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theme
On the $\hat{u}$-channel Leptoquark exchange in the $q'\bar{q} \rightarrow e^{-}\bar{\nu}$ charged current process

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Abstract

In this thesis we examine the virtual Leptoquarks (LQ) exchange in the $\hat{u}$-channel of the $q'\bar{q} \to \bar{\nu}e^-$ process, where we consider that direct LQ production is inaccessible to current colliders.

By calculating the total cross section at the partonic level, accounting for both Standard Model (W^- exchange) and New physics contributions, we investigate the impact of LQ exchange and the emergence of contact interactions (CI). Using CI energy scale bounds from Deep Inelastic Scattering (DIS) experiments, we establish upper limits on LQ coupling.

Keywords: scalar leptoquark, cross section, contact interaction.

Dans cette thèse, nous examinons l'échange de leptoquarks (LQ) virtuel dans le canal $\hat{u}$ dans le processus de diffusion $q'\bar{q} \to \bar{\nu}e^-$, où la production directe de LQ est inaccessible aux collisionneurs actuels.

En calculant la section efficace totale au niveau partonique, en tenant compte à la fois des contributions du Modèle Standard (échange de W^-) et de la nouvelle physique, nous étudions l'impact de l'échange de LQ et l'émergence des interactions de contacts. En utilisant les limites supérieures d'échelle d'énergie pour les interactions de contact issues des expériences de diffusion profondément inélastique, nous établissons des limites supérieures sur le couplage des LQ.

Mots-clés: leptoquark scalaire, section efficace, interaction de contact.

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Contents

Chapter I

Introduction

The Standard Model of particle physics is a theoretical framework that describes the fundamental particles and their interactions. It is a highly successful model that provides a comprehensive understanding of the subatomic world[1]. This Model identifies two categories of elementary particles: fermions and bosons, and recognizes three fundamental forces or interactions : the electromagnetic force, the strong nuclear force, and the weak nuclear force.

The Standard Model is based on the principles of quantum field theory, which describes particles as excitations of their respective quantum fields[2]. These fields exist everywhere in space and interact with each other based on the exchange of force-carrying particles.

Symmetries and gauge invariance in the other hand are fundamental principles in understanding particle physics. Gauge invariance ensures the consistency of the theory under local transformations, while symmetries (Lorentz, Chiral Symmetry, Flavor Symmetry, est) provide fundamental conservation laws such as conservation of energy and momentum.

The Standard Model of particle physics has been tremendously successful in explaining the behaviour of elementary particles and their interactions.
However, it also has several limitations that suggest the need for a more comprehensive theory. Here are some of its key limitations :

1. **Gravity:** The Standard Model does not incorporate gravity, which is described by Einstein's theory of general relativity[3]. Gravity is responsible for the behaviour of massive objects and the structure of the universe on a large scale. The unification of gravity with the other fundamental forces remains an open challenge.

2. **Dark Matter:** Observations suggest that a significant portion of the universe's mass is made up of dark matter, a form of matter that does not interact with light or other electromagnetic radiation. The Standard Model does not include a particle that could account for dark matter[4], and its nature remains one of the biggest mysteries in modern physics.

3. **Neutrino Masses:** The Standard Model originally assumed that neutrinos were massless particles. However, experiments have shown that neutrinos undergo oscillations[5], indicating that they have nonzero masses. The Standard Model does not provide a natural explanation for neutrino masses and requires an extension to accommodate them.

4. **Matter-Antimatter Asymmetry:** According to the Standard Model, the laws of physics should treat matter and antimatter symmetrically. However, the universe is dominated by matter, and there is a significant asymmetry between the two[6]. This phenomenon, known

as the matter-antimatter asymmetry or baryon asymmetry, is not accounted for within the framework of the Standard Model.

5. **Hierarchy Problem:** The Higgs boson, discovered in 2012, plays a crucial role in giving elementary particles their masses. However, the mass of the Higgs boson is highly unstable and susceptible to quantum corrections. This leads to a fine-tuning problem known as the hierarchy problem, where the Higgs mass is much lighter than what would be expected based on quantum corrections[7]. It suggests the existence of new physics that stabilizes the Higgs mass.

6. **Unification of Forces:** The Standard Model describes three fundamental forces. Physicists strive for a more fundamental theory that can unify these forces into a single framework[8], often referred to as "Grand Unified Theory" or "Theory of Everything."

Various theoretical frameworks and extensions to the Standard Model, such as supersymmetry [9], string theory [10], and extra dimensions [11], have been proposed to overcome its limitations and address fundamental questions in physics. Leptoquarks (LQs) are hypothetical particles introduced in these extensions, connecting leptons and quarks and suggesting a unified theory. Since they possess properties of both particles, understanding them could shed light on the mechanisms which underly particle interactions. The discovery of neutrino oscillations, indicating neutrino mass, opened new avenues beyond the Standard Model, and leptoquarks could contribute in explaining neutrino mass and flavors[12]. Additionally, leptoquarks are predicted by certain Grand Unified Theories (GUTs) aiming to unify all fundamental forces. Their discovery would provide experimental evidence and contribute to a more comprehensive understanding of the laws governing the universe.

Direct producion of Leptoquarks at existing colliders proves challenging due to their high masses, which place them beyond the kinematic reach of current experimental capabilities. As a result, indirect approaches are crucial in probing the properties and interactions of Leptoquarks.

In this master's thesis, we focus on exploring the virtual exchange of Leptoquarks in the $\hat{u}$-channel of the $q'\bar{q}e^-\bar{\nu}$ process. This later allows for the investigation of Leptoquark effects through virtual interactions, offering insights into their couplings and potential new physics signatures.

Through the subsequent chapters, we present the theoretical framework, outline the methodology employed for calculations, present our results, and discuss their implications.

Chapter II

Theoretical framework

2.1 Leptoquarks and their theoretical properties

There are no interactions involving a quark, a lepton and a boson in the Standard Model. There are vector bosons that are either coloured or electrically charged, but no boson carrying colour and electric charge. This is a reflection of the fact that classically the leptons and quarks appear to be independent and unrelated ingredients in the Standard Model. However striking symmetry between quarks and leptons in the Standard Model strongly suggests that, if there exist a more fundamental theory it should also introduces a more fundamental relation between them[13]. It would therefore seem natural to have interactions between the quarks and leptons in any extension of the Standard Model[14], and, consequently, bosons coupling to a lepton and a quark. These bosons (leptoquarks) have the following properties:

1. They carry colour charge.
2. there are leptoquarks with spin 1 (vector leptoquark) or 0 (scalar leptoquark).
3. The electrical charge of LQs is fractional, such as +2/3 or -1/3 in units of the elementary charge.
4. They carry both lepton and baryon numbers, which define an additional quantum number called fermion number F as F = L + 3B.

Leptoquarks that couple to e^+q have a fermion number of F = 0 and those coupling to e^-q have a fermion number of F = 2[15].

The coupling properties of leptoquarks describe their interactions with other particles. These properties determine how leptoquarks couple to quarks, leptons, and gauge bosons, and play a crucial role in their production and decay processes. The specific coupling properties of leptoquarks depend on the theoretical framework in which they are considered and the specific model assumptions.

Leptoquarks can be produced in high-energy particle collisions through various processes. The specific production mechanisms depend on the properties and interactions of the leptoquark, as well as the energy of the collision. Here are a few common ways in which leptoquarks can be produced:

1. Lepton-Quark Fusion: Leptoquarks can be produced in collisions between a quark and a Lepton[16]. If a sufficiently high-energy collision occurs between the two particles, the energy can be converted into the mass of a leptoquark. The quarks and lepton involved in the collision must possess the appropriate quantum numbers to combine and form the leptoquark.

2. Gluon-gluon fusion: In quantum chromodynamics (QCD), the theory of the strong force, gluons are the force-carrying particles. Leptoquarks can be produced via the fusion of gluons[17]. Gluons can interact and exchange energy, which can result in the creation of leptoquark anti-leptoquark pairs or leptoquark-gluon pairs.

3. Quark-anti-quark annihilation: Leptoquarks can also be created through the annihilation of a quark and an anti-quark. When a quark and an anti-quark collide with sufficient energy, they can annihilate, resulting in the formation of a leptoquark anti-leptoquark pair.

It's important to note that the specific production mechanisms and rates of leptoquarks depend on their properties, such as their electric charge, spin, and coupling strengths to other particles. These properties influence the probability of leptoquark production and subsequent detection.

To date, experiments at colliders, including the LHC, have not provided conclusive evidence for leptoquarks. However, these experiments have placed stringent constraints on the properties and masses of leptoquarks, ruling out certain regions of parameter space predicted by theoretical models. Experimental collaborations at colliders continue to refine search strategies and analyze more data to probe further into the existence of leptoquarks. They explore higher energies and new collision channels, aiming to increase sensitivity to possible leptoquark signals.

2.2 Leptoquarks classification

Leptoquarks are classified as first, second, or third-generation, depending on the generation of leptons to which they couple.

There are basic conditions that leptoquarks must satisfy in order to avoid the most severe indirect limits [17]:

- Leptoquarks couple diagonally meaning that leptoquarks can interact with particles of one generation of quarks and leptons but not with particles of different generations. In other words, only to quarks and leptons of the same generation, The concept of diagonal couplings is related to the principle of flavor conservation, so there are no intergenerational couplings and thus no flavor-changing neutral currents (FCNC)[18].

- Leptoquark couplings are purely chiral [18], meaning they either couple to left-handed leptons or right-handed leptons.

A general classification of leptoquark states was proposed by Buchmuller,Ruckl and Wyler [19], this model is based on the assumption that new interactions should respect the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry of the Standard Model.In addition to other assumption [20] there are 10 possible states of scalar and vector leptoquarks.

In this work we are interested in first generation scalar leptoquarks. Their interaction Lagrangian is given by[21]:

$$L_{LQ} = S_0(\lambda_{LS_0}\bar{l}i\tau_2 q^c + \lambda_{RS_0}\bar{e}u^c) + hc. \tag{1}$$

Where τ_2 is a Pauli matrix, so $i\tau_2$ provides the antisymmetric SU(2) contraction.and the SU(2) singlet leptoquarks S have subscript 0, These leptoquarks can contribute to $q\overline{q} \to \overline{\nu}e^-$ where $\bar{l}$ is the anti lepton involved, $q \in \{u, d\}$ and the superscript c indicates the color charge, λ is the coupling and the L/R index on it reflects the lepton chirality.

And the Hermitian Conjugate (hc) is :

$$(-\lambda_{LS_0}\overline{q}^c i\tau_2 l + \lambda_{RS_0}\overline{u}^c e)S_0^+.$$

Chapter III

Leptoquarks exchange in $q'\bar{q}e^-\bar{\nu}$ process

Actual colliders have sensitivity to new physics from beyond their kinematic reach, which could, for instance materialise, as extra events at high energy. Such a plateau at high centre of mass energy is commonly parameterized by a four-fermion contact interaction with coefficient $\frac{4\pi}{\Lambda^2}$ and experimental results are quoted as lower bounds on Λ.

In this section we attempt to extract bounds on a first generation scalar LQ exchanged in $\hat{u}$-channel in $q'\bar{q}e^-\bar{\nu}$ process, using experimental contact interaction bounds. DIS@HERA [1] searched for contact interaction of the form:

$$\mathcal{L}_{CI} = \frac{4\pi}{\Lambda^2}\left[u\gamma^\mu P_L \overline{d}\right]\left[\overline{\nu}\gamma_\mu P_L e\right] \tag{2}$$

and set bounds of order[22]:

$$\Lambda \geq 2.4\,\mathrm{TeV} \tag{3}$$

Since the quarks and the antiquarks involved in CI operator (2) are $\bar{u}$ and d, we will be interested to the LQ (S_O) exchanged in $\bar{u}d \to \bar{\nu}e^-$ process. So, we attempt to set limits on S_0 coupling using

$$\frac{\lambda^2}{2M_{LQ}^2} \leq \frac{4\pi}{\Lambda^2} \tag{4}$$

3.1 Standard model and new physics contributions to the total cross section

Regarding the SM contribution, the process under consideration is mediated by W^- boson in the $\hat{s}$-channel. The scattering amplitude corresponding to the associated Feynman diagram shown in figure 1, is given by:

$$i\mathcal{M}_W = \frac{G_f M_W^2}{\sqrt{2}\hat{s}}\left[\overline{v}^j(p_1)\gamma^\mu P_L u^i(p_2)\right]\left[\overline{u}(k_4)\gamma_\mu P_L v(k_3)\right] \tag{5}$$

Where, the indices $i(j)$ stand for quarks (antiquark) colors, M_W is the W mass boson, $P_L = \frac{1}{2}(1-\gamma^5)$ is for the left handed chiral projector and G_F is the Fermi constant coupling within the Weak theory. The specific quark flavours that participate in this process satisfy the conservation

[1] Deep inelastic scattering (DIS) is a process in which a lepton scatters off a nucleon with large negative four-momentum transfer Q^2. Two types of deep inelastic scattering processes are measured at HERA (Hadron Electron Ring Accelerator) located at Deutsches Elektronen-Synchrotron laboratory in Hamburg, Germany. The processes are classified according to the particles exchanged between the interacting electron and proton and are called neutral current (NC) or charged current (CC).

laws, such as the conservation of Lepton number L, Baryon number B and electric Charge Q.

At high energy, the SM contribution could be interfered with the LQ exchange in $\hat{u}$-channel as illustrated in Figure 2. Actually, the only LQ which contributes to this process is the Leptoquark S_0 involving up-quark and antidown-quark flavors. The 4-fermion vertex for this scalar leptoquark is shown in the table below[13] :

interaction	4-fermion vertex	Fierz-transformed vertex
$(\lambda_{LS_o}\bar{q}_L^c i\tau_2 \ell_L + \lambda_{RS_o}\bar{u}_R^c e_R)S_o^\dagger$	$\frac{\lambda_{LS_o}^2}{m_o^2}(\bar{u}_L^c e_L)(\bar{\nu}_L d_L^c)$	$\frac{\lambda_{LS_o}^2}{2m_o^2}(\bar{u}_L^c \gamma^\mu d_L^c)(\bar{\nu}_L \gamma_\mu e_L)$

Table 1. 4-fermion vertices for S_0 leptoquark.

The scattering amplitude associated with LQ exchange in $\hat{u}$-channel is:

$$i\mathcal{M}_{LQ} = \frac{\lambda^2}{2\Big(M_{LQ}^2 - (p_1 - k_4)^2\Big)} \Big[\overline{v}(p_1)\gamma^\mu P_L u(p_2)\Big] \Big[\overline{u}(k_4)\gamma_\mu P_L v(k_3)\Big] \tag{6}$$

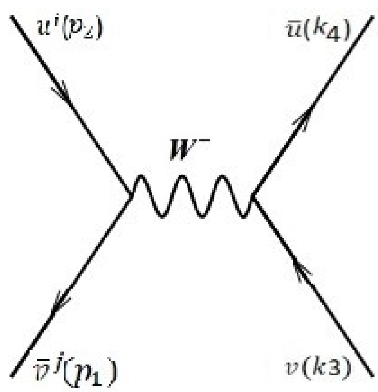

Figure 1. Feynman diagram in the $\hat{s}$ channel for the $u\overline{d} \to \overline{\nu}e^-$ interaction by the exchange of a W^- boson.

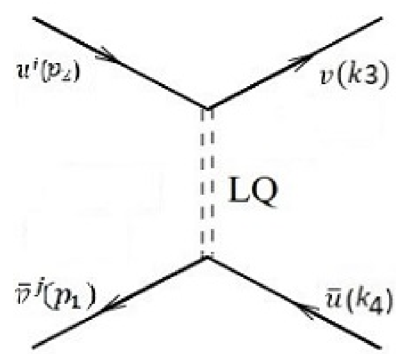

Figure 2. Feynman diagram in the $\hat{u}$ channel for the $u\overline{d} \to \overline{\nu}e^-$ interaction by the exchange of a Leptoquark.

The contact interaction induced by S_0 when $\hat{s} \ll M_{S_0}^2$ (see the third column of table 1) is reached when $\hat{u} \to 0$. The total scattering amplitude is the sum of the two contributions (eqs (5) and (6)) from both Feynman diagrams shown in Figures 1 and 2, this is given by:

$$i\mathcal{M} = \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{LQ}^2 - (p_1 - k_4)^2)}\right) \Big[\overline{v}(p_1)\gamma^\mu P_L u(p_2)\Big] \Big[\overline{u}(k_4)\gamma_\mu P_L v(k_3)\Big] \tag{7}$$

And the partonic cross section (see appendix A1) will be :

$$\frac{d\sigma}{d\hat{u}} = \frac{\overline{|i\mathcal{M}|}^2}{16\pi\hat{s}^2} \tag{8}$$

The bar on scattering amplitude squared indicates that the cross section is averaged over initial colors and spins.

$$\overline{|i\mathcal{M}|}^2 = (\frac{1}{2})^2 \sum_{\text{in spin}} \sum_{\text{out spin}} (\frac{1}{3})^2 \sum_{\text{in color}} \sum_{\text{out color}} |i\mathcal{M}|^2 \tag{9}$$

To calculate the cross-section, we need to follow a set of Feynman rules [23],we can notice that the two amplitudes in (5) and (6) have the same spinor part, so it is practical to write $i\mathcal{M}$ as spinor part ($\mathcal{S}$) and propagator part ($i\mathcal{P}$) such as:

$$i\mathcal{M} = (\mathcal{S})(i\mathcal{P}) \tag{10}$$

where,

$$i\mathcal{P} = \frac{G_f M_W^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{S_0}^2 - (p_1 - k_4)^2)} \tag{11}$$

and,

$$\mathcal{S} = \Big[u^i(p_2)\gamma^\mu P_L \overline{v}^j(p_1)\Big]\Big[\overline{u}(k_4)\gamma_\mu P_L v(k_3)\Big] \tag{12}$$

so,

$$\overline{|i\mathcal{M}|}^2 = \overline{|\mathcal{S}|}^2 |i\mathcal{P}|^2 \tag{13}$$

Let's calculate first the term $\overline{|\mathcal{S}|}^2$:

$$|\mathcal{S}|^2 = \Big[\overline{v}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p_2)\Big]\Big[\overline{u}(k_4)\gamma_\mu \frac{1}{2}(1-\gamma^5)v(k_3)\Big] \\ \Big[\overline{v}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p_2)\Big]^*\Big[\overline{u}(k_4)\gamma_\mu \frac{1}{2}(1-\gamma^5)v(k_3)\Big]^* \tag{14}$$

Using the definition in equation (9) and the calculation details in Appendix A2, we get,

$$\overline{|\mathcal{S}|}^2 = \frac{3}{4\times 9} Tr\Big[\gamma^\mu \frac{1}{2}(1-\gamma^5)(\gamma^\alpha p_{\alpha 2} + m_2)\overline{\gamma}^\nu \frac{1}{2}(1-\gamma^5)(\gamma^\beta p_{\beta 1} + m_1)\Big] \\ Tr\Big[\gamma_\mu \frac{1}{2}(1-\gamma^5)(\gamma^\lambda k_{\lambda 4} + m_4)\overline{\gamma}_\nu \frac{1}{2}(1-\gamma^5)(\gamma^\sigma k_{\sigma 3} + m_3)\Big] \tag{15}$$

Knowing that $P_L = \frac{1}{2}(1-\gamma^5)$, $(P_L)^2 = P_L$, $P_L\gamma^\mu = -\gamma^\mu P_L$, $\overline{\gamma}^\nu = \gamma^\nu$.

And because $m_1 = m_2 = m_3 = m_4 = 0$. (relativistic limit)

$$\overline{|\mathcal{S}|}^2 = \frac{1}{12} Tr\Big[\gamma^\mu P_L P_L \gamma^\alpha \gamma^\nu \gamma^\beta\Big] p_{\alpha 2} p_{\beta 1} Tr\Big[\gamma_\mu P_L P_L \gamma^\lambda \gamma_\nu \gamma^\sigma\Big] k_{\lambda 4} k_{\sigma 3} \tag{16}$$

$$\overline{|\mathcal{S}|}^2 = \frac{1}{48} Tr\Big[\gamma^\mu (1-\gamma^5) \gamma^\alpha \gamma^\nu \gamma^\beta\Big] p_{\alpha 2} p_{\beta 1} Tr\Big[\gamma_\mu (1-\gamma^5) \gamma^\lambda \gamma_\nu \gamma^\sigma\Big] k_{\lambda 4} k_{\sigma 3} \tag{17}$$

for the first trace term

$$\begin{aligned}
&= p_{\alpha 2} p_{\beta 1}\Big[Tr[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta] - Tr[\gamma^5 \gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta]\Big] \\
&= p_{\alpha 2} p_{\beta 1}\Big[4(g^{\mu\alpha} g^{\nu\beta} - g^{\mu\nu} g^{\alpha\beta} + g^{\mu\beta} g^{\alpha\nu}) + 4i\epsilon^{\mu\alpha\nu\beta}\Big] \\
&= 4\Big[(g^{\mu\alpha} p_{\alpha 2} g^{\nu\beta} p_{\beta 1} - g^{\mu\nu} g^{\alpha\beta} p_{\alpha 2} p_{\beta 1} + g^{\mu\beta} p_{\beta 1} g^{\alpha\nu} p_{\alpha 2}) + i\epsilon^{\mu\alpha\nu\beta} p_{\alpha 2} p_{\beta 1}\Big] \\
&= 4\Big[(p_2^\mu p_1^\nu - g^{\mu\nu} p_2^\beta p_{\beta 1} + p_1^\mu p_2^\nu) + i\epsilon^{\mu\alpha\nu\beta} p_{\alpha 2} p_{\beta 1}\Big] \\
&= 4\Big[(p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)) + i\epsilon^{\mu\alpha\nu\beta} p_{\alpha 2} p_{\beta 1}\Big]
\end{aligned} \tag{18}$$

so,

$$Tr\Big[(\gamma^\mu - \gamma^\mu \gamma^5)(\gamma^\alpha)\gamma^\nu(\gamma^\beta)\Big] p_{\alpha 2} p_{\beta 1} = 4\Big[(p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)) + i\epsilon^{\mu\alpha\nu\beta} p_{\alpha 2} p_{\beta 1}\Big] \tag{19}$$

The same is for the second term,

$$Tr\Big[(\gamma_\mu - \gamma_\mu \gamma^5)(\gamma^\lambda)\gamma_\nu(\gamma^\sigma)\Big] k_{\lambda 4} k_{\sigma 3} = 4\Big[(k_{4\mu} k_{3\nu} + k_{3\mu} k_{4\nu} - g_{\mu\nu}(k_4 \cdot k_3)) + i\epsilon_{\mu\lambda\nu\sigma} k_4^\lambda k_3^\sigma\Big] \tag{20}$$

Replacing (19) and (20) in (17) we get,

$$\begin{aligned}
\overline{|\mathcal{S}|}^2 = \frac{1}{48} 4\Big[(p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)) + i\epsilon^{\mu\alpha\nu\beta} p_{\alpha 2} p_{\beta 1}\Big] \\
4\Big[(k_{4\mu} k_{3\nu} + k_{3\mu} k_{4\nu} - g_{\mu\nu}(k_4 \cdot k_3)) + i\epsilon_{\mu\lambda\nu\sigma} k_4^\lambda k_3^\sigma\Big]
\end{aligned} \tag{21}$$

so after expanding equation (21) and calculating the Levi-Civita terms (see Appendix A3) the equation becomes ;

$$\begin{aligned}
&= \frac{1}{12}\Big[2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3) - 2!(\delta_{\alpha\lambda}\delta_{\beta\sigma} - \delta_{\alpha\sigma}\delta_{\beta\lambda}) p_{\alpha 2} p_{\beta 1} k_4^\lambda k_3^\sigma\Big] \\
&= \frac{1}{12}\Big[2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3) - 2!(\delta_{\alpha\lambda}\delta_{\beta\sigma} p_{\alpha 2} p_{\beta 1} k_4^\lambda k_3^\sigma - \delta_{\alpha\sigma}\delta_{\beta\lambda} p_{\alpha 2} p_{\beta 1} k_4^\lambda k_3^\sigma)\Big] \\
&= \frac{1}{12}\Big[2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3) - 2!(p_{\alpha 2} p_{\beta 1} k_4^\alpha k_3^\beta - p_{\alpha 2} p_{\beta 3} k_4^\beta k_3^\alpha)\Big] \\
&= \frac{1}{12}\Big[2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3) - 2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3)\Big]
\end{aligned}$$

Finally,

$$\overline{|\mathcal{S}|}^2 = \frac{1}{12}\Big[4(p_1 \cdot k_4)(p_2 \cdot k_3)\Big] \tag{22}$$

Using Mandelstam variable in the relativistic limit (See Appendix A4).

$$\overline{|\mathcal{S}|}^2 = \frac{\hat{u}^2}{3}. \tag{23}$$

3.2 Including the Leptoquark propagator

To deal with the scattering probability of equation (13), we should also, calculate the amplitude coming from the propagator part, i.e, equation (10).
So,

$$\overline{|i\mathcal{P}|}^2 = i(-i)\left[\frac{G_f M_w^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{LQ}^2 - (p_2 - k_4))^2}\right]\left[\frac{G_f M_w^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{LQ}^2 - (p_2 - k_4))^2}\right] \tag{24}$$

$$\overline{|i\mathcal{P}|}^2 = \underbrace{\left[\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right]^2}_{W\ boson\ exchange} + \underbrace{\left[\frac{\lambda^2}{2(M_{LQ}^2 - (p_2 - k_4))^2}\right]^2}_{New\ physics\ term(LQ^2)} + \underbrace{\frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2 - (p_2 - k_4))^2}}_{Interference\ term(W \times LQ)} \tag{25}$$

Now, substituting (25) and (23) in (13), then the result in (8) we get,

$$\frac{d\sigma_{LQ+W}}{d\hat{u}} = \frac{\hat{u}^2}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 + \left(\frac{\lambda^2}{2(M_{LQ}^2 - \hat{u})}\right)^2 + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2 - \hat{u})}\right] \tag{26}$$

Its important to note that the Mandelstam variables are linked as follows:

$$\hat{s} + \hat{t} + \hat{u} = (p_1 + p_2)^2 + (p_1 + k_3)^2 + (p_1 + k_4)^2$$

To find the total cross section, we integrate over the kinematic variable $\hat{u}$,

$$\sigma_{LQ+W} = \int_0^{\hat{s}} \frac{|\overline{M}|^2}{16\pi\hat{s}^2} d\hat{u} \tag{27}$$

$$\sigma_{LQ+W} = \int_0^{\hat{s}} \frac{\hat{u}^2}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 + \left(\frac{\lambda^2}{2(M_{LQ}^2 - \hat{u})}\right)^2 + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2 - \hat{u})}\right] d\hat{u} \tag{28}$$

$$\sigma_{LQ+W} = \frac{1}{48\pi\hat{s}^2}\left[\int_0^{\hat{s}} \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \hat{u}^2 d\hat{u} + \int_0^{\hat{s}} \left(\frac{\lambda^2}{2(M_{LQ}^2 - \hat{u})}\right)^2 \hat{u}^2 d\hat{u} + \int_0^{\hat{s}} \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2 - \hat{u})} \hat{u}^2 d\hat{u}\right]$$

$$(29)$$

$$\sigma_{LQ+W} = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \int_0^{\hat{s}} \hat{u}^2 d\hat{u} + \left(\frac{\lambda^2}{2}\right)^2 \int_0^{\hat{s}} \frac{\hat{u}^2}{(M_{LQ}^2 - \hat{u})^2} d\hat{u} + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \int_0^{\hat{s}} \frac{\hat{u}^2}{(M_{LQ}^2 - \hat{u})} d\hat{u}\right] \tag{30}$$

To integrate (30), we perform a change of variable (see Appendix A5). Accordingly, we get the total cross section of the $q'\bar{q} \to \bar{\nu}e^-$ process, including the Leptoquark propagator in $\hat{u}$-channel.

$$\sigma_{LQ+W} = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \frac{\hat{s}^3}{3} + \left(\frac{\lambda^2}{2}\right)^2 \left[M_{LQ}^2 + \hat{s} - \frac{M_{LQ}^4}{M_{LQ}^2 + \hat{s}} - 2M_{LQ}^2 ln(1 + \frac{\hat{s}}{M_{LQ}^2})\right] + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}}\left[\frac{\hat{s}^2}{2} - M_{LQ}^2\hat{s} + M_{LQ}^4 ln(1 + \frac{\hat{s}}{M_{LQ}^2})\right]\right] \tag{31}$$

And for $\hat{s} << M_{LQ}^2$ equation (31) becomes :

$$= \frac{1}{48\pi}\left[\left(\frac{G_f M_w^2}{\sqrt{2}}\right)^2 \frac{1}{3\hat{s}} - \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}} M_{LQ}^2\right] \tag{32}$$

And for $\hat{s} >> M_{LQ}^2$ equation (31) becomes :

$$= \frac{1}{48\pi}\left[\left(\frac{G_f M_w^2}{\sqrt{2}}\right)^2 \frac{1}{3\hat{s}} + \left(\frac{\lambda^2}{2}\right)^2 \left[\hat{s} - 2M_{LQ}^2 ln(1 + \frac{\hat{s}}{M_{LQ}^2})\right] + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}}\left[\frac{1}{2\hat{s}} - \hat{s} + M_{LQ}^4 ln(1 + \frac{\hat{s}}{M_{LQ}^2})\right]\right]$$

we evaluate (31) where $\sqrt{\hat{s}} \in$ [400,2000] GeV, $G_f = 1.1663787 * 10^{-5} GeV^{-2}$,$M_w = 80.379\frac{GeV}{c^2}$, $\lambda = 1$, and $M_{LQ} = 500\frac{GeV}{c^2}$.

$\sqrt{\hat{s}}(GeV)$	$\frac{\hat{s}}{M_{LQ}^2}$	$\sigma(GeV^{-2}).(10^{-10})$
400	0.64	8.6101
500	1	9.1564
600	1.44	9.20378
700	1.96	8.90864
800	2.56	8.41855
1000	4	7.23541
1200	5.76	6.09135
1400	7.84	5.11264
1600	10.24	4.31004
1800	12.96	3.66027
2000	16	3.13445

Table 2. the cross section (σ_{LQ+W}) in function of $\sqrt{\hat{s}}$.

3.3 Contact interaction induced by Leptoquarks

To analyse the calculated data in table 2, it will be interesting to compare at the partonic level, as a first approximation, the contributions of new physics (LQ exchange) to the total effective cross-section for contact interaction $\lambda^2/2M_{LQ}^2$ [2], shown in figure 3.

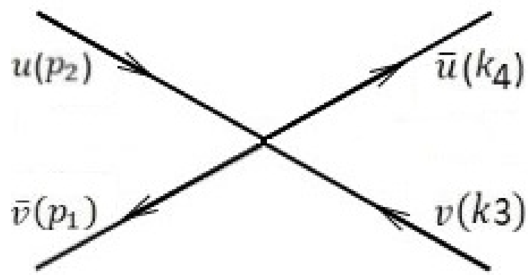

Figure 3. Contact interaction for the $u\overline{d} \to \overline{\nu}e^-$ interaction induced by leptoquark (S_0).

In this approximation, the total scattering amplitude is therefore:

$$|i\mathcal{M}_{CI+W}| = \frac{\lambda^2}{2(M_{LQ}^2)}\left[\overline{v}(p_1)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_2)\right]\left[\overline{u}(k_4)\gamma_\mu\frac{1}{2}(1-\gamma^5)v(k_3)\right] \tag{33}$$

and as same as before,

$$\overline{|i\mathcal{M}_{CI+W}|}^2 = |\mathcal{S}|^2|\overline{i\mathcal{P}_{CI+W}}|^2 \tag{34}$$

where,

$$\overline{|\mathcal{S}|}^2 = \frac{\hat{u}^2}{3} \tag{35}$$

and,

$$\overline{|i\mathcal{P}_{CI+W}|}^2 = i(-1)\left[\frac{G_f M_w^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{LQ}^2)}\right]\left[\frac{G_f M_w^2}{\sqrt{2}\hat{s}} + \frac{\lambda^2}{2(M_{LQ}^2)}\right] \tag{36}$$

substituting (35),(36) in (34) we get:

$$\overline{|i\mathcal{M}_{CI+W}|}^2 = \frac{\hat{u}^2}{3}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 + \left(\frac{\lambda^2}{2(M_{LQ}^2)}\right)^2 + \frac{G_f M_w^2\lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2)}\right] \tag{37}$$

[2]Contact interaction, also known as zero-range interaction, is a type of theoretical interaction in particle physics. It is a short-range interaction between particles, which is assumed to occur instantaneously when two particles come into contact with each other. Contact interactions are often used in theoretical models to describe the behaviour of particles at very high energies or very short distances.They are particularly useful in describing the behaviour of particles that are too massive to be directly produced in particle accelerators, such as leptoquarks.

and the total cross section,

$$\sigma_{CI+W} = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \int_0^{\hat{s}} \hat{u}^2 d\hat{u} + \left(\frac{\lambda^2}{2(M_{LQ}^2)}\right)^2 \int_0^{\hat{s}} \hat{u}^2 d\hat{u} + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2)} \int_0^{\hat{s}} \hat{u}^2 d\hat{u}\right] \tag{38}$$

$$\sigma_{CI+W} = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \frac{\hat{s}^3}{3} + \left(\frac{\lambda^2}{2(M_{LQ}^2)}\right)^2 \frac{\hat{s}^3}{3} + \frac{G_f M_w^2 \lambda^2}{2\sqrt{2}\hat{s}(M_{LQ}^2)} \frac{\hat{s}^3}{3}\right] \tag{39}$$

And for $\hat{s} << M_{LQ}^2$ equation (39) becomes :

$$= \frac{1}{48\pi}\left[\left(\frac{G_f M_w^2}{\sqrt{2}}\right)^2 \frac{1}{3\hat{s}} + \frac{G_f M_w^2 \lambda^2}{6\sqrt{2}} \frac{1}{M_{LQ}^2}\right] \tag{40}$$

And for $\hat{s} >> M_{LQ}^2$ equation (39) becomes :

$$= \frac{1}{48\pi}\left[\left(\frac{G_f M_w^2}{\sqrt{2}}\right)^2 \frac{1}{3\hat{s}} + \left(\frac{\lambda^2}{2}\right)^2 \frac{\hat{s}}{3M_{LQ}^4} + \frac{G_f M_w^2 \lambda^2}{6\sqrt{2}} \frac{1}{M_{LQ}^2}\right] \tag{41}$$

$\sqrt{\hat{s}}$	$\frac{\hat{s}}{M_{LQ}^2}$	$\sigma(GeV^{-2}).(10^{-9})$
400	0.64	1.68951
500	1	2.47116
600	1.44	3.43610
700	1.96	4.58093
800	2.56	5.90422
1000	4	9.08379
1200	5.76	12.9723
1400	7.84	17.5690
1600	10.24	22.8734
1800	12.96	28.8854
2000	16	35.6049

Table 3. the cross section (σ_{CI+W}) in function of $\sqrt{\hat{s}}$.

3.4 Limits

We plot the cross sections curves in the same plot (see figure 4) to compare the SM (W^-), CI and LQ cross sections. For better comparison we set a log scale for both σ, and $\frac{\hat{s}}{M_{LQ}}$.

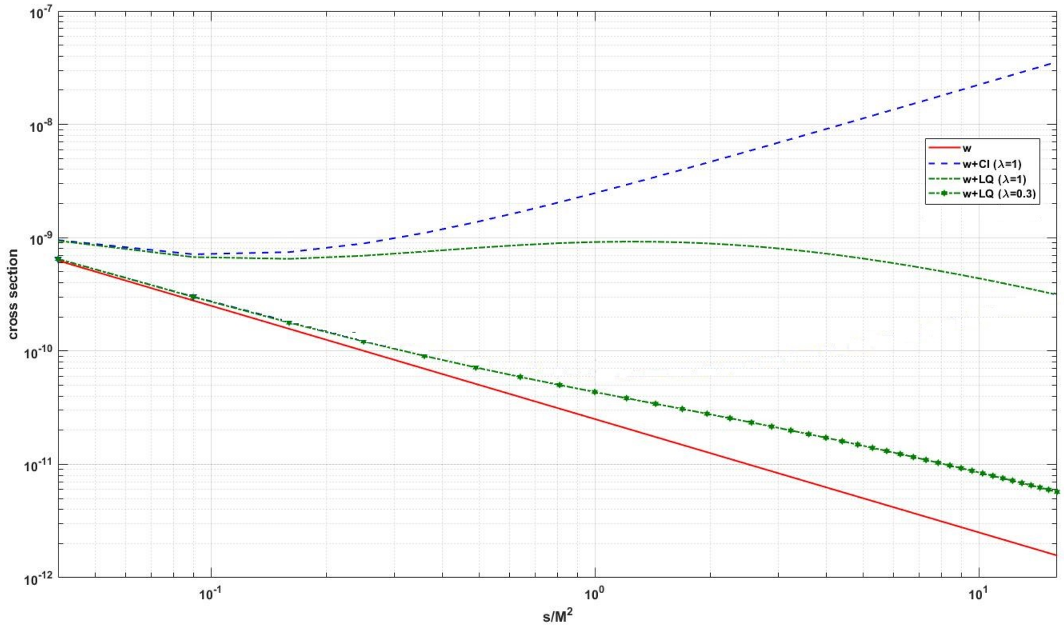

Figure 4. the cross sections as functions of $\frac{\hat{s}}{M_{LQ}^2}$ for different values of LQ couplings λ.

As we can see from figure 4, the cross section for LQ exchange (green dashed line) is almost independent of $\hat{s}$ at low energy $\hat{s} \ll M_{LQ}^2$ (this the interference between LQ and W^-), and decreases in $1/\hat{s}$ for $\hat{s} \gg M_{LQ}^2$. However, the CI cross section (blue dashed line) continues its increase with $\hat{s}$ as expected. Therefore the increase of the LQ exchange cross section at high energy, upon which CI bounds rely, is absent. However, we still want to know if the search for contact interactions at HERA can exclude massive leptoquarks with high couplings.
We observe in figure 4, that for coupling $\lambda > g_W$ ($g_W \approx 0.65107$), the LQ exchange makes a plateau over W^- exchange descent (red line), and result, around $\hat{s} \approx M_{LQ}$, in a significant excess of events similar to contact interactions. Therefore, a first way to set bounds on leptoquarks in the $\hat{u}$-channel is to calculate the ratio $C = \sigma_{LQ+W}/\sigma_{CI+W}$ and assume that the bound on contact interactions comes from $\hat{s} \approx M_{LQ}^2$. The results are shown in both table 4, and figure 5.

$\sqrt{\hat{s}}(TeV)$	C
0.4	0.50962
0.5	0.37053
0.6	0.26785
0.7	0.19447
0.8	0.14259
1.0	0.07965
1.2	0.04696
1.4	0.02910
1.6	0.01884
1.8	0.01267
2	0.00880

Table 4. the ratio C as a function of the center of mass energy $\sqrt{\hat{s}}$.

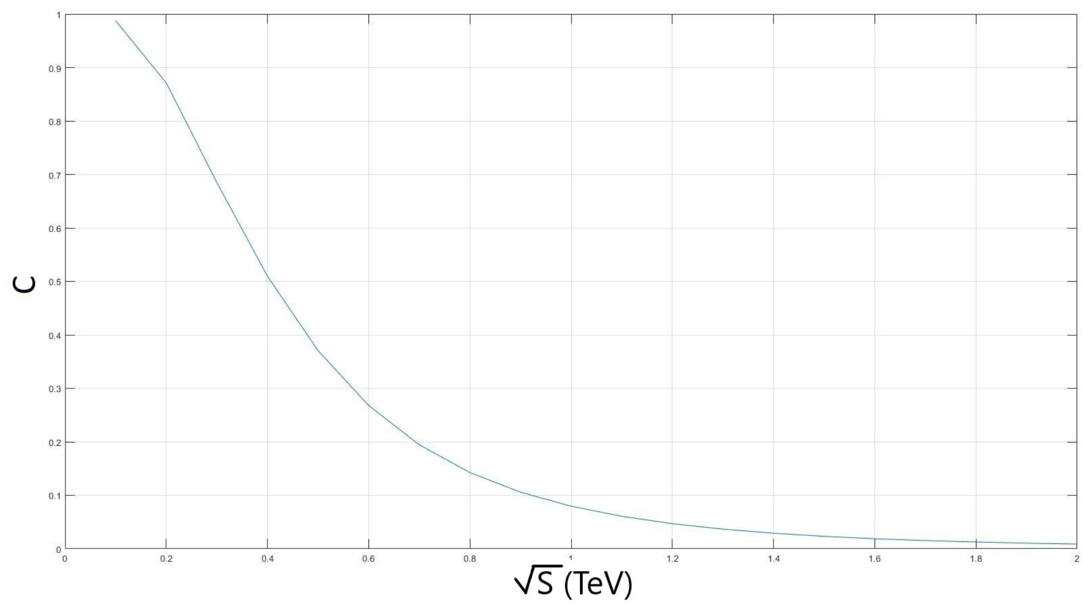

Figure 5. the ratio C in function of the center of Mass energy $\sqrt{\hat{s}}$.

This ratio is of order $C \sim 0.37$ for $\hat{s} \approx M_{LQ}^2$, then the effect of the LQ propagator would be to affect the bound on λ, in (4), by a factor of $\sim C^{0.25}$,

$$C^{1/2} \frac{\lambda^2}{2M_{LQ}^2} \leq \frac{4\pi}{\Lambda^2} \tag{42}$$

Then,

$$\lambda \leq \sqrt{\frac{8\pi}{C^{1/2}}} \frac{M_{LQ}}{\Lambda} \tag{43}$$

And,

$$\lambda \leq 1.34 \tag{44}$$

So, with $\Lambda = 2.4$ TeV and $M_{S_0} \approx 0.5$ TeV, we exclude couplings $\lambda > 1.34$. Our result is illustrated in Figure 6.

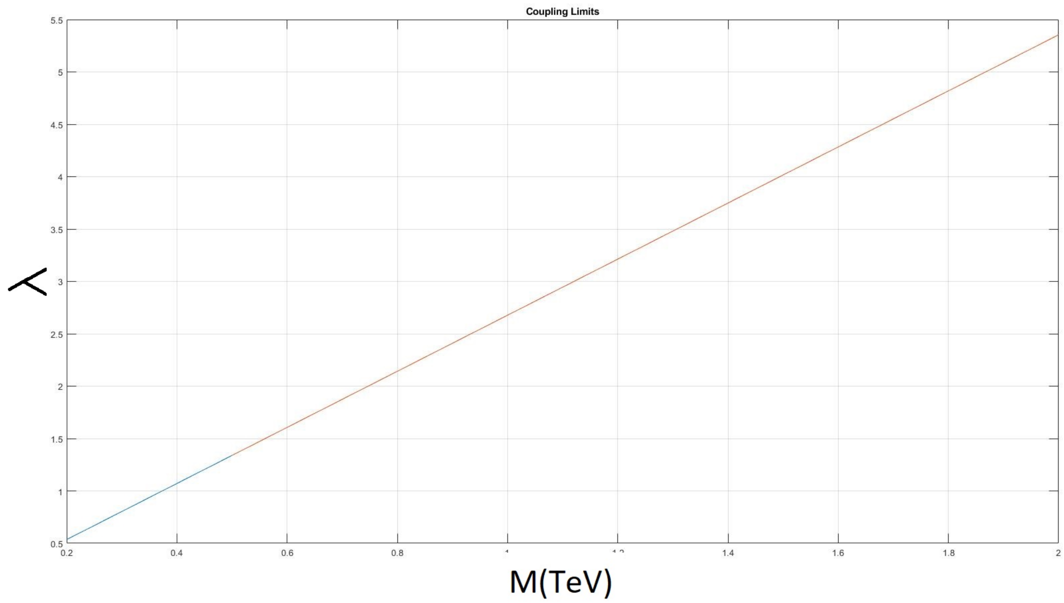

Figure 6. Bound on $\hat{u}$ channel LQ exchange, obtained from HERA contact interaction searches $\Lambda = 2.4$ TeV.

Conclusion

In conclusion, this thesis focused on analyzing the cross section behavior in the context of high-energy particle interactions. The main objectives were to investigate the impact of virtual leptoquark (LQ) exchange in the $q^{'}\overline{q} \to \overline{\nu} e^{-}$process and to explore the emergence of contact interactions (CI) at the partonic level. Through rigorous calculations, we calculated the total cross section, accounting for both Standard Model (W^{-}exchange) and new physics coming from LQ contributions. The results revealed interesting insights into the presence of LQ exchange and its potential coupling strengths. We established upper limits on LQ coupling by using energy scale bounds $\Lambda = 2.4$ TeV from Deep Inelastic Scattering (DIS) experiments.

Our findings indicated that the contact interaction model holds promise for understanding the dynamics of particle interactions at high energies. Moreover, the limitations imposed by the energy scales provided crucial information for future research in particle physics.

It is essential to acknowledge certain limitations. The calculations were conducted under specific collider conditions, not taking into account many other factors (Parton Distribution Functions (PDFs), Luminosity), and further investigations with different experimental setups are warranted to validate our results.

In summary, while our study reveals no evidence for new physics signals, the set upper limits on Leptoquarks masses contribute to the ongoing search for new physics phenomena. By addressing the limitations and considering future improvements, we pave the way for more accurate future investigations.

Appendix

5.1 Appendix A1

Cross section Scattering is essentially colliding two (or more) particles and monitoring the result in terms of cross sections.

The likelihood of a particular collision event $A + B = C + D$ is the scattering cross-section σ_i, were the total or inclusive cross-section for $A + B$ is

$$\sigma_{tot} = \sum_{i}^{n} \sigma_i .$$

So σ_{tot} is the measure of all different outcomes, but for a particular outcome $(C+D)$ for example we measure σ_i.
Suppose particles 1 and 2 collide, producing particles 3, 4, . . . n. The scattering cross section is given by the formula:

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |M|^2 (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - ......p_n)$$
$$\Pi_{j=3}^{n} 2\pi \delta(p_j^2 - m_j^2 c^2) \Theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

Where S is a statistical factor that corrects for double counting when there are identical particles in the final state.
The term : $(2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - ......p_n)$) is a delta function so $(p_1 + p_2 - p_3 - ......p_n)$ must equal 0 for the integral to be equal to 1 any other values and the integral is 0, So $p_1 + p_2 = p_3 + p_4 + ... + p_n$. Were p_1 and p_2 are the 4-momenta of the in-going particles and $p_3, p_4, ..., p_n$ are the 4-momenta of the outgoing particles, so the δ function is the condition for momentum conservation.
The term: $\Pi_{j=3}^{n} 2\pi \delta(p_j^2 - m_j^2 c^2)$ is a δ function so the same as before $(p_j^2 = m_j^2 c^2)$.
We know that $p_\mu p^\mu = \frac{E^2}{c^2} - \vec{p}^2$ but for the rest frame $p_\mu p^\mu = \frac{E^2}{c^2} - \vec{0} = m^2 c^2$ and because $p_\mu p^\mu$ is invariant across all references $\frac{E^2}{c^2} - \vec{p}^2 = m^2 c^2$ witch is known as the mass-shell condition for real particles, so the $\delta(p_j^2 - m_j^2 c^2)$ is a condition for the outgoing particles to be real particles and each particles has to satisfy this condition.
The term $\Theta(p_j^0)$ is a function that for all negative values it will be 0 and for all positive values its 1 so it's a condition for p_j^0 (energy) of all outgoing particles to have a positive value.
The term M contains the dynamics (the forces involved)[24].

$$\sigma = \frac{S\hbar^2}{64\pi^2\sqrt{(p_1\cdot p_2)^2-(m_1m_2c^2)^2}}\int |M|^2\delta^4(p_1+p_2-p_3-p_4)$$
$$\frac{1}{\sqrt{(\vec{p_3}^{\,2}-m_3^2c^2)}}\frac{1}{\sqrt{(\vec{p_4}^{\,2}-m_4^2c^2)}}d^3\vec{p_3}d^3\vec{p_4}.$$

in the CM frame $\vec{p_2} = -\vec{p_1}$

$$\begin{aligned}
\sqrt{(p_1\cdot p_2)^2-(m_1m_2c^2)^2} &= \sqrt{\left(\frac{E_1E_2}{c^2}+\vec{p_1}\vec{p_1}\right)^2-(\frac{E_1^2}{c^2}-\vec{p_1}^{\,2})(\frac{E_2^2}{c^2}-\vec{p_1}^{\,2})} \\
&= \sqrt{\left((\frac{E_1E_2}{c^2})^2+\vec{p_1}^{\,4}+2\frac{E_1E_2}{c^2}\vec{p_1}^{\,2}\right)-(\frac{E_1^2}{c^2}-\vec{p_1}^{\,2})(\frac{E_2^2}{c^2}-\vec{p_1}^{\,2})} \\
&= \sqrt{\left((\frac{E_1E_2}{c^2})^2+\vec{p_1}^{\,4}+2\frac{E_1E_2}{c^2}\vec{p_1}^{\,2}\right)-(\frac{E_1^2E_2^2}{c^4}-\vec{p_1}^{\,2}\frac{E_1^2}{c^2}-\vec{p_1}^{\,2}\frac{E_2^2}{c^2}+\vec{p_1}^{\,4})} \\
&= \sqrt{(\frac{E_1E_2}{c^2})^2+\vec{p_1}^{\,4}+2\frac{E_1E_2}{c^2}\vec{p_1}^{\,2}-\frac{E_1^2E_2^2}{c^4}+\vec{p_1}^{\,2}\frac{E_1^2}{c^2}+\vec{p_1}^{\,2}\frac{E_2^2}{c^2}-\vec{p_1}^{\,4}} \\
&= \sqrt{(\frac{E_1^2E_2^2}{c^4})+\vec{p_1}^{\,4}+2\frac{E_1E_2}{c^2}\vec{p_1}^{\,2}-\frac{E_1^2E_2^2}{c^4}+\frac{\vec{p_1}^{\,2}}{c^2}(E_1^2+E_2^2)-\vec{p_1}^{\,4}} \\
&= \sqrt{+2\frac{E_1E_2}{c^2}\vec{p_1}^{\,2}+\frac{\vec{p_1}^{\,2}}{c^2}(E_1^2+E_2^2)} \\
&= \sqrt{\frac{\vec{p_1}^{\,2}}{c^2}(+2E_1E_2+E_1^2+E_2^2)} \\
&= \sqrt{\frac{\vec{p_1}^{\,2}}{c^2}(E_1+E_2)^2} \\
&= \sqrt{\left(\frac{\vec{p_1}}{c}(E_1+E_2)\right)^2} \\
&= \frac{|\vec{p_1}|}{c}(E_1+E_2)
\end{aligned}$$

$$\sigma = \frac{S\hbar^2c}{64\pi^2(E_1+E_2)|\vec{p_1}|}\int |M|^2\delta^4(p_1+p_2-p_3-p_4)\frac{1}{\sqrt{(\vec{p_3}^{\,2}-m_3^2c^2)}}\frac{1}{\sqrt{(\vec{p_4}^{\,2}-m_4^2c^2)}}d^3\vec{p_3}d^3\vec{p_4}.$$

The δ function can be written as fallow

$$\begin{aligned}
\delta(p_1+p_2-p_3-p_4) &= \delta(p_1^0+p_2^0-p_3^0-p_4^0)\delta^3(\vec{p_1}+\vec{p_2}-\vec{p_3}-\vec{p_4}) \\
&= \delta(\frac{E_1+E_2}{c}-p_3^0-p_4^0)\delta^3(-\vec{p_3}-\vec{p_4})
\end{aligned}$$

So σ becomes

$$\sigma = \frac{S\hbar^2 c}{64\pi^2(E_1+E_2)|\vec{p_1}|}\int |M|^2\delta(\frac{E_1+E_2}{c} - p_3^0 - p_4^0)\delta^3(-\vec{p_3} - \vec{p_4})$$
$$\frac{1}{\sqrt{(\vec{p_3}^2 - m_3^2c^2)}}\frac{1}{\sqrt{(\vec{p_4}^2 - m_4^2c^2)}}d^3\vec{p_3}d^3\vec{p_4}.$$

From the mass-shell condition we get

$$p_3^0 = \sqrt{(\vec{p_3}^2 + m_3^2c^2)}$$

Doing the same for p_4^0 we get

$$\sigma = \frac{S\hbar^2 c}{64\pi^2(E_1+E_2)|\vec{p_1}|}\int |M|^2\delta\left(\frac{E_1+E_2}{c} - \sqrt{(\vec{p_3}^2 + m_3^2c^2)} - \sqrt{(\vec{p_3}^2 + m_4^2c^2)}\right)$$
$$\frac{1}{\sqrt{(\vec{p_3}^2 - m_3^2c^2)}}\frac{1}{\sqrt{(\vec{p_3}^2 - m_4^2c^2)}}d^3\vec{p_3}.$$

In our case we have no identical particles in the final state so $S = 1$
we didn't really want σ in the first place, what we're after is $\frac{d\sigma}{d\hat{u}}$.

$$\frac{d\sigma}{d\hat{u}} = \frac{|M|^2}{16\pi\hat{s}^2}$$

5.2 Appendix A2

$$G = \Big[\overline{v}(a)\Gamma_1 u(b)][\overline{v}(a)\Gamma_2 u(b)\Big]^*$$

we know that

$$\begin{aligned}\overline{u}(a) &= u^+(a)\gamma^0.\\ (\gamma^0)^+ &= \gamma^0.\\ (\gamma^0)^2 &= 1\\ \Big[\overline{v}(a)\Gamma_2 u(b)\Big]^* &= \Big[\overline{v}(a)\Gamma_2 u(b)\Big]^+\end{aligned}$$

because $\Big[\overline{v}(a)\Gamma_2 u(b)\Big]$ is a scalar.

$$\begin{aligned}\Big[\overline{v}(a)\Gamma_2 u(b)\Big]^+ &= \Big[v^+(a)\gamma^0\Gamma_2 u(b)\Big]^+\\ &= \Big[u^+(b)\Gamma_2^+(\gamma^0)^+ v(a)\Big]\\ &= \Big[u^+(b)\gamma^0\gamma^0\Gamma_2^+(\gamma^0)^+ v(a)\Big]\\ &= \Big[\overline{u}(b)\overline{\Gamma}_2 v(a)\Big]\end{aligned}$$

Where $\overline{\Gamma}_2 = \gamma^0\Gamma_2^+\gamma^0$ so G becomes $\Big[\overline{v}(a)\Gamma_1 u(b)\Big]\Big[\overline{u}(b)\overline{\Gamma}_2 v(a)\Big]$
Because $u(b)\overline{u}(b)$ are next to each other we can sum over the spin orientations of particle b.

$$\sum_{bspins} G = \left[\overline{v}(a)\Gamma_1\left(\sum_{s_b=1,2} u(b)\overline{u}(b)\right)\overline{\Gamma}_2 v(a)\right]$$

Using the completeness relation:

$$\sum_{bspins} G = \Big[\overline{v}(a)\Gamma_1\left(\gamma^\mu p_\mu + m_b c\right)\overline{\Gamma}_2 v(a)\Big]$$

In order to sum over the spin orientations of particle a $\overline{v}(a)v(a)$ must be next to each other but $\Gamma_1\overline{\Gamma}_2\gamma^\mu$ are in the same spin space. But noticing that $\Big[\overline{v}(a)\Gamma_1\left(\gamma^\mu p_\mu + m_b c\right)\overline{\Gamma}_2 v(a)\Big]$ is a scaler

$$\sum_{bspins} G = Tr\Big[\Gamma_1\left(\gamma^\mu p_\mu + m_b c\right)\overline{\Gamma}_2\overline{v}(a)v(a)\Big]$$

witch gives

$$\sum_{aspins}\sum_{bspins} G = Tr\Big[\Gamma_1\left(\gamma^\mu p_\mu + m_b c\right)\overline{\Gamma}_2\left(\gamma^\nu p_\nu + m_a c\right)\Big]$$

5.3 Appendix A3

Expanding (21) and calculating the Levi-Civita terms

$$= \frac{1}{12} 4\Big[(p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2\cdot p_1)) + i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}\Big]$$
$$4\Big[(k_{4\mu}k_{3\nu} + k_{3\mu}k_{4\nu} - g_{\mu\nu}(k_4\cdot k_3)) + i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma\Big]$$

$$= \frac{1}{3}\Big[p_2^\mu p_1^\nu k_{4\mu}k_{3\nu} + p_2^\mu p_1^\nu k_{3\mu}k_{4\nu} - p_2^\mu p_1^\nu g_{\mu\nu}(k_4\cdot k_3) + p_2^\mu p_1^\nu i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma + p_1^\mu p_2^\nu k_{4\mu}k_{3\nu} + p_1^\mu p_2^\nu k_{3\mu}k_{4\nu}$$
$$- p_1^\mu p_2^\nu g_{\mu\nu}(k_4\cdot k_3) + p_1^\mu p_2^\nu i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma - g^{\mu\nu}(p_2\cdot p_1)k_{4\mu}k_{3\nu} - g^{\mu\nu}(p_2\cdot p_1)k_{3\mu}k_{4\nu} + g^{\mu\nu}(p_2\cdot p_1)g_{\mu\nu}(k_4\cdot k_3)$$
$$- g^{\mu\nu}(p_2\cdot p_1)i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma + i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}k_{4\mu}k_{3\nu} + i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}k_{3\mu}k_{4\nu} - i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}g_{\mu\nu}(k_4\cdot k_3)$$
$$- \epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma\Big]$$

$$= \frac{1}{3}\Big[(p_2\cdot k_4)(p_1\cdot k_3) + (p_2\cdot k_3)(p_1\cdot k_4) - (p_2\cdot p_1)(k_4\cdot k_3) + p_2^\mu p_1^\nu i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma$$
$$+ (p_1\cdot k_4)(p_2\cdot k_3) + (p_1\cdot k_3)(p_2\cdot k_4) - (p_1\cdot p_2)(k_4\cdot k_3) + p_1^\mu p_2^\nu i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma$$
$$- (p_2\cdot p_1)(k_4\cdot k_3) - (p_2\cdot p_1)(k_3\cdot k_4) + 4(p_1\cdot p_2)(k_3\cdot k_4) - g^{\mu\nu}(p_1\cdot p_2)i\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma$$
$$+ i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}k_{4\mu}k_{3\nu} + i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}k_{3\mu}k_{4\nu} - i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}g_{\mu\nu}(k_3\cdot k_4) - \epsilon^{\mu\alpha\nu\beta}p_{\alpha 2}p_{\beta 1}\epsilon_{\mu\lambda\nu\sigma}k_4^\lambda k_3^\sigma\Big]$$

For the Levi-Civita symbol we know that

$$\epsilon_{\mu\lambda\nu\sigma} = \begin{cases} +1, \text{ if } \mu\lambda\nu\sigma \text{ is an even permutation of 0123} \\ \text{-1, if } \mu\lambda\nu\sigma \text{ is an odd permutation of 0123} \\ \text{0, if any two indices are the same .} \end{cases}$$

$$\epsilon^{\mu\lambda\nu\sigma} = \begin{cases} \text{-1, if } \mu\lambda\nu\sigma \text{ is an even permutation of 0123} \\ +1, \text{ if } \mu\lambda\nu\sigma \text{ is an odd permutation of 0123} \\ \text{0. if any two indices are the same.} \end{cases}$$

$$i\epsilon^{\mu\alpha\nu\beta}p_{\alpha j}p_{\beta i}p_{k\mu}p_{l\nu} = i\Big(p_{1j}p_{3i}p_{0k}p_{2l} + p_{2j}p_{1i}p_{0k}p_{3l} + p_{3j}p_{2i}p_{0k}p_{1l} + p_{0j}p_{2i}p_{1k}p_{3l} + p_{2j}p_{3i}p_{1k}p_{0l}$$
$$+ p_{3j}p_{0i}p_{1k}p_{2l} + p_{0j}p_{3i}p_{2k}p_{1l} + p_{1j}p_{0i}p_{2k}p_{3l} + p_{3j}p_{1i}p_{2k}p_{0l} + p_{0j}p_{1i}p_{3k}p_{2l} + p_{1j}p_{2i}p_{3k}p_{0l} + p_{2j}p_{0i}p_{3k}p_{1l}$$
$$- p_{2j}p_{3i}p_{0k}p_{1l} - p_{3j}p_{1i}p_{0k}p_{2l} - p_{1j}p_{2i}p_{0k}p_{3l} - p_{0j}p_{3i}p_{1k}p_{2l} - p_{2j}p_{0i}p_{1k}p_{3l} - p_{3j}p_{2i}p_{1k}p_{0l}p_{0j}p_{1i}p_{2k}p_{3l}$$
$$- p_{1j}p_{3i}p_{2k}p_{0l} - p_{3j}p_{0i}p_{2k}p_{1l} - p_{0j}p_{2i}p_{3k}p_{1l} - p_{1j}p_{0i}p_{3k}p_{2l} - p_{2j}p_{1i}p_{3k}p_{0l}\Big)$$

$$i\epsilon^{\mu\alpha\nu\beta}p_{\alpha j}p_{\beta i}p_{l\mu}p_{k\nu} = i\Big(p_{1j}p_{3i}p_{0l}p_{2k} + p_{2j}p_{1i}p_{0l}p_{3k} + p_{3j}p_{2i}p_{0l}p_{1k} + p_{0j}p_{2i}p_{1l}p_{3k} + p_{2j}p_{3i}p_{1l}p_{0k}$$
$$+ p_{3j}p_{0i}p_{1l}p_{2k} + p_{0j}p_{3i}p_{2l}p_{1k} + p_{1j}p_{0i}p_{2l}p_{3k} + p_{3j}p_{1i}p_{2l}p_{0k} + p_{0j}p_{1i}p_{3l}p_{2k} + p_{1j}p_{2i}p_{3l}p_{0k} + p_{2j}p_{0i}p_{3l}p_{1k}$$
$$- p_{2j}p_{3i}p_{0l}p_{1k} - p_{3j}p_{1i}p_{0l}p_{2k} - p_{1j}p_{2i}p_{0l}p_{3k} - p_{0j}p_{3i}p_{1l}p_{2k} - p_{2j}p_{0i}p_{1l}p_{3k} - p_{3j}p_{2i}p_{1l}p_{0k}p_{0j}p_{1i}p_{2l}p_{3k}$$
$$- p_{1j}p_{3i}p_{2l}p_{0k} - p_{3j}p_{0i}p_{2l}p_{1k} - p_{0j}p_{2i}p_{3l}p_{1k} - p_{1j}p_{0i}p_{3l}p_{2k} - p_{2j}p_{1i}p_{3l}p_{0k}\Big)$$

We see that for every term in the fist one we have its negative term in the second one so

$$i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 1}p_{\beta 2}k_{3\mu}k_{4\nu} = -i\epsilon^{\mu\alpha\nu\beta}p_{\alpha 1}p_{\beta 2}k_{4\mu}k_{3\nu}$$

The same could be said for

$$p_1^\mu p_2^\nu i\epsilon_{\mu\lambda\nu\sigma}k_3^\lambda k_4^\sigma = -p_2^\mu p_1^\nu i\epsilon_{\mu\lambda\nu\sigma}k_3^\lambda k_4^\sigma$$

The remainng term is

$$= \Big[2(p_1 \cdot k_3)(p_2 \cdot k_4) + 2(p_1 \cdot k_4)(p_2 \cdot k_3) - 4(p_1 \cdot p_2)(k_3 \cdot k_4) + 4(p_1 \cdot p_2)(k_3 \cdot k_4)$$
$$- \epsilon^{\mu\alpha\nu\beta}p_{\alpha 1}p_{\beta 2}\epsilon_{\mu\lambda\nu\sigma}k_3^\lambda k_4^\sigma\Big]$$

We have

$$\epsilon^{\mu\alpha\nu\beta}\epsilon_{\mu\lambda\nu\sigma} = 2!(\delta_{\alpha\lambda}\delta_{\beta\sigma} - \delta_{\alpha\sigma}\delta_{\beta\lambda})$$

5.4 Appendix A4

We know that

$$\hat{u} = (p_1 - k_4)^2 = (p_2 - k_3)^2$$

$$\begin{aligned}
\frac{\hat{u}^2}{3} &= \frac{(p_1 - k_4)^2 (p_2 - k_3)^2}{3} \\
&= \frac{\Big((p_1)^2 + (k_4)^2 - 2(p_1)(k_4)\Big)\Big((p_2)^2 + (k_3)^2 - 2(p_2)(k_3)\Big)}{3}
\end{aligned}$$

where $(p_i)^2 = m_i^2 c^2$

$$= \frac{\Big(m_1^2 c^2 + m_4^2 c^2 - 2(p_1)(k_4)\Big)\Big(m_2^2 c^2 + m_3^2 c^2 - 2(p_2)(k_3)\Big)}{3}$$

And because $m_1 = m_2 = m_3 = m_4 = 0$. (relativistic limit)

$$\begin{aligned}
\frac{\hat{u}^2}{3} &= \frac{\Big(-2(p_1)(k_4)\Big)\Big(-2(p_2)(k_3)\Big)}{3} \\
\frac{\hat{u}^2}{3} &= \frac{1}{3}\Big[4(p_1 \cdot k_4)(p_2 \cdot k_3)\Big]
\end{aligned}$$

5.5 Appendix A5

$$\sigma = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\int_0^{\hat{s}}\hat{u}^2 d\hat{u} + \left(\frac{\lambda^2}{2}\right)^2\int_0^{\hat{s}}\frac{\hat{u}^2}{(M_{LQ}^2-\hat{u})^2}d\hat{u} + \frac{G_F M_w^2\lambda^2}{2\sqrt{2}\hat{s}}\int_0^{\hat{s}}\frac{\hat{u}^2}{(M_{LQ}^2-\hat{u})}d\hat{u}\right]$$

to simplify the integral we put $Q^2 = -\hat{u}$ so it reduces to

$$\sigma = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\int_0^{\hat{s}}Q^4 dQ^2 + \left(\frac{\lambda^2}{2}\right)^2\int_0^{\hat{s}}\frac{Q^4}{(M_{LQ}^2+Q^2)^2}dQ^2 + \frac{G_F M_w^2\lambda^2}{2\sqrt{2}\hat{s}}\int_0^{\hat{s}}\frac{Q^4}{(M_{LQ}^2+Q^2)}dQ^2\right]$$

we define $x = M_{LQ}^2 + Q^2$

$$\sigma = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}(x-M_{LQ}^2)^2 dx + \left(\frac{\lambda^2}{2}\right)^2\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}\frac{(x-M_{LQ}^2)^2}{(x)^2}dx + \right.$$
$$\left.\frac{G_F M_w^2\lambda^2}{2\sqrt{2}\hat{s}}\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}\frac{(x-M_{LQ}^2)^2}{x}dx\right]$$

$$\sigma = \frac{1}{48\pi\hat{s}^2}\left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}x^2 + M_{LQ}^4 - 2M_{LQ}^2 x dx + \left(\frac{\lambda^2}{2}\right)^2\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}\frac{x^2+M_{LQ}^4-2M_{LQ}^2 x}{(x)^2}dx + \right.$$
$$\left.\frac{G_F M_w^2\lambda^2}{2\sqrt{2}\hat{s}}\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}\frac{x^2+M_{LQ}^4-2M_{LQ}^2 x}{x}dx\right]$$

we solve each integral separately

$$\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}x^2 + M_{LQ}^4 - 2M_{LQ}^2 x dx = \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\left[\frac{x^3}{3} + M_{LQ}^4 x - M_{LQ}^2 x^2\right]_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}$$

$$= \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\left[\frac{(M_{LQ}^2+\hat{s})^3}{3} + M_{LQ}^4(M_{LQ}^2+\hat{s}) - M_{LQ}^2(M_{LQ}^2+\hat{s})^2\right.$$
$$\left.-\frac{(M_{LQ}^2)^3}{3} - M_{LQ}^4 M_{LQ}^2 + M_{LQ}^2(M_{LQ}^2)^2\right]$$

$$= \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2\left[\frac{(M_{LQ}^6+\hat{s}^3+3M_{LQ}^2\hat{s}(M_{LQ}^2+\hat{s}))}{3} + M_{LQ}^6 + M_{LQ}^4\hat{s} - M_{LQ}^2(M_{LQ}^4+\hat{s}^2+2M_{LQ}^2\hat{s})\right.$$
$$\left.-\frac{M_{LQ}^6}{3} - M_{LQ}^6 + M_{LQ}^6\right]$$

$$= \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \left[\frac{M_{LQ}^6}{3} + \frac{\hat{s}^3}{3} + M_{LQ}^4\hat{s} + M_{LQ}^2\hat{s}^2 + M_{LQ}^6 + M_{LQ}^4\hat{s} \right.$$

$$\left. -M_{LQ}^6 - M_{LQ}^2\hat{s}^2 - 2M_{LQ}^4\hat{s} - \frac{M_{LQ}^6}{3} - M_{LQ}^6 + M_{LQ}^6\right]$$

$$= \left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}}\right)^2 \left[+\frac{\hat{s}^3}{3}\right]$$

for the second one

$$\left(\frac{\lambda^2}{2}\right)^2 \int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}} \frac{x^2 + M_{LQ}^4 - 2M_{LQ}^2 x}{x^2} dx = \left(\frac{\lambda^2}{2}\right)^2 \int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}} (1 + \frac{M_{LQ}^4}{x^2} - \frac{2M_{LQ}^2}{x}) dx$$

$$= \left(\frac{\lambda^2}{2}\right)^2 \left[(x - \frac{M_{LQ}^4}{x} - 2M_{LQ}^2 ln(x)\right]_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}$$

$$= \left(\frac{\lambda^2}{2}\right)^2 \left[M_{LQ}^2 + \hat{s} - \frac{M_{LQ}^4}{M_{LQ}^2 + \hat{s}} - 2M_{LQ}^2 ln(M_{LQ}^2 + \hat{s}) - M_{LQ}^2 + \frac{M_{LQ}^4}{M_{LQ}^2} + 2M_{LQ}^2 ln(M_{LQ}^2)\right]$$

$$= \left(\frac{\lambda^2}{2}\right)^2 \left[+M_{LQ}^2 + \hat{s} - \frac{M_{LQ}^4}{M_{LQ}^2 + \hat{s}} - 2M_{LQ}^2 \left(ln(M_{LQ}^2 + \hat{s}) - ln(M_{LQ}^2)\right)\right]$$

$$= \left(\frac{\lambda^2}{2}\right)^2 \left[+M_{LQ}^2 + \hat{s} - \frac{M_{LQ}^4}{M_{LQ}^2 + \hat{s}} - 2M_{LQ}^2 ln(1 + \frac{\hat{s}}{M_{LQ}^2})\right]$$

for the final inregral

$$\frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}} \frac{x^2 + M_{LQ}^4 - 2M_{LQ}^2 x}{x} dx = \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \int_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}} (x + \frac{M_{LQ}^4}{x} - 2M_{LQ}^2) dx$$

$$= \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[\frac{x^2}{2} + M_{LQ}^4 ln(x) - 2M_{LQ}^2 x\right]_{M_{LQ}^2}^{M_{LQ}^2+\hat{s}}$$

$$= \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[\frac{(M_{LQ}^2 + \hat{s})^2}{2} + M_{LQ}^4 ln(M_{LQ}^2 + \hat{s}) - 2M_{LQ}^2(M_{LQ}^2 + \hat{s}) \right.$$

$$\left. -\frac{(M_{LQ}^2)^2}{2} - M_{LQ}^4 ln(M_{LQ}^2) + 2M_{LQ}^2 M_{LQ}^2\right]$$

$$= \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[\frac{M_{LQ}^4}{2} + \frac{\hat{s}^2}{2} + M_{LQ}^2 \hat{s} + M_{LQ}^4 ln(M_{LQ}^2 + \hat{s}) - 2M_{LQ}^4 - 2M_{LQ}^2 \hat{s}) \right.$$

$$\left. - \frac{M_{LQ}^4}{2} - M_{LQ}^4 ln(M_{LQ}^2) + 2M_{LQ}^4 \right]$$

$$= \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[+ \frac{\hat{s}^2}{2} - M_{LQ}^2 \hat{s} + M_{LQ}^4 \Big(ln(M_{LQ}^2 + \hat{s}) - ln(M_{LQ}^2) \Big) \right]$$

$$= \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[+ \frac{\hat{s}^2}{2} - M_{LQ}^2 \hat{s} + M_{LQ}^4 ln(1 + \frac{\hat{s}}{M_{LQ}^2}) \right]$$

assembling all three part of the integral we get

$$\sigma = \frac{1}{48\pi\hat{s}^2} \left[\left(\frac{G_f M_w^2}{\sqrt{2}\hat{s}} \right)^2 \frac{\hat{s}^3}{3} + \left(\frac{\lambda^2}{2} \right)^2 \left[M_{LQ}^2 + \hat{s} - \frac{M_{LQ}^4}{M_{LQ}^2 + \hat{s}} - 2M_{LQ}^2 ln(1 + \frac{\hat{s}}{M_{LQ}^2}) \right] + \right.$$

$$\left. \frac{G_F M_w^2 \lambda^2}{2\sqrt{2}\hat{s}} \left[\frac{\hat{s}^2}{2} - M_{LQ}^2 \hat{s} + M_{LQ}^4 ln(1 + \frac{\hat{s}}{M_{LQ}^2}) \right] \right]$$

References

[1] W. N. Cottingham and D. A. Greenwood, An introduction to the standard model of particle physics, Second Edition, Cambridge University Press, 2007.

[2] J. C. Baez and I. E. Segal and Z. Zhou, Intoduction to algebraic and constructive quantum field theory, Princeton University press, 1992.

[3] Ø. Grøn and S. Hervik, Einstein's general theory of relativity, Springer, 2004.

[4] K. Freeman and G. Mcnamara, In search of dark matter, Prasix puplishing, 2006

[5] C. P. Burgess and G. D. Moor, The Standard Model: A Primer, Cambridge University Press, 2012.

[6] B. A. Robson, The Matter-Antimatter Asymmetry Problem, Journal of High Energy Physics, Gravitation and Cosmology, 4, 166-178, 2018.

[7] Z. Habibolahi and K. Ghorbani and P. Ghorbani, Hierarchy problem and the vacuum stability in two-scalar dark matter model, Physical Review D, 5, 106, 2022.

[8] G. Djordjevic and L. Nesic, Mathematical Theoretical and Phenomenological Challenges Beyond the Standard Model, World Scientific Publishing, 2003.

[9] H. E. Haber and G. L. Kane, the search for supersymmetry: probing physics beyound the standard model, north-holland Publishing, 1984.

[10] B. Zwiebach, A first course in string theory, Cambridge University Press, 2004.

[11] I. Bars and J. Terning, Extra Dimensions in Space and Time, Springer, 2010.

[12] I. Doršner and S. Fajfer and N. Košnik, Leptoquark mechanism of neutrino masses within the grand unification framework, University of Split, 2017.

[13] S. Davidson and D. Bailey and B. A. Campbell, Model independent constraints on leptoquarks from rare processes, Zeitschrift fur Physik C, 61, 613-643, 1994.

[14] J. Wudka, Composite leptoquarks, Physics Letters B, 167, 337-342, 1986 .

[15] H. Pirumov, QCD Analysis of Neutral and Charged Current Cross Sections and Search for Contact Interactions at HERA, physical institute, 2013.

[16] A. Greljo and N. Selimovic, Lepton-Quark Fusion at Hadron Colliders. precisely,Journal of High Energy Physics, 2103, 279-305, 2021

[17] J. Ohnemus and S. Rudaz and T.F. Walsh and P.M. Zerwas, Single leptoquark production at hadron colliders, Physics Letters B, 334, 203-207, 1994.

[18] G. Aad and ATLAS Collaboration, Search for pair production of scalar leptoquarks decaying into first- or second-generation leptons and top quarks in proton–proton collisions at $sqrt(\hat{s}) = 13$ Tev with the ATLAS detector, The European Physical Journal C, 81, 313-354, 2021.

[19] W.Buchmuller and R.Ruckl and D.Wyler, Leptoquarks in lepton-quark collisions, Physics Letters B, 191, 320, 1987.

[20] A.F.Zarnecki, Leptoquark signal from global analysis, The European Physical Journal C, 17, 695-706, 2000.

[21] A. Bessaa and S. Davidson, Constraints on t-channel leptoquark exchange from LHC contact interaction searches, The European Physical Journal C, 75-97, 2015.
[22] J. R. Gilmor, Search for Contact Interaction in Deep Inelastic Scattering at ZEUS, The Ohio State University, 2001.
[23] A. Crivellin and L. Schnell, Complete Lagrangian and set of Feynman rules for scalar leptoquarks, Computer Physics Communications, 271, 108-188, 2022.
[24] D. Griffiths, Introduction to Elementary Particles, Wiley-VCH publisher, 2008.