μ_r .
The geometrical characteristics of the magnetic circuit are:
The cross-sectional area S = const = 6 cm^2
$L_{ab} = L_{bc} = L_{ed} = L_{fe}$ = 8 cm ;$L_{af} = L_{be} = L_{cd}$ =5 cm



Homogeneous forked circuit

- If N = 50, determine the current III required to establish a flux
- Ø= 4. 10^{-4}Wbin both yokes.
- From the magnetization curve, we are given: B = 0.67 T si H = 100 At/m ; B = 1.33 T si H = 650 At/m

Exercise 04

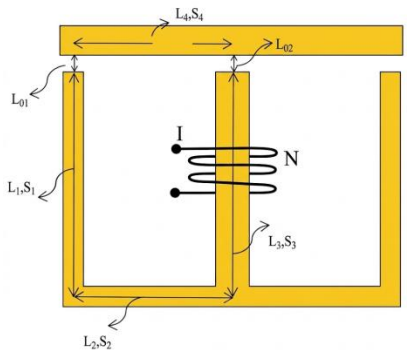
Consider the magnetic circuit (shown in the figure) made of mild steel laminations.

This circuit is symmetrical with respect to the central core.

The geometrical parameters of the different sections are as follows:

$S_1 = S_2 = 2.812\ 10^{-4}\ m^2$, $S_3 = 5.625\ 10^{-4} m^2$, $S_4 = 3.04\ 10^{-2}\ m^2$

$L_1 = L_3 = 43.75$ mm, $L_2 = 25$ mm, $L_1 = 38.5$ mm, $L_{01} = L_{02} = 0.15$ mm.



Heterogeneous forked circuit

-Calculate the number of turns required to obtain in the core B = 1T, I = 100 mA

The magnetization curve is given as follows:

B (T)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
H (Atr/m)	30	65	110	155	220	390	520	3500	13800

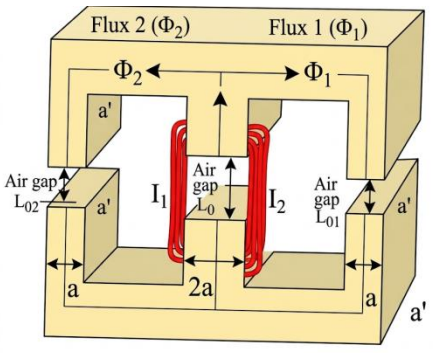
Exercise 05

The magnetic circuit shown in the figure is excited by a coil with

1000 turns and supplied with a current of 0.78A.

The magnetic induction in the air gap of the central core is B_0=0.02T

$L_{01} = 25.0^{-3}$ m, $L_{02} = 15.10^{-3}$ m



Heterogeneous bifurcated circuit

1. In electrical engineering, what type of material is generally used?
2. Plot the magnetization curve of this material, **B=f(H)**, and briefly explain this curve.

3. Assuming that the reluctance of the iron is negligible, calculate the magnetic inductions in the air gaps of the lateral cores, $\mathbf{B_{01}}$ and $\mathbf{B_{02}}$, as well as the length $\mathbf{l_0}$ of the air gap of the central core. Take $\boldsymbol{\mu_0 = 4\pi\ 10^{-7}}$.

Chapter V

Transformers

V.1 Introduction

The transformer allows transferring energy (in alternating form) from a source to a load, while modifying the voltage value. The voltage can be either increased or decreased depending on the intended use. The change from one voltage level to another is achieved through the effect of e and a magnetic field.

Among the applications of transformers, we note:

Electronics: low-voltage power supplies, impedance matching.

Electrical engineering: voltage transformation for power transmission and distribution, low-voltage power supply (e.g., halogen lamps).

V.2 Operating Principle

The transformer consists of two windings (or more) coupled on a magnetic core (a ferromagnetic material).

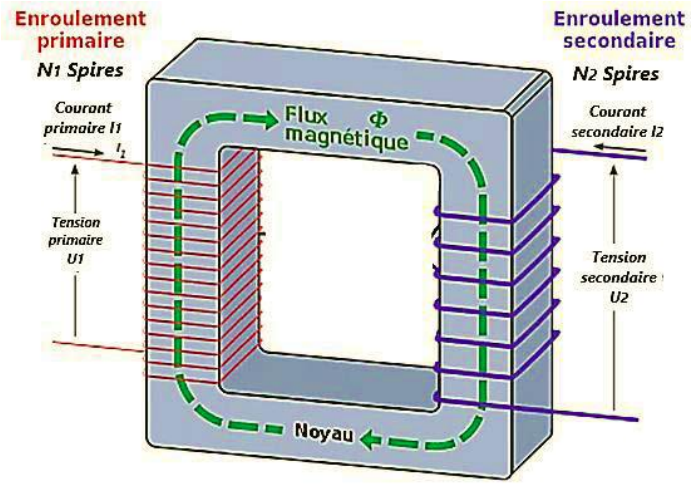


Figure V.1Transformer — Operating Principle (Primary and Secondary)

The side of the source is called the primary. The side of the load is called the secondary. The flux Ø is the mutual flux. It should be noted that there is no electrical connection between the primary and the secondary. All the coupling between the two windings is magnetic.

When an alternating voltage is applied to the source, this creates an alternating flux in the magnetic core. According to Faraday's law, this flux creates electromotive forces (EMF) in the coils. The induced electromotive force is proportional to the number of turns in the coil and to the rate of change of flux.

Depending on the turns ratio between primary and secondary, the secondary supplies the load with a voltage different from that of the source.

If U1 < U2, the transformer is called a step-up transformer;

If U1 > U2, the transformer is called a step-down transformer;

If U1 = U2, the transformer is an isolation transformer;

V.3 Ideal Transformer

An ideal transformer is defined with the following characteristics:

1. The resistance in the wires (at primary and secondary) is *zero*.
2. The magnetic core is perfect ($\mu r = \infty$).

In this case the reluctance of the core will be zero, and therefore there are no leakages. The flux is thus completely contained inside the core. The magnetic coupling between primary and secondary is perfect; all the flux from the primary reaches the secondary. The efficiency in this case is equal to 100%.

The equivalent circuit of the ideal transformer is therefore:

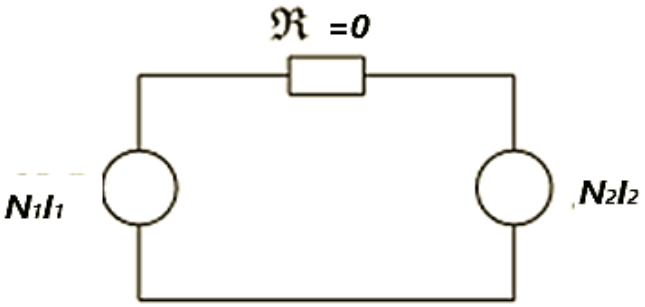


Figure V.2 Equivalent circuit of the ideal transformer

The transformation ratio

$$K = \frac{N_2}{N_1} = \frac{U_2}{U_1} = \frac{I_1}{I_2}$$

V.4 Real Transformer

The real transformer does not have perfect characteristics like the ideal transformer. We must take into account:

1. Magnetic core. The core has a non-linear B(H) characteristic, with hysteresis, and a non-infinite permeability ($\mu r \neq \infty$).
2. Windings. The windings are made of copper, having a non-zero resistivity ($\rho \neq 0$).

Taking into account these characteristics, we can deduce six sources of losses in the transformer:

1. Since the permeability of the core is non-infinite, the reluctance of the core will not be zero. There are therefore flux leakages: (a) at the primary (b) at the secondary.
3. Losses in the iron due to Hysteresis and eddy current leakages.
4. The resistivity of the copper wires implies an internal resistance at the primary and secondary and results in Joule losses.

The consequences of these parasitic phenomena are:

- The efficiency of the transformer is *less than 100%.*
- The voltage ratio between primary and secondary will not be exactly equal to the turns ratio.

$$K = \frac{N_2}{N_1} \neq \frac{U_2}{U_1} \neq \frac{I_1}{I_2}$$

V.4.1 Equivalent Circuit of the Real Transformer

With all the parasitic phenomena seen in the previous section, we can represent these losses by equivalent circuit elements.

V.4.1.1 Core losses

Since the permeability of the core is finite, the reluctance will be non-zero. Consequently, to create the flux ϕ in the core, a current Im is required. This can be represented by an inductance Lm, called a magnetizing inductance. The losses in the core are represented by a resistance Rm in parallel with the magnetizing inductance Lm.

V.4.1.2 Primary and secondary leakages

These losses are represented by inductive reactances X_1 and X_2, for the primary and secondary, respectively.

V.4.1.3 Joule effect losses

The resistance of the copper wires is represented by resistances R_1 and R_2 for the primary and secondary, respectively.

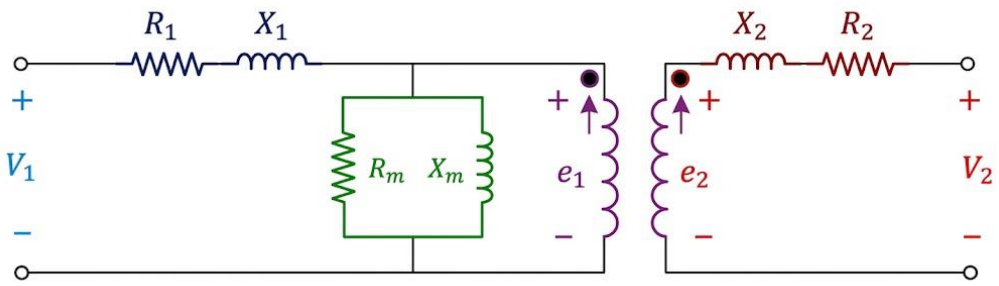


Figure V.3 Complete Equivalent Circuit of the Real Transformer

To facilitate circuit analysis, the secondary impedances are referred to the primary. Similarly, the primary parameters can be referred to the secondary. We then obtain:

V.4.1.4 Primary parameters referred to the secondary

The transformer resistance referred to the secondary.

$$R_s = R_2 + k^2 R_1$$

The magnetic leakage reactance referred to the secondary.

$$X_s = X_2 + k^2 X_1$$

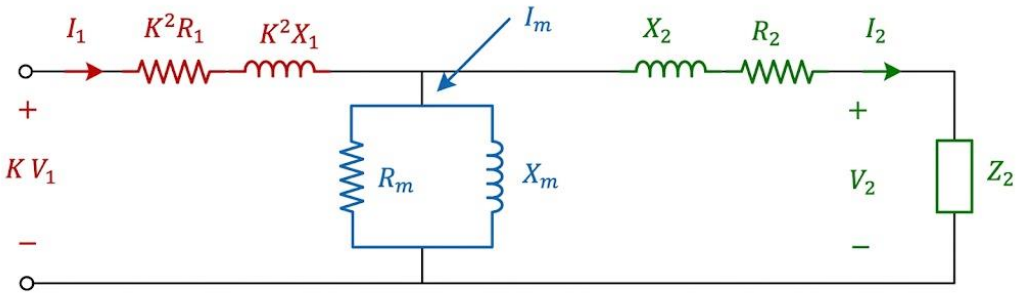


Figure V.4 Primary Parameters Referred to the Secondary

From this circuit, we define:

$$X'_1 = k^2 X_1 R'_1 = k^2 R_1 V'_1 = k V_1 I'_1 = \frac{I_1}{k}$$

V.4.1.5 Secondary parameters referred to the primary

The transformer resistance referred to the primary.

$$R_p = R_1 + \frac{R_2}{k^2}$$

The magnetic leakage reactance referred to the primary.

$$X_p = X_1 + \frac{X_2}{k^2}$$

The equivalent circuit in the case of secondary parameters referred to the primary:

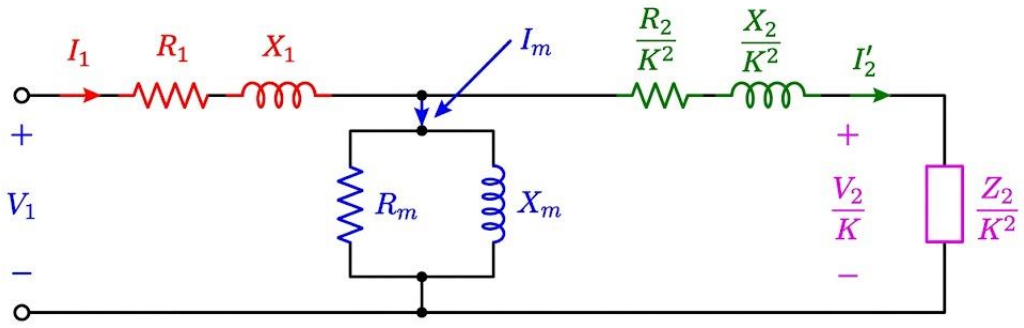


Figure V.5 Secondary Parameters Referred to the Primary

From this circuit, we define:

$$X'_2 = \frac{X_2}{k^2} R'_2 = \frac{R_2}{k^2} V'_2 = \frac{V_2}{k} I'_2 = kI_2$$

In a typical transformer, the current I0 is *only 2 to 4% of the value of current I1*.

To simplify the analysis, we can therefore *neglect the current* I_0. We will therefore assume that the core has negligible iron losses and high permeability.

We then obtain the following circuit:

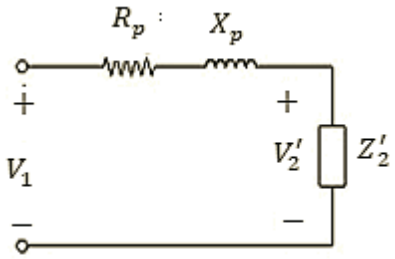


Figure V. 6 Simplified equivalent circuit of the transformer.

$$X_p = X_1 + \frac{X_2}{k^2} \; ; \quad R_p = R_1 + \frac{R_2}{k^2} \; ; \quad Z'_2 = \frac{Z_2}{k^2} \; ; \quad V'_2 = \frac{V_2}{k^2}$$

We then obtain the following relations:

$$V_1 = R_p I_1 + jX_p I_1 + V'_2$$
$$I'_2 = I_1$$

and thus a simplified phasor diagram.

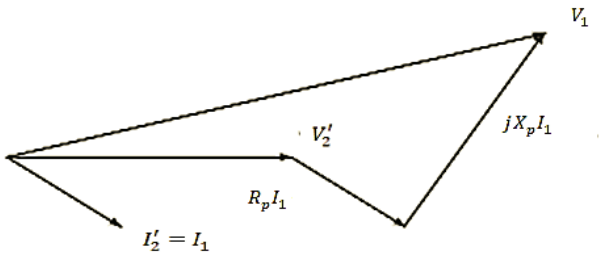


Figure V. 7 Phasor diagram - Simplified transformer.

V.5 Other Transformers

V.5.1 Isolation Transformer

A transformer creates galvanic isolation between its primary and secondary; this property is used specifically in isolation transformers. They serve to ensure the safety of an installation by protecting against electric shock, for example. Galvanic separation also eliminates part of the electrical noise, which is useful for certain sensitive electronic devices. These transformers have almost the same number of turns at the primary and secondary.

They are, for example, widely used in operating theaters: each room of the block is equipped with its own isolation transformer, to prevent a fault in one block from affecting the others.

Another benefit is the ability to change the neutral regime (case of use of computer equipment and/or sensitive electronic equipment in an IT installation).

V.5.2 Pulse Transformer

This type of transformer is used for controlling thyristors, triacs, and transistors. It presents, compared to the optocoupler, the following advantages: possible operation at high frequency, simplified assembly, ability to supply a large current, good voltage withstand.

V.5.3 Autotransformer

The autotransformer consists of a primary and secondary winding wound on the same magnetic circuit. In other words, it is a single-winding transformer. The two windings share a common part, and there is no galvanic isolation between the two windings.

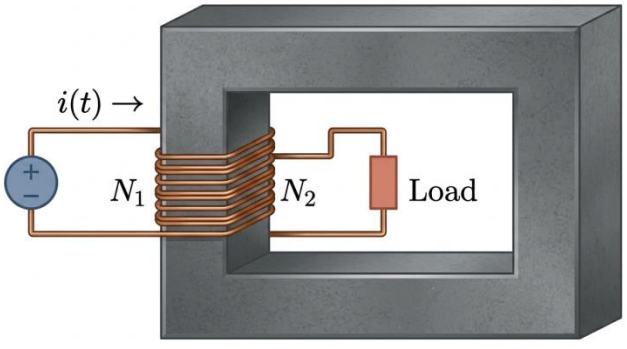


Figure V.8 Pulse Transformer Circuit

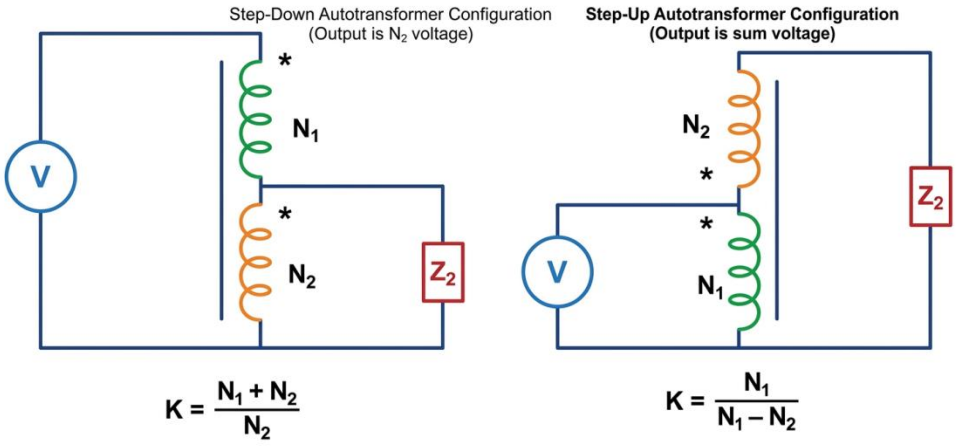


Figure V.9 Autotransformer — Circuit Diagram

An autotransformer is used for:

Starting induction motors and synchronous motors in order to supply up to 50 to 60% of the total voltage to the motor stator during starting.

- Giving a slight boost to cable distribution, by compensating for voltage drops.
- It is also used as a voltage regulator.

V.5.4 Three-Phase Transformers

A three-phase transformer can consist of three single-phase transformers. The figure shows the primary windings coupled in star. This solution leads to large bulk and underutilization of iron, but it is sometimes used.

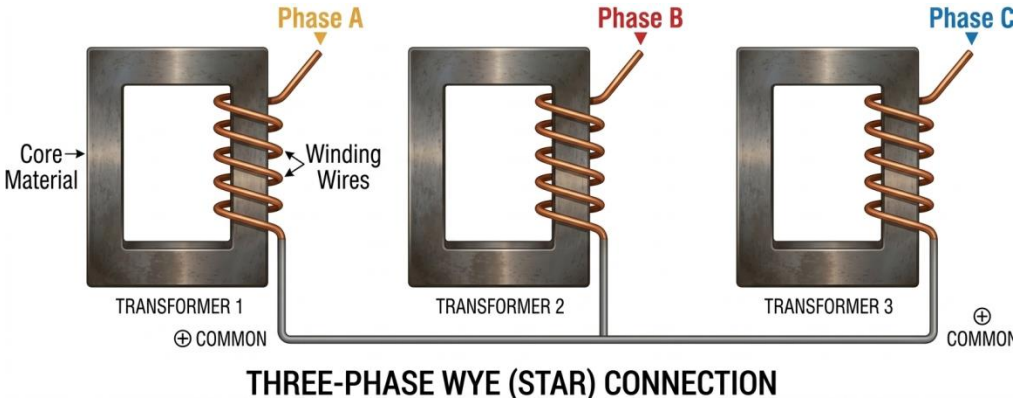


Figure V.10 Three-Phase Transformer — Magnetic Circuit

It is possible to produce the magnetic circuit shown (the primary and secondary windings are placed on the peripheral vertical columns): Each primary winding has n_1 turns, the

secondaryn_2 turns. The magnetic circuit of each winding closes in the central column; the fluxes are independent.

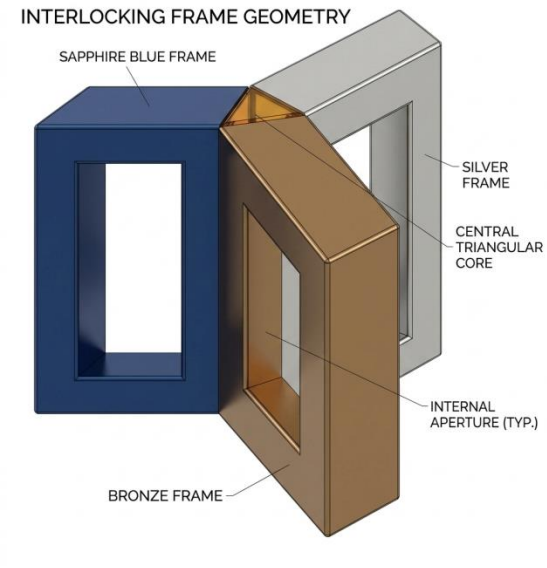


Figure V.11 InterlockingThree-Phase Magnetic Core Frame Geometry

$$V_1 = jn_2\omega\emptyset_1 \ , \qquad V_2 = jn_2\omega\emptyset_2 \ , \qquad V_3 = jn_3\omega\emptyset_3$$

Hence the flux in the central column:

$$\emptyset = \emptyset_1 + \emptyset_2 + \emptyset_3 = \frac{1}{jn_1\omega}(V_1 + V_2 + V_3)$$

If $V_1 + V_2 + V_3 = 0$, it is possible to remove the central column. This type of transformer is encountered very rarely due to the complexity of construction compared to the benefit obtained.

To simplify construction, coplanar magnetic circuits are used (only one winding per column is shown in the diagram): the sum of the fluxes in each of the vertical columns is zero $\emptyset_1 + \emptyset_2 + \emptyset_3 = 0$if flux leakages are neglected.

The magnetic circuit has linked fluxes: the different fluxes cannot establish themselves independently of each other.

Exercises

Exercise 01

A single-phase control and signaling transformer has the following characteristics: 230 V / 24 V, 50 Hz, 630 VA.

1. The total losses at rated load are 54.8 W. Calculate the rated efficiency of the transformer for cos $\Phi_2 = 1$ and cos $\Phi_2 = 0.3$.
2. Calculate the rated secondary current I_{2n}.
3. The no-load losses (iron losses) are 32.4 W. Deduce the Joule losses at rated load.

Deduce RS, the winding resistance referred to the secondary.

Exercise 02

A single-phase transformer has the following nameplate data: S = 2200 VA, $\eta = 0.95$, Primary $V_{1n} = 220$ V, Secondary $V_{2n} = 127$ V.

1. Calculate the rated primary current I_{1n}.
2. Calculate the rated secondary current I_{2n}.
3. The efficiency is specified for a load absorbing the rated current at rated secondary voltage and presenting a power factor cos$\varphi = 0.8$ lagging. Calculate the value of the losses in the transformer under these conditions.

Chapter VI

Introduction to Electrical Machines

VI.1 General Overview of Direct Current Machines (DC Machines)

A direct current machine is an energy converter that is fully reversible. It can operate either as a motor, converting electrical energy into mechanical energy, or as a generator, converting mechanical energy into electrical energy. In both cases, a magnetic field is required for the different conversion processes. Therefore, this machine is an electromechanical energy converter.

VI.2 Structure of the Machine

A direct current machine consists of three main parts:

1. Stator: the fixed, stationary part (it is a magnetic circuit).
2. Rotor: the rotating element (it is also a magnetic circuit).
3. Commutator system and brushes.

The rotor is connected to the transmission shaft. The stator and rotor are separated by an air gap.

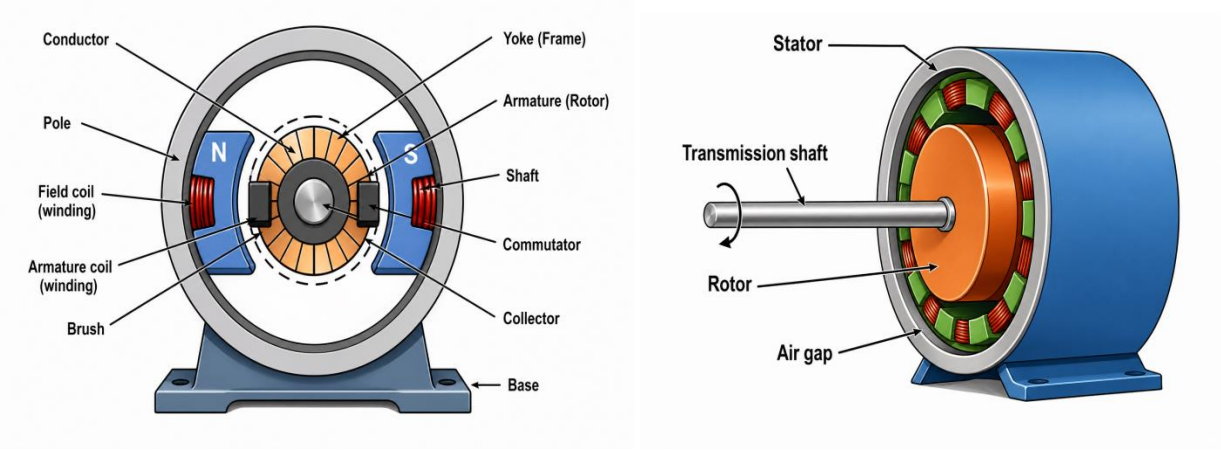


Figure VI.1 Structure of a DC Machine — General View

Depending on the machine:

- **Stator** = field system (contains the excitation circuit) or armature (where the induced voltage appears).
- **Rotor** = field system or armature.

VI.2.1 The Field System (or Excitation Circuit)

It is either a permanent magnet or an electromagnet (a winding supplied with direct current).

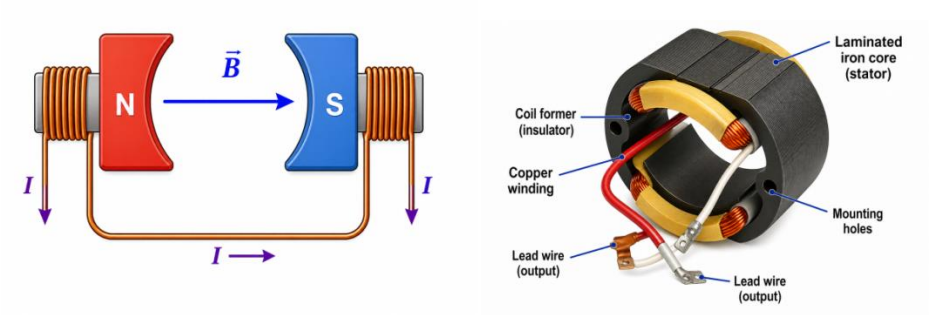


Figure VI.2 Field System (Excitation Circuit) — Permanent Magnet and Electromagnet

In a direct current machine, the field system is located in the stationary part of the machine (the stator). Its role is to create a magnetic field (the "field flux") in the rotor.

VI.2.2 The Armature (Power Circuit)

The armature is located on the rotor (the rotating part of the machine). It consists of a winding through which direct current flows (armature current).

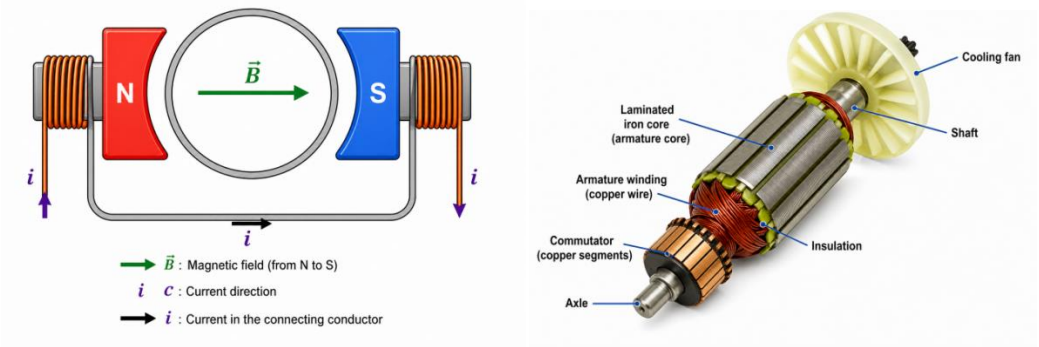


Figure VI.3 DC Machine Magnetic Field and Current Flow

The armature is the electromagnetic component responsible for receiving the induction from the field system and converting it into electrical energy (in generator mode) or into mechanical force (in motor mode).

VI.2.3 The Commutator and Brushes

The commutator–brush system consists of components that enable an electrical connection between the stationary part (stator) and the rotating part (rotor). In this case, direct wiring between the two parts is not possible.

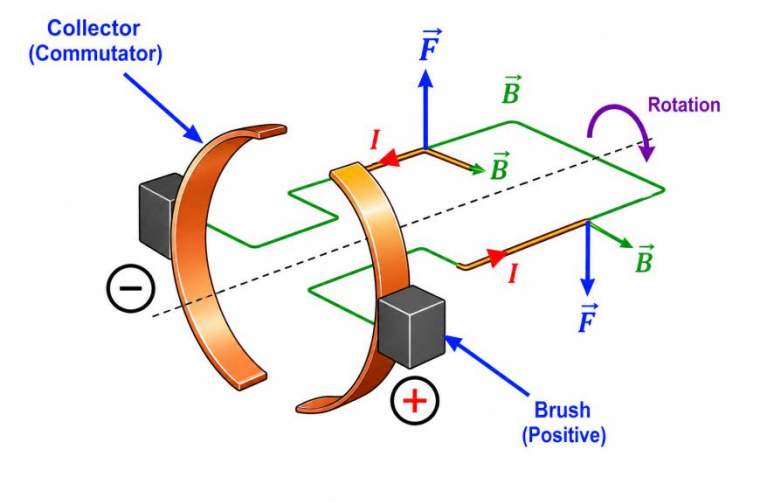


Figure VI.4 Commutator and Brush System

The commutator consists of a conductive ring (generally made of copper), divided into an even number of segments insulated from each other, and mounted with an insulating spacer on the machine shaft.

The electrical connection is established between the conductive segments and the fixed part on the stator (terminal block) through brushes made of carbon. Electrical power is supplied to the rotor winding through these contacts (motor operation), or conversely, the electrical energy produced by the rotor winding is collected through them (generator operation).

VI.3 Operating Principle

A direct current machine has a number N of active conductors. When the field circuit (the stator in our case) is supplied, these conductors are immersed in a magnetic field.

If we consider the rotor (the armature in our case), two operating modes are possible:

VI.3.1 Motor operation: the rotor conductors carry an electric current while being placed inside a magnetic field. They are then subjected to an electromagnetic force **F**.

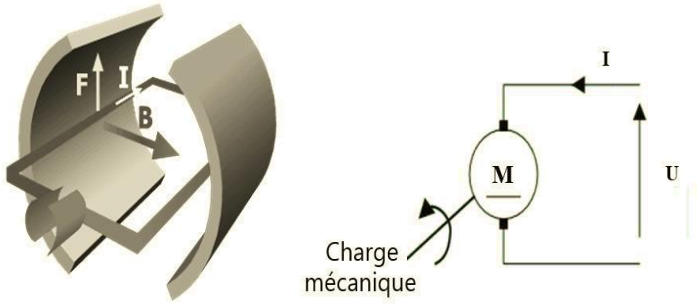


Figure VI.5 Motor Operation — Electromagnetic Force and Torque

A magnetic field **B** (field flux) combined with the armature current produces:

- Electromagnetic forces (Laplace forces)
- Electromagnetic torque
- Rotation of the rotor

VI.3.2 Generator operation (dynamo): the rotor conductors are both rotating and immersed in a magnetic field. They are then subjected to an electromotive force (EMF), i.e., a voltage.

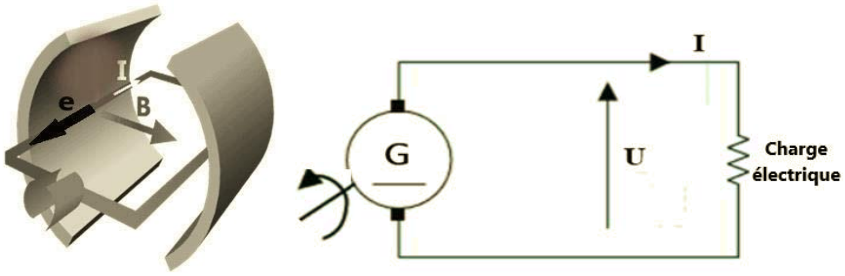


Figure VI.6 Generator Operation — Induced EMF

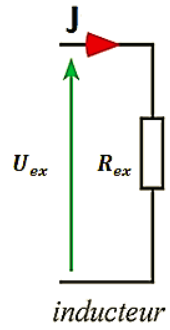
VI.4 Equivalent circuit of direct current machines

Since the stator and rotor windings are supplied with direct current, the only electrical component involved in the equivalent electrical circuit is electrical resistance.

VI.4.1 Equivalent circuit of the field system

The field coil (inductor coil) is supplied by a voltage called the excitation voltage UeU_eUe (to produce the magnetic flux). It is then traversed by an excitation current J. The inductor has a resistance called the inductor resistance or excitation resistance Re.

The voltage across the terminals of the inductor is:



$$U_{ex} = R_{ex}\, J$$

Figure VI.7 Equivalent Circuit of the Field System (Inductor)

VI.4.2 Equivalent circuit of the armature

In both cases, whether generator or motor, the armature winding is traversed by an electric current called the armature current I. The direction of this current is entering in the case of a motor and leaving in the case of a generator. The voltage across the armature terminals is U. In both cases, the rotor is rotating in a magnetic field. Therefore, the armature is always the seat of a voltage E (induced electromotive force).

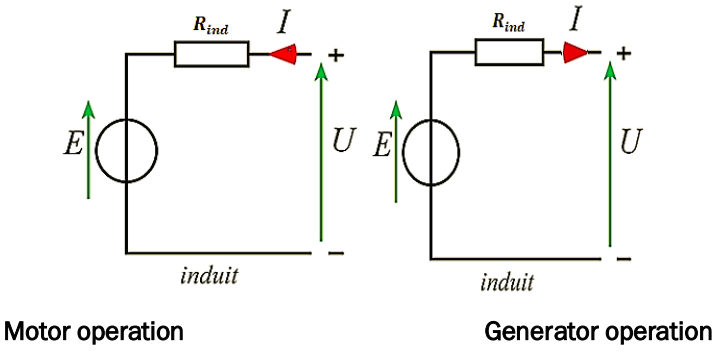


Figure VI.8 Equivalent Circuit of the Armature — Motor and Generator

Therefore, the equivalent circuit is:

- **Motor case**

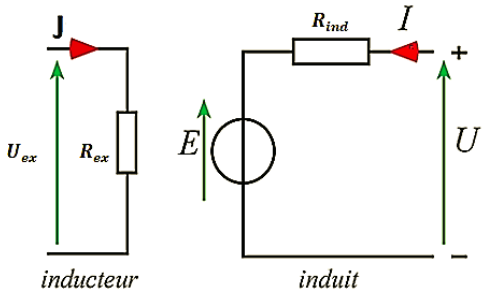


Figure VI.9 DC Motor — Equivalent Circuit

$$-U + R_{ind}\, I + E = 0$$

Thus:

$$U_{ex} = R_{ex}\, J$$

$$U = E + R_{ind}\, I$$

- **Generator case**

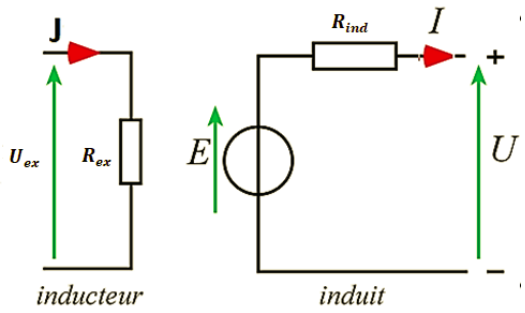


Figure VI.10 DC Generator — Equivalent Circuit

$$-U + R_{ind}\, I + E = 0$$

Thus:

$$U = E - R_{ind}\, I$$

$$U_{ex} = R_{ex}\, J$$

In practice

$$R_{ex} \gg R_{ind} \quad et \quad J \ll I$$

VI.5 Electromotive force and torque in a DC machine

VI.5.1 Expression of the induced EMF

The electromotive force induced in a machine is given by the expression (Faraday's law):

$$E = k_e \emptyset \Omega$$

E: induced EMF (DC voltage in volts)

Φ: magnetic flux produced under one pole by the field winding

Ω: rotational speed (in rad/s)$\Omega = \frac{2\pi n}{60}$ wheren is the speed in revolutions per minute rpm)

K_e: constant depending on the considered machine

VI.5.2 Expression of the electromagnetic torque

The electromagnetic torque in a machine is given by the expression (Laplace's law):

$$C_{em} = K_c \phi I$$

Cem: electromagnetic torque (in N·m)

I: armature current (in A)

Kc: constant depending on the machine

VI.6 Power balance and efficiency

VI.6.1 Motor case

The motor converts the electrical energy it absorbs into mechanical energy in order to drive a mechanical load (pump, propeller, etc.). A part of the absorbed energy is lost within the motor.

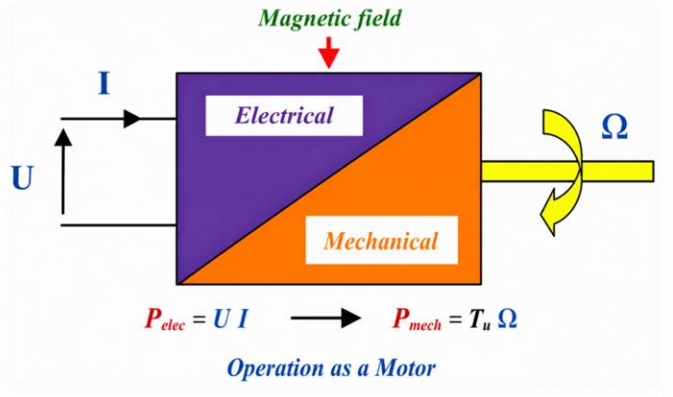


Figure VI.11 DC Motor

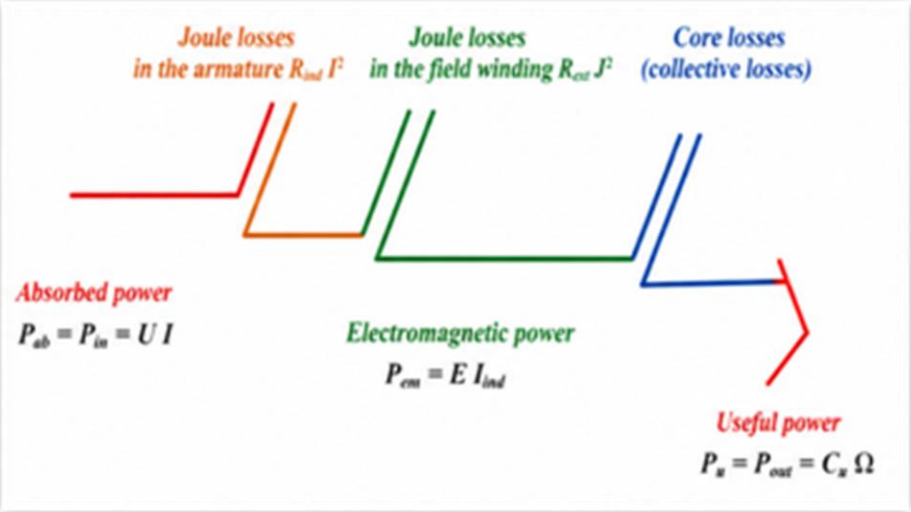


Figure VI.12 Power Balance Diagram

3. Induit : $PJ_{induit} = R_{ind} I^2$

4. Inducteur : $PJ_{induct} = R_{ext} J^2 = U_{ext} J$

The powers:

1. Absorbed power = electrical power: $P_{elect} = U. I$
2. Useful power = mechanical power: $P_{mech} = T. \Omega$

Joule losses:

3. Armature: $PJ_{armature} = R_{ind.} I^2$
4. Field winding (exciter): $PJ_{field} = R_{ext.} J^2 = U_{ext.} J$

Constant (or collective) losses:

1. Mechanical losses (friction, vibrations, ventilation, etc.)
2. Iron losses (in the ferromagnetic material)

The efficiency of the motor is:

$$\eta = \frac{P_{mec}}{P_{elec}} = \frac{P_u}{P_u + \sum pertes}$$

VI.6.2 Generator case

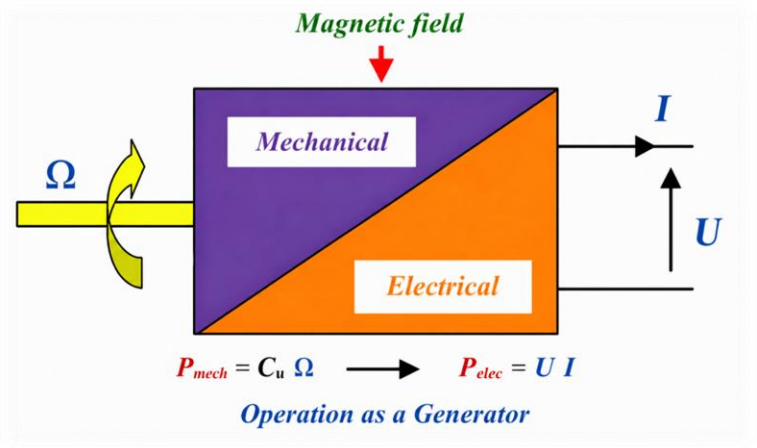


Figure VI.13 DC Generator

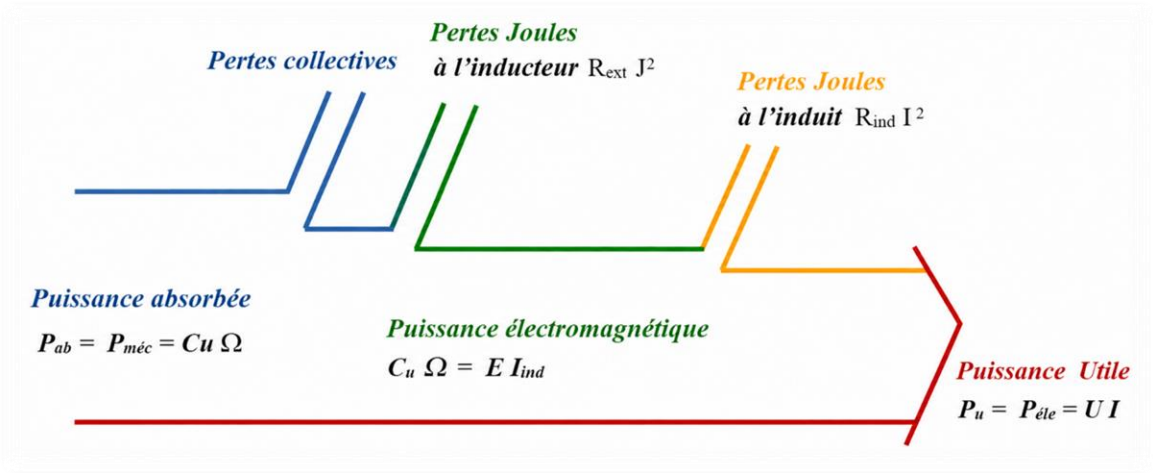


Figure VI.14 Power Balance and Efficiency

The powers:

1. Absorbed power = mechanical power: $P_{mech}=C\cdot\Omega$
2. Usefulpower = electrical power:$P_{elec}=U\cdot I$

Joule losses:

3. Armature:$PJ_{armature}=R_{ind}\cdot I_2$
4. Fieldwinding(exciter):$PJ_{field}=R_{ext}$

Constant (or collective) losses:

1. Mechanical losses (friction, vibrations, ventilation, etc.)
2. Iron losses (in the ferromagnetic material)

The efficiency of the generator is:

$$\eta = \frac{P_{elec}}{P_{mec}} = \frac{P_u}{P_u + \sum pertes}$$

Exercises

Exercise 01

An extraction machine is driven by a DC motor. The field winding is supplied by a voltage Uext=600 V and is traversed by a constant excitation current: J=30 A.

The armature, with resistance Ra=0.012 Ω, is supplied by a source providing an adjustable voltage from 0 V up to its nominal value: Un=600 V. The armature current I has a nominal value: In=1.50 kA. The nominal rotational speed is n=30 rpmn.

1. Write the relationship between U, E, and I.
2. Express the power absorbed by the motor armature and calculate its numerical value.
3. Express the total power absorbed by the motor and calculate its numerical value.
4. Express the total power lost due to Joule effect and calculate its numerical value.
5. Knowing that the other losses are equal to 27 kW, express and calculate the useful power and the efficiency of the motor.
6. Express and calculate the useful torque Cu and the electromagnetic torque Cem.

Given:Ω=2πn

Exercise 02

A separately excited DC motor operates under a voltage U=160 V. The current absorbed by the motor is I=8.2 A. The rotational speed is 1420 rpm; its useful shaft torque is u=7.1 Nm. The field current is J=0.7 A. The armature resistance is Rind=0.45 Ω and the field resistance is Rex=220 Ω. Calculate:

1. The back electromotive force EEE.
2. The electromagnetic torque of the motor.
3. The efficiency of the motor.

Conclusion

This course, Fundamentals of Electrotechnics I, has provided a comprehensive introduction to the core principles and methods of electrical engineering as applied to circuits, magnetic systems, transformers, and rotating machines. Through a progressive and structured approach, students have developed the analytical tools necessary to model, analyze, and understand a wide variety of electrotechnical systems encountered in engineering practice.

Starting from the mathematical framework of complex numbers, the course established the foundation for the study of sinusoidal signals and complex impedances. This allowed for a rigorous treatment of both DC and AC electrical circuits, including the calculation of power in its various forms — active, reactive, and apparent — and the application of fundamental theorems such as Kirchhoff's laws and Boucherot's theorem.

The study of magnetic circuits extended the electrical circuit analogy to magnetic phenomena, enabling the understanding of electromagnetic induction, inductance, and the behavior of ferromagnetic materials. This knowledge provided the necessary basis for the analysis of transformers, which are indispensable components in modern power distribution systems. The different types of transformers — isolation, pulse, autotransformer, and three-phase — were examined both theoretically and in terms of their practical applications.

Finally, the introduction to direct current machines offered an insight into the operating principles of electromechanical energy conversion. The concepts of induced EMF, electromagnetic torque, and power balance in both motor and generator modes are fundamental to the understanding of electrical drives and generation systems.

Key Takeaways

- Complex numbers are an indispensable mathematical tool for the analysis of sinusoidal signals and AC circuits.
- Kirchhoff's laws and the concept of impedance allow systematic analysis of any electrical circuit in steady-state.
- Power in AC circuits involves three complementary quantities: active power (P), reactive power (Q), and apparent power (S).
- Magnetic circuits obey laws analogous to those of electrical circuits, facilitating their calculation and understanding.
- The transformer is a key device for voltage adaptation, galvanic isolation, and efficient energy transmission in power systems.
- DC machines are fully reversible electromechanical converters, capable of operating as both motors and generators.

References bibliographies :

[1]. J.P Perez, Electromagnétisme Fondements et Applications, 3eme Edition, 1997.

[2]. A. Fouillé, Electrotechnique à l'Usage des Ingénieurs, 10e édition, Dunod, 1980.

[3]. C. François, Génie électrique, Ellipses, 2004

[4]. L. Lasne, Electrotechnique, Dunod, 2008

[5]. J. Edminister, Théorie et applications des circuits électriques, McGraw Hill, 1972

[6]. D. Hong, Circuits et mesures électriques, Dunod, 2009

[7]. M. Kostenko, Machines Electriques - Tome 1, Tome 2, Editions MIR, Moscou, 1979.

[8]. M. Jufer, Electromécanique, Presses polytechniques et universitaires romandes- Lausanne, 2004.

[9]. A. Fitzgerald, Electric Machinery, McGraw-Hill Higher Education, 2003.

[10].J. Lesenne, Introduction à l'électrotechnique approfondie. Technique et Documentation, 1981.

[11].P. Maye, Moteurs électriques industriels, Dunod, 2005.

[12]. S. Nassar, Circuits électriques, Maxi Schaum.

[13] S. J. Chapman, *Electric Machinery Fundamentals*, 6th ed. New York, NY, USA: McGraw-Hill, 2022.

[14] C. K. Alexander and M. N. O. Sadiku, *Fundamentals of Electric Circuits*, 7th ed. New York, NY, USA: McGraw-Hill, 2023.

[15] J. W. Nilsson and S. A. Riedel, *Electric Circuits*, 11th ed. Boston, MA, USA: Pearson, 2022.

[16] E. Kreyszig,*Advanced Engineering Mathematics*, 10th ed. Hoboken, NJ, USA: Wiley, 2020.

[17] R. L. Boylestad, *Introductory Circuit Analysis*, 13th ed. Boston, MA, USA: Pearson, 2021.