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Adaptive modulation for OFDM systems with partial CSI feedback.

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Acronyms

ASK Amplitude Shift Keying

AWGN Additif White Gaussien Noise

BER Bit Error Rate

CSI Channel State Infomation

DFE Decision-feedback channel estimation

DFT Discrete Furrier Transform

FDM Frequency Division Multiplexing

ISI Inter Symbol Interference

MMSE Minimum Mean-Square Estimator

LTE Long Term Evolution

LOS Line of sight

LSE Linear Square Estimator

MIMO Multiple Input Multiple Output

ML Maximum Likelihood

MSE Mean Square Error

NLOS Non Line of sight

OFDM Orthogonal Frequency Division Multiplexing

PAM Pulse Amplitude Modulation

PAPR Peak-to-average power ratio

PDP Power delay profile

PSK Phase Shift Keying

QAM Quadrature Amplitude Modulation

RMS Root Mean Square

SNR Signal to Noise Ratio

UMTS Universal Mobile Telecommunications Standards Institute

WLAN Wireless Local Area Networks

WSS Wide Sense Stationary

ADC Analogue to Digital convertor

DAC Digital to Analogue convertor

RFID Radio Frequency IDentification

HSDPA High Speed Downlink Packet Access

MC Multi-Carrier

SQNR Signal to Quantization Noise Ratio

STC Space-Time Code

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Dedication

I dedicate my dissertation work

To my dear Mother for her unbiased support,

To my beloved Father who have made me who i am,

To my sisters Afef and Nabila and their husbands,

To my nephew Abdelbari and my neice Arwa,

To my teacher and my example Saida,

To my best friend Soraya,

To all my english classmates,

To my colleagues at Networks and Communications class,

To all my friends of Electronic department,

To everyone who impacted my life,

And to those who believe in me

 … I dedicate this thesis.

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I am dedicating this work to my father Emmanuel , my mother Esther for they always wanted me to become the best version of myself. Through their encouragement, support, and taking care of me, I reached the level that when I look back I and future I see their trust and encouragement. May God bless them

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ملخص

هذه الأطروحة تهتم بلوغاريتم التوزيع الفعال لوحدة المعلومة للأنظمة المتعددة النواقل اعتمادا على المعرفة غير الكاملة لمعلومات وضع القناة ما يقود إلى التوزيع المتساوي للطاقة. أخذت بعين الاعتبار عدة قيود مثل قدرة القناة الأعظمية و احتمال الخطأ في مجموعة وحدة المعلومة، و هذا بهدف جعل الكفاءة الطيفية للطاقة أعظمية قدر المستطاع. علاوة على ذلك، قمنا بمقارنة مختلف التصورات كالتحوير المكيف و غير المكيف من أجل معرفة كيفية التوزيع المثالي لوحدة المعلومة. نتائج المحاكاة تثبت انه يمكننا توزيع عدد معين من وحدات المعلومة باستخدام المعلومات المتوفرة حول وضع القناة.

كلمات مفتاحية: نظام تعدد النواقل، قدرة القناة، الكفاءة الطيفية للطاقة، التحوير المكيف، توزيع وحدات المعلومة.

Abstract

In this dissertation we treated the adaptive modulation problem for a time-varying and frequency selective channel when the channel state information is imperfect. The imperfections of channel state information are due to errors in channel estimation or in delay caused by the Doppler spread. By using a uniform power allocation of every sub-channel, the non linear problem considered was to maximize the spectral power efficiency and keeping the same targeted instantaneous bit error rate for all subchannels. Furthermore we compared different scenarios such as adaptive and non-adaptive modulation in order to know how to efficiency adapt the modulation. The simulation results show how it is important to underload the system when errors in channel state information are detected.

Keywords : OFDM, Channel capacity, Spectral efficiency, Adaptive modulation, Bit allocation.

Résumé

Dans ce mémoire, nous avons traité le problème de modulation adaptative pour un canal sélectif en fréquence et variant dans le temps lorsque les informations sur l'état du canal sont supposées imparfaites. Les imperfections du canal sont dues à des erreurs dans l'estimation du canal ou au retard provoqué par l'étalement Doppler. En utilisant une répartition uniforme de la puissance sur chaque sous-canal, le problème non linéaire considéré, était de maximiser l'efficacité spectrale et de maintenir le même taux d'erreur instantané cible pour tous les sous-canaux. En outre, nous avons comparé différents scénarios tels que la modulation adaptative et non adaptative afin de savoir comment adapter efficacement la modulation. Les résultats de la simulation montrent comment il est important de décharger le système lorsque des erreurs sur le canal sont détectées.

Mots clés : OFDM, Capacité du canal, Efficacité spectral, Modulation adaptative, Allocation des bits..

General Introduction

Over several years, there has been a high demand of high data rates that are to be supported by wireless communication applications especially due to the increase of mobile users and advancement of technology. Different solutions have been proposed by the research communities to adapt to this new demand, the utilization of Orthogonal Division Multiplexing (OFDM) systems arises as one of the best candidates due to the fact that it works better in frequency selective channels especially in mitigating ISI in multipath environment . In order to effectively adapt the information to the frequency selective channel through better selection of OFDM modulation scheme, Different channel parameters and scenarios have to be measured so as to better understand the channel variations and characteristics. The term CSI (Channel state information) which characterize the status of the channel between transmitter and receiver helps in the knowledge and modeling of the communication link.

In this dissertation we deal with the analysis of CSI for OFDM communication systems for various degrees of quality and quantity of channel state information. The analysis section is devoted to the study of capacity and achievable rates and the part that deals with design is aimed at the synthesis of practical communication systems that maximize the channel spectral efficiency for frequency selective channels.

Firstly, in first chapter,we focus on the understanding of basic transmission chain with different blocks that characterize different processes that are done in the respective step. Also we discuss different modulation schemes since its choice is very important in order to select the convenient modulation scheme depending on the actual state of channel,

factors such as minimum distance, error probability and symbol constellation are also discussed because they help in the choice of which modulation scheme is to be used.

Secondly, we discuss OFDM system, its principle, implementation, advantages and disadvantages are some of the main focus in this second chapter. Some interferences such as Inter Symbol Interference (ISI) and Inter Carrier Interference which are problems associated with multipath and OFDM systems are discussed with the solutions in order to combat these effects. Also Peak to Average Power Ratio (PARP) which is the main short coming of OFDM is briefly described with the solution to mitigate it.

Thirdly, CSI and channel capacity are discussed in third chapter. Different concepts such as pilot symbol, pilot insertion, feedback, perfect CSI, partial CSI and no CSI are discussed and their respective contribution towards characterization of wireless link. Also different capacity types are considered, we see the effect of error probability on the capacity and how its understanding helps in the designing of devices so as to better adapt the modulation schemes with respect to channel parameters and characteristics. Lastly, we compare different simulation graphs that show how the channel capacity varies in relation to different degree of information that we have on channel and hence the discussion is made in order to show that, better knowledge about the channel facilitates better adaptation of OFDM modulation scheme so as to improve the channel spectral efficiency.

Chapter 1

Fundamentals

1.1   Introduction

The radio propagation is not as smooth as in wired transmission since the received signal is not only coming directly from the transmitter, but the combination of reflected, diffracted, and scattered copies of the transmitted signal. It is interesting and rewarding to examine the effects of propagation to a radio signal since consequences determine data rate, range, and reliability of the wireless system.

In this chapter, different concepts such as digital transmission chain and the common key modeling parameters like path loss, delay spread, coherence bandwidth, Doppler spread, and coherence time will be discussed. All these factors, characterize and are considered in designing of wireless communication systems. Since communication is a hugely important aspect, not only for people around the world, but also for small and large businesses therefore we need a better way to relay messages from one end to the other. Probably businesses would be lost without the current technological advancements and a lot of companies would cease to exist. But this is not the only benefit that

telecommunications can bring. With these advancements also comes science. Without telecommunications, we would be unable to fly on planes and helicopters or effectively navigate in the seas. Besides this, space travel would be nearly impossible. In saying so, the need of having the best information transmission that has minimum errors is inevitable. The following are some of the basic concepts in understanding how Telecommunication works.

1.2   Basic digital communication system

Actually digital transmission systems are designed to transmit reliable information anytime, anywhere and to everyone. Thanks to its flexibility, robustness and low cost, many applications, technologies and services depend on digital communication systems. In this section, we wish to describe the basic aspects in digital communication design. As observed from the figure 1.1, digital transmission chain comprises of the blocs whose processes are done on the information in order to insure that it arrives with minimum decay at the receiving end. In a digital transmission chain, the digital signal can be obtained by converting the origianl signal (e.g acoustic signal) into an electric signal which is then transformed through an analogue to digital conversion. Typically, the digital conversion and subsequent source encoding will be selected to obtain the desired trade-off between the transmitted signal quality and the amount of information to be transmitted. Once the signal has been digitized, it can be transmitted as any digital information through a transmission chain that potentially includes channel/forward error coding, mapping of the channel encoded information to a modulation scheme, digital to analogue conversion of the modulated signal, transmission of the radio-frequency signal, analogue to digital conversion of the received signal followed by demodulation and finally decoding of the channel/forward error correcting code. Such a digital transmission chain may or may not involve a retransmission mechanism in case the packet is not error free at reception.
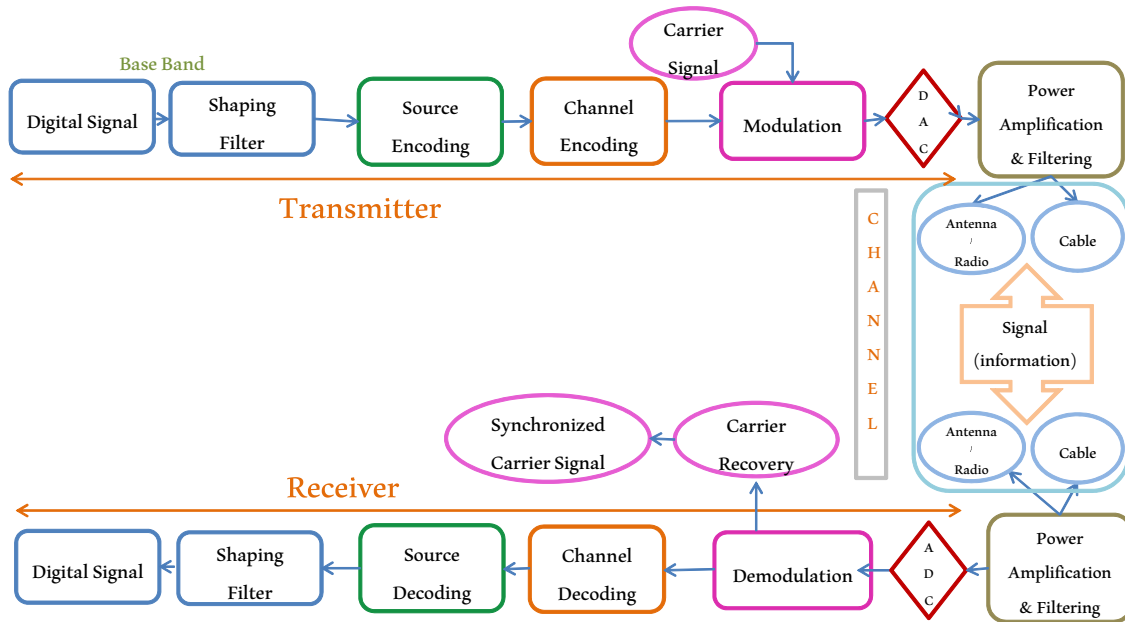
Figure 1.1: Basic block diagram of a digital communication

The information undergoes signal processing and a series of modification. Here are some of the most common processes that are done at the transmitting end.

1.2.1   Sampling and quantization :

The first step in analogue-to-digital conversion is sampling the continuous time signal at discrete values of time. In order to sample the signal, it is necessary that the sampling frequency (F_s) follows the theorem of Shannon, which states that $F_s \geq 2F_{max}$, where F_{max} is the maximum frequency of the transmitting signal.

Quantization in digital signal processing is the process where by different samples are attributed to a specific value chosen from a finite set.
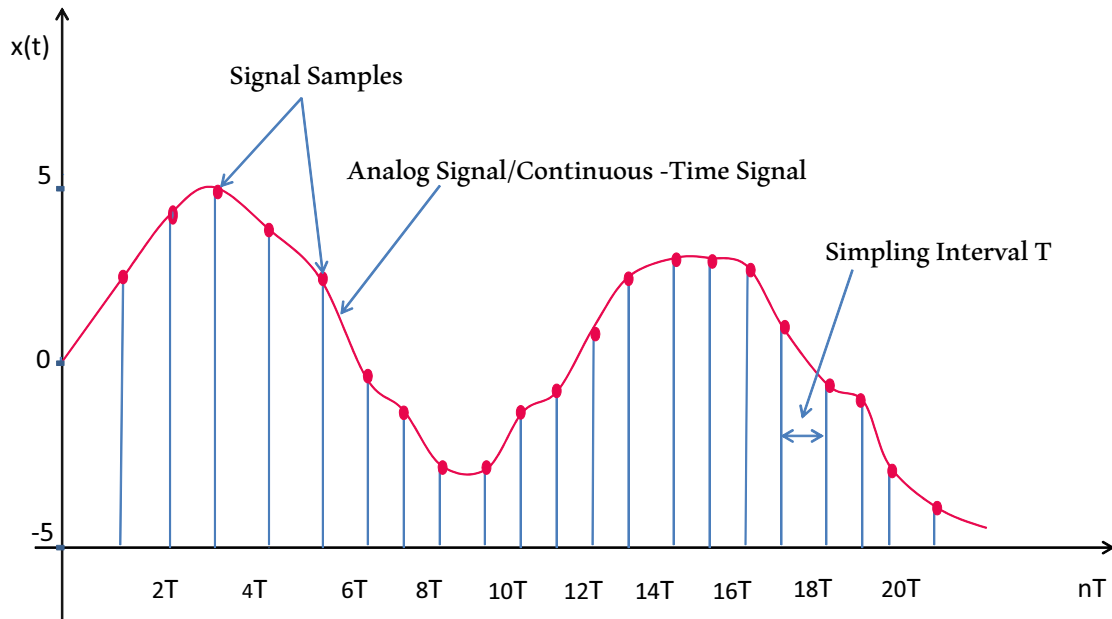
Figure 1.2: Display of the analog (continuous) signal and display of digital samples versus the sampling time instants.

1.2.2   Source Encoding:

Source encoding is used to compress the signals or to reduce the redundancy and increasing the bit rate by removing the repeating bits. This step bases on the analysis and statistics on the properties of the transmitting signal. The mapping is generally performed in sequences or groups of information and alphabetical symbols. Also, it must be performed in such a manner that it guarantees the exact recovery of the information symbol back from the alphabetical symbols otherwise it will destroy the basic theme of the source encoding.

The source encoding is called lossless compression if the information symbols are exactly recovered from the alphabetical symbols otherwise it is called lossy compression. The Source coding also known as compression or bit-rate reduction process. It is the process of removing redundancy from the source symbols, which essentially reduces data size. Source coding is a vital part of any communication system as it helps to use disk space and transmission bandwidth efficiently. The source encoding can either be lossy

or lossless. In case of lossless encoding, error free reconstruction of source symbols is possible, whereas, exact reconstruction of source symbols is not possible in case of lossy encoding. The minimum average length of codewords as function of entropy is restricted between an upper and lower bound of the information source symbols by the Shannon's source coding theorem.

1.2.3 Channel Encoding:

This is another type of coding that consists of the addition of redundancy for the purpose of protecting or correcting the errors. The choice of codes mainly depends on the error probability during the signal transmission. Different codes such as linear codes, linear bloc codes, convolutional codes and turbo codes can be used in different applications.

The channel encoding is a framework of increasing reliability of data transmission at the cost of reduction in information rate. This goal is achieved by adding redundancy to the information symbol vector resulting in a longer coded vector of symbols that are distinguishable at the output of the channel.

1.2.4 Modulation:

Modulation is the process of varying one or more properties (Amplitude, phase, frequency) of a periodic waveform, called the carrier signal, with a modulating signal that typically contains information to be transmitted. In other words, modulation is the process of conveying a message signal, for example a digital bit stream or an analog audio signal, inside another signal that can be physically transmitted. Modulation of a sine waveform transforms a base-band message signal into a bandpass signal. There are
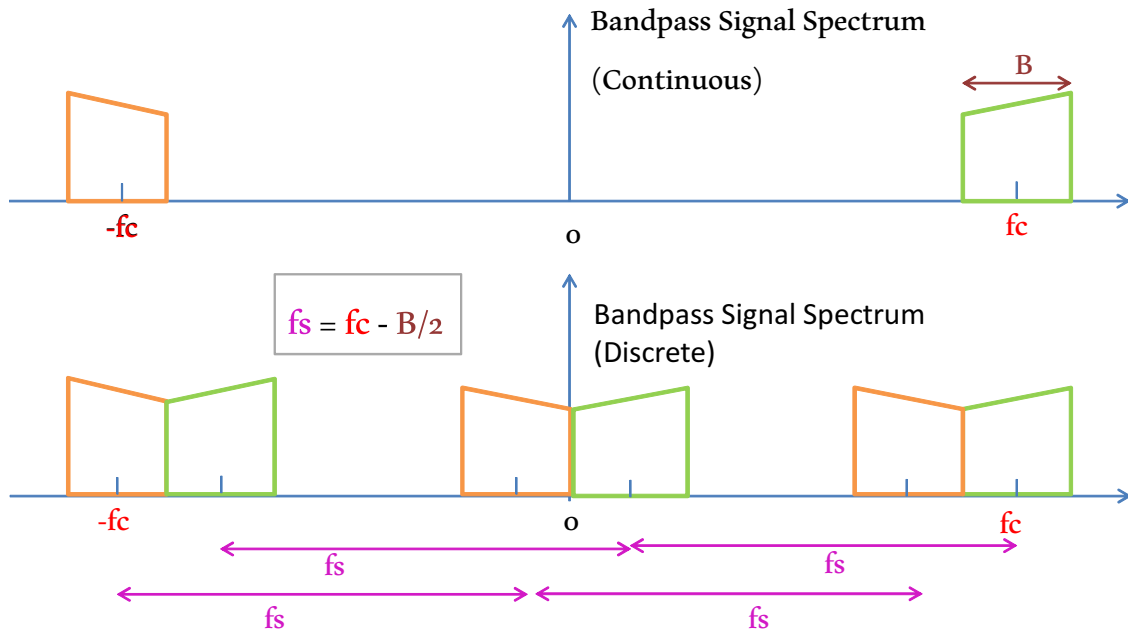
Figure 1.3: Bandpass Signal Spectrum

two types of modulation which are: **digital modulation** and **analogue modulation**. In this master's dissertation, digital modulation, which is highly used in high data rate transmission, will be discussed.

In digital modulations use generally the **Gray coding** for binary mapping, where two successive values differ in only one bit. These codes are widely used to facilitate error correction in digital communications. An Example of 16QAM constellation with Gray coding is given by figure 1.4. In many of today's communication systems, the modulation type and size are not fixed parameters but are chosen according to instantaneous channel conditions and data rate demands of the users. These adaptive modulation schemes require some kind of channel knowledge at the transmitter. As an example, the SNR can be used to determine the highest possible modulation size under the constraint of a desired error probability. Adaptive modulation schemes are already applied in wireless local area networks WLAN using the IEEE 802.11g standard [1].
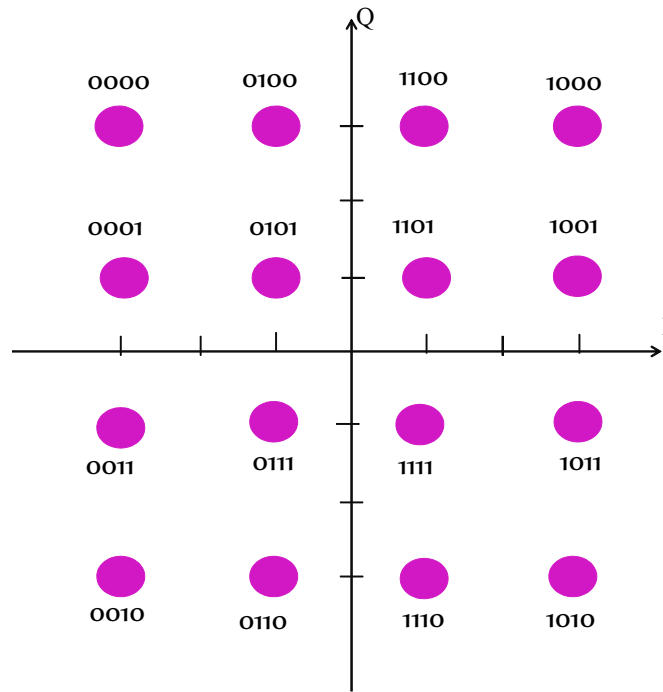
Figure 1.4: Example of 16QAM constellation with Gray coding

1.3 Digital modulation:

In Digital communication, digital modulations are very important to increase the spectrum efficiency and to transform bits to analog waveforms that can be sent over a physical channel. Some of useful different types of digital modulations as Pulse Amplitude Modulation (PAM), Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM) are presented.

1.3.1 Pulse Amplitude Modulation (PAM):

PAM is the special case of Amplitude Shift Keying (ASK) it also involves the variations in Amplitude if the amplitude of real valued symbols bears the information. This case is only true when the neighbouring symbols are equidistant from each

other. Figure 1.5 shows an example of ASK modulation. The amplitudes are chosen to $X_\mu = (2\mu + 1 - M)e$, For $0 \le \mu < M$ in order to have equal distances between the neighbouring symbols. The general formula of binary error probability of M- PAM is

Baseband data

1

0   0

1

Modulated ASK signal

Acos(ωt)   Acos(ωt)
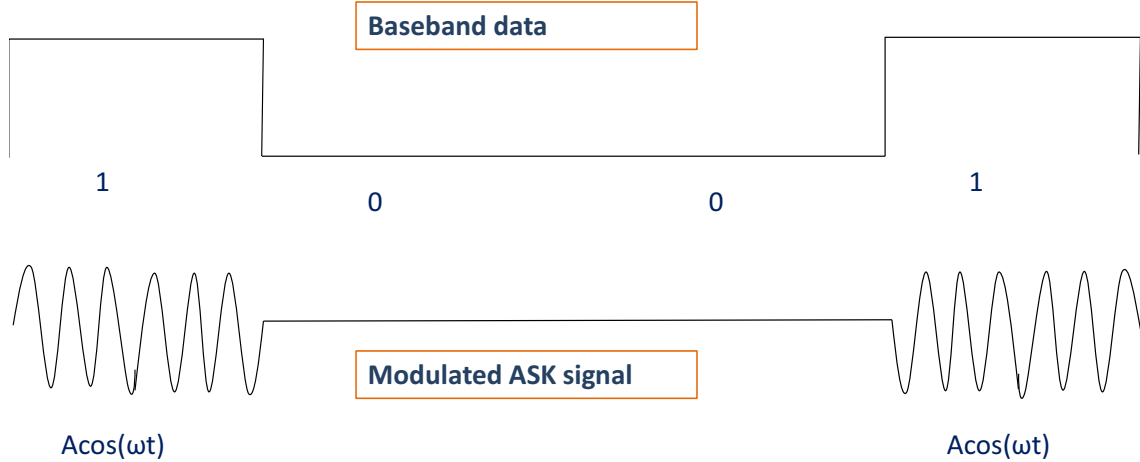
Figure 1.5: Amplitude Shift Keying (ASK)

given by [2]:

$$P_{e,bin}(\texttt{M-PAM}) = \frac{2(M-1)}{M\log_2(M)} Q\left(\sqrt{\frac{6\log_2(M)}{(M^2-1)}\frac{E_b}{N_0}}\right) \tag{1.1}$$

where M is the number of symbols which determines the modulation level; N_0 is the power spectral density of noise and E_b is the average energy per bit given by :

$$E_b = E_s/\log_2(M) \tag{1.2}$$

1.3.2   Phase shift keying modulation (PSK):

Is another type of digital modulation that involves the changing of phase of a transmitting signal, it is done by varying the sine and cosine waves of the input at precise times. It has a circular constellation and mostly deployed in Wireless Local Area Networks (WLAN), Radio Frequency IDentification (RFID) and Bluetooth communications. Different cases of PSK modulation are illustrated in figure 1.7.
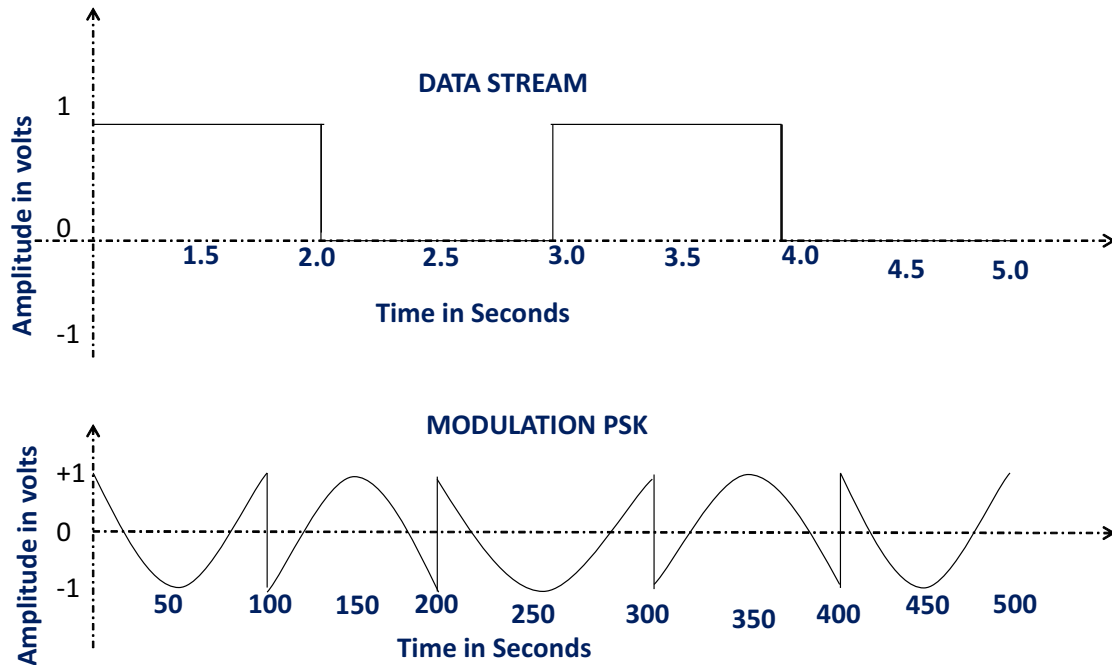
Figure 1.6: phase shift keying (PSK)

1.3.3 Binary Phase Shift Keying modulation (BPSK):

Is the special case of PSK that uses two phases that are separated by 180° , this modulation is the most robust of all the PSKs because the demodulator can only make the incorrect decision when the noise or distortion is of highest level. BPSK is functionally equivalent to 2-PAM and the binary error probability is given by [2]

$$P_{e,bin}(\texttt{BPSK}) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) \tag{1.3}$$

1.3.4 Quadrature Phase Shift Keying(QPSK):

This is another case of PSK which uses four points in constellation diagram. These points are equispaced around a circle with four phases. This modulation can be used to double the data rate of BPSK while maintaining the same bandwidth of the signal,

it can also be used to maintain the data rate of BPSK while halving the bandwidth. QPSK and 4-QAM have the same resulted modulated signal waves although the root concepts are different. The general binary error probability formula for M-PSK when $(M > 2)$ is given by [2]

$$P_{e,bin}(\texttt{M-PSK}) = \frac{2}{\log_2(M)} Q\left(\sqrt{2\log_2(M)\frac{E_b}{N_0}}\sin(\frac{\pi}{M})\right) \qquad (1.4)$$
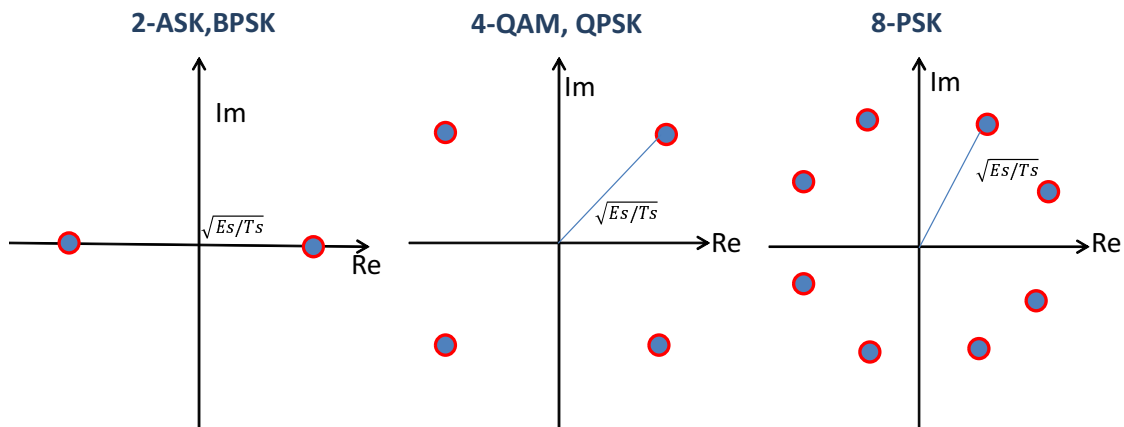
Figure 1.7: Symbol alphabets of digital phase modulation

1.3.5   Quadrature Amplitude Modulation(QAM):

The term Quadrature is due to the fact that the two carrier waves of the same frequency (usually sinusoids) are out of phase from each other by 90°.
This type of modulation is highly deployed in WLAN, High Speed Downlink Packet Access (HSDPA)/ Universal Mobile Telecommunications Standards Institute (UMTS) and LTE. The real and imaginary symbols can be chosen independently from each other. The first carrier wave modulates the real symbols, and the second carrier wave modulates the imaginary symbol. The combination of both parts results in a square arrangement, hence the combination of ASK and PSK is possible in QAM. The equation

that shows the relation between error probability and symbol energy is given below [2]
:

$$P_{e,bin}(\texttt{M-QAM}) = \frac{4\left(\sqrt{M}-1\right)}{\log_2(M)\sqrt{M}}Q\left(\sqrt{\frac{3\log_2(M)}{M^2-1}\frac{E_b}{N_0}}\right) \tag{1.5}$$
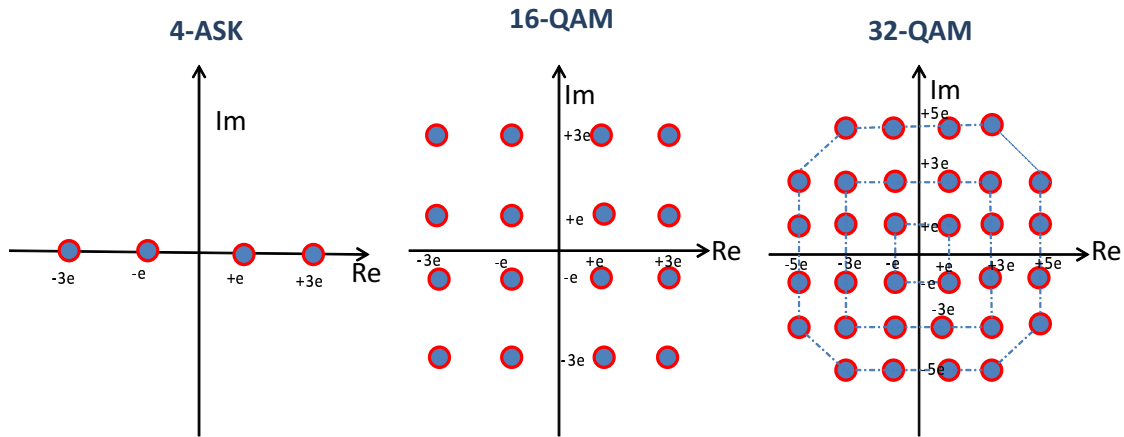
Figure 1.8: Symbol alphabets of linear amplitude modulation

1.4 Radio Channel Characteristics:

1.4.1 Radio Channel:

Radio channel is of time-varying nature in which the parameters randomly change with respect to time. Therefore understanding the various characteristics of such channel and understanding their physical significance is of great importance. For the optimization of system parameters, the choice of its transmission system architecture and the dimensioning of equipment and components depend on the appropriate channel modeling in different environments such as urban, mountainous, indoor, outdoor, mobile, fixed *etc....* In this part, different factors that contribute to the signal perturbations

will be discussed.

1.4.2 Multipath Propagation:

The multipath propagation phenomenon occurs when the radio signals arrive at the receiving antenna by two or more paths. It is due to multiple scatterers such as atmospheric ducting, ionospheric reflection and refraction, reflection from water bodies and terrestrial objects such as mountains and as well as man-made factors i.e. moving objects and buildings. Reflection occurs when the signal hits a surface and partial energy is reflected and the remaining is transmitted into the surface. Reflection coefficient is a coefficient that determines the ratio of reflection and transmission, it depends on the material properties. Diffraction occurs when the signal is obstructed by a sharp object which derives secondary waves. Each path in the multipath fading model is associated with a corresponding attenuation factor (a_n) and the path-delay (τ_n). In continuous time, the complex path attenuation suffered by a signal on a given path is given by

$$\tilde{a}_n(t) = a_n(t) \exp\left(-j2\pi f_c \tau_n(t)\right) \tag{1.6}$$

The complex channel response is given by [3]

$$h(t, \tau) = \sum_{n=0}^{N-1} \tilde{a}_n(t)\delta(t - \tau_n(t)) \tag{1.7}$$

The average delay is equal to the first moment of the Power delay profile (PDP) $p(\tau)$. For a discrete channel it is calculated as [4]

$$\bar{\tau} = \frac{\sum \tau p(\tau)}{\sum p(\tau)} \tag{1.8}$$

When the given PDP values are continuous in terms of time delays, the summation is simply replaced with integral and integrate it with respect to $d\tau$. The second central
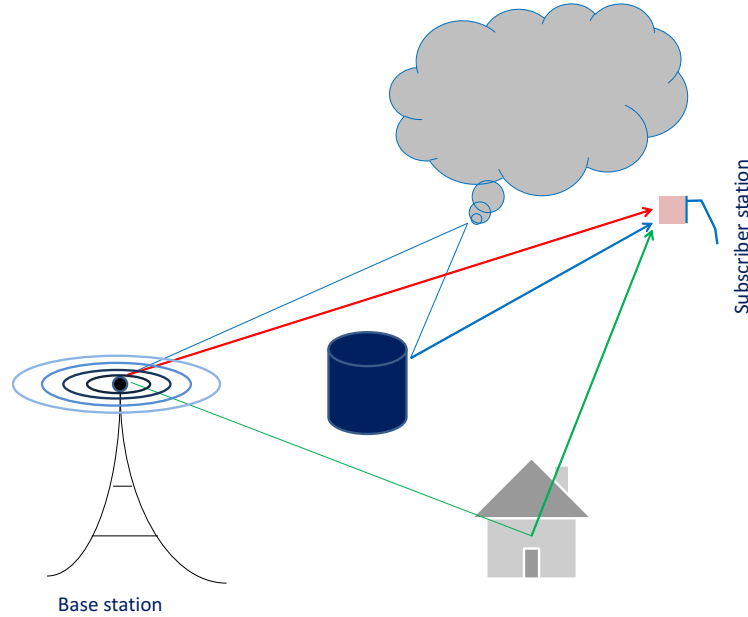
Figure 1.9: Multipath channel.

moment of power delay profile $p(\tau)$ is equal to Root Mean Square (RMS) delay spread σ_τ. It is similar to the standard deviation of a statistical distribution. For a discrete channel the RMS delay spread is given by [4]

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \tag{1.9}$$

where,

$$\overline{\tau^2} = \frac{\sum \tau^2 p(\tau)}{\sum p(\tau)} \tag{1.10}$$

The summation is again replaced with integral and integrate it with respect to $d\tau$ if the given PDP values are continuous in terms of time delays. The ratio of RMS delay spread and symbol time duration quantifies the strength of Inter Symbol Interference (ISI).

A multipath channel, can generate a constructive or destructive interference. Constructive interference occurs when the resulted wave has a bigger amplitude than either of the forming waves. Destructive interference is when the resulted wave has a smaller amplitude than either if the signals, it is also known as fading.

In general, two models represent the magnitudes of the signals arriving from different paths. The Rayleigh distribution is basically the magnitude of the sum of two equal independent orthogonal Gaussian random variables. Rayleigh distribution which is given by the following formula [4] :

$$P(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{\sigma^2}\right) \qquad \text{for} \quad 0 \leq r \leq \infty \qquad (1.11)$$

When one component (mostly, a line of sight component) dominates, the Rician distribution provides a more accurate model, which is known as Rician fading given by the formula below [4]:

$$P(r) = \frac{r}{\sigma^2} \exp\left(\frac{-(r^2 + A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) \qquad for \quad r \geq 0, A \geq 0 \qquad (1.12)$$

1.4.3 Doppler Spread:

Doppler spread f_d is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel and is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. When a pure sinusoidal tone of frequency f_c is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range $(f_c - f_d)$ to $(f_c + f_d)$,where f_d is the Doppler shift which is relative to mobile velocity. Doppler frequency is given by the the following formula [4] :

$$f_d = \frac{1}{2\pi}\frac{\delta\phi}{\delta t} = \frac{v}{\lambda}cos(\phi) = f_m cos(\phi) \qquad (1.13)$$

where f_m is the maximum Doppler Frequency (Hz).

In a small-scale region, the time varying nature of the channel is described by Doppler spread and coherence time.

1.4.4   Coherence Bandwidth:

The multipath spread of the channel is the difference in time between the arrival of the earliest and latest multipath component. We denote this channel parameter as τ_{max}. The coherence bandwidth B_c is described as the bandwidth over which the frequency channel response function remains virtually constant.Coherence bandwidth is used to quantify the channel frequency response and it is inversely proportional to RMS delay spread. Coherence bandwidth is used to measure how flat the channel bandwidth is. The shorter the delay spread, the larger is the coherence bandwidth. Thus, we define the channel parameter :

$$B_c = \frac{1}{\tau_{max}} \tag{1.14}$$

If the transmitted signal has a bandwidth $B_s < B_c$, the channel is called frequency nonselective (or flat-fading channel. On the other hand, if the transmitted signal has a bandwidth $B_s > B_c$, the channel is said to be frequency selective.

1.4.5   Shadowing:

Shadowing is the result of more or less strong attenuation of signal strength, this is due to the obstruction of signal waves by objects such as mountains, trees and buildings as shown in figure 1.10. This phenomena causes slow fading. In other words Shadowing is the effect that the received signal power fluctuates due to objects obstructing the propagation path between transmitter and receiver.

1.4.6   Path loss:

Occurs when the average signal power decays with the distance between the emitter and the receiver, it can also be influenced by other environmental variations such us
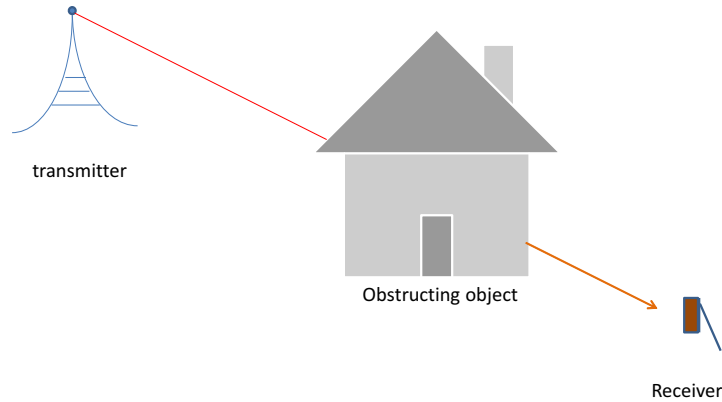
Figure 1.10: Shadowing

urban or rural. In free space the mean power decreases with the square of the distance between the two terminals, in environments Non Line of sight (NLOS) the mean power decreases with a power higher than two and it is generally in the order of three to five.Usually to model real environments the shadowing effects cannot be neglected. If the shadowing effect is neglected, the Path Loss is simply a straight line. In free space, the path loss from isotropic antennae is given by Friis Transmission equation stated below : [2]

$$L_s(dB) \quad = \quad 20\log\left(\frac{4\pi d}{\lambda}\right) \tag{1.15}$$

$$= \quad 92.4 + 20\log\left(d_{km}\right) + 20\log\left(f_{GHz}\right) \tag{1.16}$$

In other environments with NLOS it is given as:

$$
\begin{aligned}
L_s(\text{dB}) = 69.55 \quad &+ \quad 26.16\log(f) - 13.82\log(h_b) - A(h_m) \\
&+ \quad (44.9 - 6.55\log(h_b))\log(d)
\end{aligned}
\tag{1.17}
$$

where L_s is the pathloss in urban areas (dB), h_b is the height of base station antenna (m), h_m is the height of mobile station antenna(m), f is the frequency of transmission and d is the distance between base station and the mobile station (km). The Okumura-Hata model is : [2]

- Small and medium-sized cities :

$$A(h_m)(\texttt{dB}) = 3.2 \left[\log(11.75 h_m) \right]^2 - 4.79 \tag{1.18}$$

- Large cities:

$$A(h_m)(\texttt{dB}) = [1.1 \log(f) - 0.7) h_m] - [1.56 \log(f) - 0.8] \tag{1.19}$$

where $A(h_m)$ is the pathloss with respect to environment.

Because of all these variations in signal intensity it is important to clearly understand the channel fading parameter so as to better simulate and transmit the signals in different times, geographical locations, and radio frequency. These variations can be conquered by power control.

1.5   Fading channel:

In the following part the channel is going to be discussed according to its fading characteristics. The magnitude change and phase change occur in a certain period of time, the minimum time in which these changes change is referred to as coherence time. It is given by :

$$T_c = 1/f_d \tag{1.20}$$

Where f_d is the Doppler frequency.

Moreover, Fading is term used to describe the variations in a received signal strength as a result of multipath components. Multiple copies of the signal arrive at the receiver, having transmitted through different propagation paths, each signal copy will experience differences in attenuation, delay and phase shift while traveling from the source to the receiver. This can result in either constructive or destructive interference. The fading can be defined as fast fading or slow fading. Additionally, fading can be defined as flat or frequency selective fading.

1.5.1 Fast fading:

This phenomenon is used to describe the channel characterized by rapid variations over very short distances. This fading is due to scattering from nearby objects and thus is termed small-scale fading. When there is no direct path or NLOS, a **Rayleigh distribution** tends to best fit this fading scenario, thus fast fading is sometimes referred to as Rayleigh fading.the complex impulse response h(t) of the flat fading channel as follows :

$$h(t) = h_I(t) + jh_Q(t) \tag{1.21}$$

where $h_I(t)$ and $h_Q(t)$ are zero mean Gaussian distributed. Therefore the fading envelope is Rayleigh distributed and is given by

$$|h(t)| = \sqrt{|h_I(t)|^2 + |h_Q(t)|^2}[5] \tag{1.22}$$

Fast fading occurs when there is a direct path or a dominant path (Line of sight (LOS)), fast fading can be modeled with a **Rician distribution** [5]. Fast Fading results might be due to the following:

- High Doppler Spread f_d

- Coherence Time $T_c \ll$ Symbol Period T_s

- Channel impulse response changes rapidly within the symbol duration.

1.5.2 Slow fading channels:

Describe the characteristics of a channel by slow variations in the mean value of the signal. This fading is due to scattering from more distant larger objects and thus is termed large-scale fading. It does not vary quickly with the frequency. It originates due to effect of mobility. A log-normal distribution tends to best fit this fading scenario,

thus slow fading is sometimes referred to as log-normal fading. Slow fading might also be due to the following:

- Low Doppler spread

- Coherence time $T_c \gg$ symbol period T_s.

- Impulse response changes much slower than the transmitted signal.

A superimposed plot of fast and slow fading is developed and is shown below:
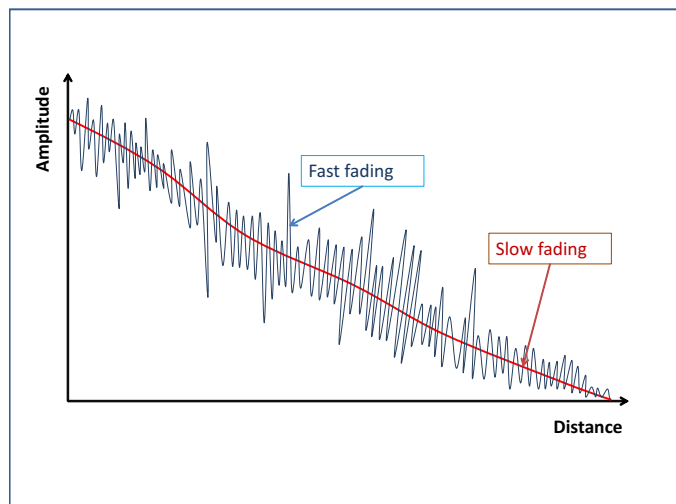
Figure 1.11: Comparison between fast and slow fading

1.5.3   Flat fading Channels:

They are amplitude varying channels and are sometimes referred to as Narrow-band channels since the bandwidth of the applied signal is narrow as compared to the coherence bandwidth of channel. Since the coherence bandwidth of the channel is

larger than that of the signal, all the components of a signal will experience the same magnitude of fading.

$$B_c \gg B_s \tag{1.23}$$

There are various ways to reduce the fading distortion. Diversity is one way which leverages multiple independent channels. Since the channels are independent they have lower probability to experience fades at the same time. Space diversity is one of the most efficient diversity technique compared to time or frequency [5].

1.5.4 Frequency Selective Fading

This type of fading occurs when the channel bandwidth is smaller than the signal bandwidth.

$$B_c \ll B_s \tag{1.24}$$

Channel bandwidth limits the signal bandwidth which leads to overlapping of the successive symbols in the time domain due to the convolution and hence the overall symbol duration becomes more than the actual symbol duration. This phenomenon is called ISI. There are several ways to combat ISI. Equalization is one way that tries to invert the effects of the channel. In OFDM, each subcarrier undergoes flat fading since its bandwidth is less than the coherence bandwidth. Therefore ISI is eliminated within an OFDM symbol but OFDM symbols may overlap in time [5].

1.5.5 Additive White Gaussian Noise

The Additif White Gaussien Noise (AWGN) is basic and generally accepted model for thermal noise in communication channels. The name is due to the following set of assumptions :

- the noise is additive which means that the received signal equals the transmit signal plus some noise, where the noise is statistically independent of the signal.

- the noise is white which means that the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset.

- the noise samples have a Gaussian distribution. Generally it is assumed that the channel is Linear and Time Invariant. The most basic results further assume that it is also frequency non-selective.

1.6 Conclusion:

As discussed in this first chapter one can observe that in transmission chain, different types of modulations were discussed, generally modulation is very important in digital transmission since it conveys a multitude of advantages such as multiplexing, signal bandwidth adjustment, avoidance of signal mixing, reception quality and SNR improvement, reduction of antennae size, and the increase of communication range. Furthermore, its counterpart, demodulation, has relative advantages too since the modulated wave has a large frequency and consist of carrier and sideband frequencies. If the modulated wave is directly fed to the operating device, it wouldn't at all be able to respond to those high frequencies that's why the separation of higher carrier frequency is needed. However The choice of the best type of modulation for a particular communications system requires the simultaneous consideration of many factors evaluated for the basic requirements of that system. Such factors include compatibility, effective range, bandwidth, signal to noise performance, interference rejection capabilities, distortion characteristics, required stability, required transmitter power, and resulting circuit complexity. It is also observed that there mostly two common types of variations of a mobile radio signal. First, the average signal strength at any point depends on its distance from the transmitter, the carrier frequency, the type of antennas used, antenna

heights, atmospheric conditions, and so on, this signal strength may also fluctuate due to shadowing caused by terrain and clutter such as hills, buildings, and other obstacles. Also, it is remarked that this type of signal fluctuation which is commonly observable over relatively long distances, such as, a few tens or hundreds of wavelengths of the radio frequency RF carrier, has a log normal distribution and is classified in the literature as a large-scale variation. The second type of variation is caused by multipath reflections. In urban or dense urban areas, there may not be any direct line-of-sight path between a mobile and a base station antenna. Instead, the signal may arrive at a mobile station over a number of different paths after being reflected from tall buildings, towers, and so on. Because the signal received over each path has a random amplitude and phase, the instantaneous value of the composite signal is found to vary randomly about a local mean. A multipath fading is said to occur in such conditions.

Since different factors affect the transmission of information in different ways, several approaches have been applied to combat the perturbations and as remarked that the radio channel has multiple perturbations. Better error minimization techniques are needed in order to insure that the information is well received. Many techniques exist but until now especially for high data rates transmission OFDM is the best technique that fits well and it will be described in second chapter.

Chapter 2

Orthogonal Frequency Division Multiplexing (OFDM)

2.1 Introduction:

The idea of multicarrier transmission dates back to the beginning of 1950's, the notions such as Frequency Division Multiplexing FDM existed ever since, more than one low rate signals were carried over relatively wide bandwidth channel using separate carrier frequency over each signal, the resulted spectra were relatively low because the carrier frequencies were spaced far from each other to avoid the overlapping and hence to facilitate the separation of signals at the receiving terminal. The typical example of this system is Telegraph [5].

In 1970's and 1980's the high spectral efficiency and low cost implementation of these techniques became possible by the application of Discrete Furrier Transform DFT. Orthogonal Frequency Division Multiplexing OFDM originates from frequency division multiplexing (FDM), in which more than one low rate signal is carried over separate

carrier frequencies too. But in FDM the resulting spectral efficiency is very low as compared with OFDM.

The comparison that shows an analogy of OFDM against single carrier and FDM in terms of spectral efficiency is depicted in figure 2.1.
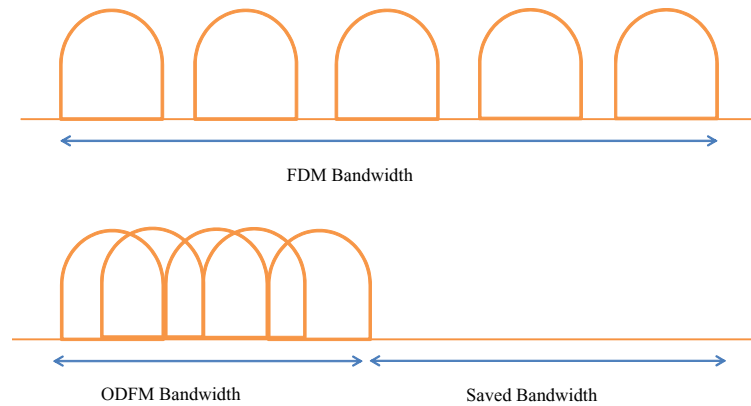
Figure 2.1: Comparison of OFDM and FDM

2.2   Orthogonal Frequency Division Multiplexing :

OFDM is a Frequency Division Multiplexing scheme used as a digital multicarrier method where a large number of closely spaced orthogonal sub-carrier signals are used to carry data on several parallel data streams or channels. Instead of carrying separate messages as it was done in telegraph, the different frequency carriers can carry different bits of a single higher rate message.

The source may be in such a parallel format, or a serial source can be presented to a serial-to-parallel converter whose output is fed to the multiple carriers. If the parallel formats were to be processed by different transmitters and receivers it would be complex and costly to implement. It would then require the implementation of

complex equalizers. In spite of its high cost and bandwidth inefficiency, High data rates over a dispersive channel used the parallel technique as preferred means of transmission and the advantage of this is the relatively reduced susceptibility to various impulse noise forms. The introduction of Fast Fourier Transform (FFT) technique to the modulation and demodulation process brought a major contribution to OFDM complexity problem. The technique involved assembling the input information into blocks of N complex numbers, one for each sub channel. An inverse FFT is performed on each block, and the resultant transmitted serially. At the receiver, the information is recovered by performing an FFT on the received block of signal samples. This form of OFDM is often referred to as Discrete Multi-Tone (DMT) i.e. binary data are transmitted serially as a pulse train.

FFT reduces the spacing between the orthogonal sub carriers. Consider for example a sinusoid of duration T seconds and bandwidth $\frac{k}{T}$ where k is the positive integer. Two sinusoids of the same parameters can become orthogonal to each other by separating them far apart in frequency domain so that their spectra do not overlap, however this is a waste of spectrum because ideally two such sinusoid which are spaced $\frac{1}{T}$ apart in frequency are orthogonal. FFT done on sampled signal at proper rate can insure this and data is loaded on subcarriers spaced $\frac{1}{T}$ apart in frequency domain. A communication system with multi-carrier modulation transmits N complex-valued source symbols in parallel onto N sub-carriers. The source symbols may, for instance, be obtained after source and channel coding, interleaving, and symbol mapping. The source symbol duration T of the serial data symbols results after serial-to-parallel conversion in the OFDM symbol duration

$$T = NT_s \tag{2.1}$$

Generally, an OFDM signal can be represented as

$$\textbf{OFDM signal}: c(t) = \sum_{n=0}^{N-1} s_n(t) \sin(2\pi f_n t) \tag{2.2}$$

The principle advantage of OFDM application is that multi-carrier modulation can be implemented in the discrete domain by using an IDFT, or a more computationally

efficient IFFT. The diagram of OFDM modulation scheme is given by the figure 2.2.
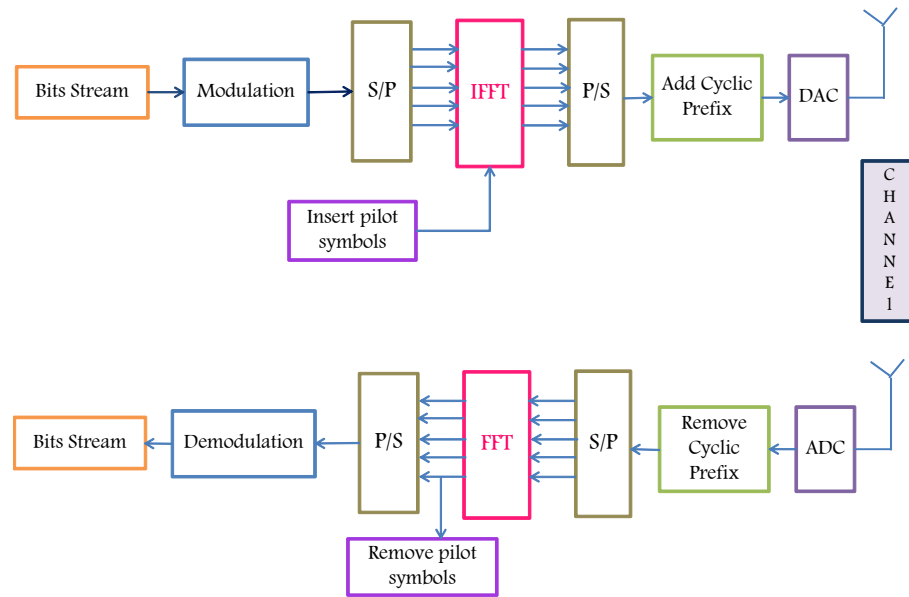
Figure 2.2: OFDM modulation scheme

2.3   The basic principle of OFDM:

The basic principle of OFDM is based on splitting a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the symbol duration increases for lower rate parallel subcarriers, the relative amount of dispersion in time caused by multipath delay spread is decreased.

The analog implementation of OFDM can be extended to the digital domain by using the discrete Fourier Transform (DFT) and its inverse (IDFT). These operations are widely used in order to transform data between the time-domain and frequency-domain. These transforms make the mapping of data onto orthogonal subcarriers. The example of mapping is when the IDFT correlates the frequency-domain input data with its orthogonal basis functions, which are sinusoids at certain frequencies. In practice,

OFDM systems are implemented using a combination of fast Fourier Transform (FFT) and inverse fast Fourier Transform (IFFT) blocks that are mathematically equivalent versions of the DFT and IDFT, respectively, but more efficient to implement. An OFDM system treats the source symbols (e.g., the QPSK or QAM symbols that would be present in a single carrier system) at the transmitter as though they are in the frequency-domain. These symbols are used as the inputs to an IFFT block that converts the signal into the time-domain. FFT is represented by

$$X(k) = \sum_{n=0}^{N-1} x(n) \sin(\frac{2\pi kn}{N}) + j \sum_{n=0}^{N-1} x(n) \cos(\frac{2\pi kn}{N}) \tag{2.3}$$

where as its dual , IFFT is given by

$$x(n) = \sum_{n=0}^{N-1} X(k) \sin(\frac{2\pi kn}{N}) - j \sum_{n=0}^{N-1} X(k) \cos(\frac{2\pi kn}{N}) \tag{2.4}$$

The IFFT takes in N symbols at a time where N is the number of subcarriers in the system. Each of these N input symbols has a symbol period of T_s seconds. Recall that the basic functions for an IFFT are N orthogonal sinusoids. These sinusoids each have a different frequency and the lowest frequency is DC. Each input symbol acts like a complex weight for the corresponding sinusoidal basis function. Since the input symbols are complex, the value of the symbol determines both the amplitude and phase of the sinusoid for that subcarrier.

The IFFT output is the summation of all N sinusoids. Thus, the IFFT block provides a simple way to modulate data onto N orthogonal subcarriers. The block of N output samples from the IFFT make up a single OFDM symbol. The length of the OFDM symbol is $T = NT_s$. After some additional processing, the time-domain signal that results from the IFFT is transmitted across the channel. At the receiver, an FFT block is used to process the received signal and bring it into the frequency-domain. Ideally, the FFT output will be the original symbols that were sent to the IFFT at the transmitter [6].

2.4 Principle of orthogonality:

Obviously the spectra of the subcarriers are not separated but overlap. The reason why the information transmitted over the subcarriers can still be separated is the so called orthogonality relation giving the method its name. The subcarriers orthogonality is illustrated by figure 2.3. By using an IFFT for modulation we implicitly chose the spacing of the subcarriers in such a way that at the frequency where we evaluate the received signal all other signals are zero. In order for this orthogonality to be preserved the following must be true [7] :

1. The receiver and the transmitter must be perfectly synchronized. This means they both must assume exactly the same modulation frequency and the same time-scale for transmission (which usually is not the case).

2. The analog components, part of transmitter and receiver, must be of very high quality.

3. There should be no multipath channel.
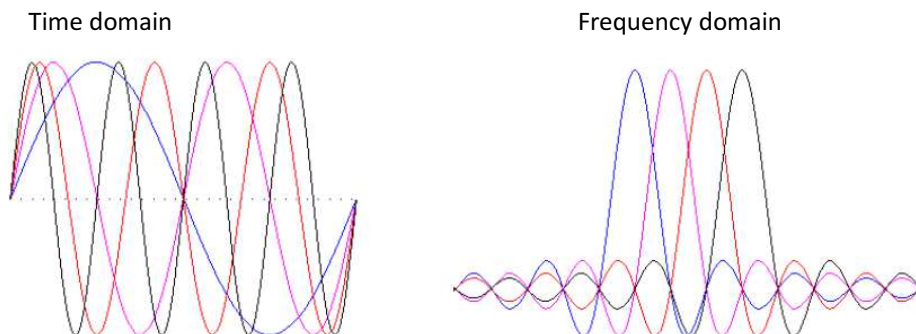
Figure 2.3: Orthogonal subcarriers

2.5   Inter-symbol interference:

The inter-symbol interference occurs as the effect of filtering part of the transmitted signal by the channel. The filtering causes the transmitted pulses to get mixed together, meaning that a pulse that is transmitted between time instants will smear into adjacent pulses affecting the process of data detection and possibly causing errors not as a result of noise but as a result of symbols mixing together. Unlike analog signals, which are usually smooth in nature, digital signals are composed of pulses with often vertical transitions. The fact that digital signals sometimes have vertical transitions increases their bandwidth significantly since it requires infinite bandwidth to represent a signal with vertical transitions.

Any communication system has limited bandwidth to transmit digital data, this indicates that certainly a transmitted square pulse will be received differently at the receiver as the channel will filter some components of it. The difference depends on how narrow the bandwidth of the channel compared to that of the signal is [8]. The typical representation of ISI is illustrated by the figure 2.4.
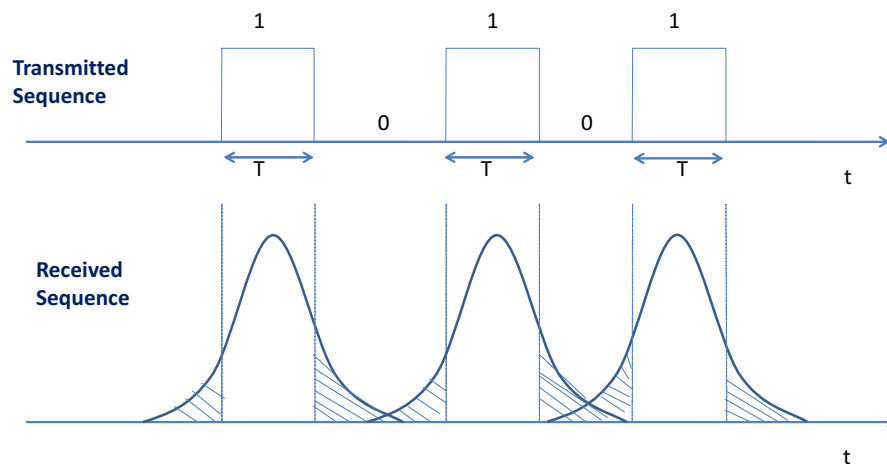
Figure 2.4: Typical representation of ISI

2.6   Inter-carrier interference:

The random frequency errors (offset) in OFDM system distort orthogonality between subcarriers. As a result, inter-carrier interference (ICI) occurs. These undesired changes degrade the performance of the system. The carrier frequency offset is produced at the receiver because of the local oscillator instability and variability of operating conditions at transmitter and receiver, Doppler shifts caused by the relative motion between the transmitter and receiver, or the phase noise introduced by other channel impairments.

The degradation is caused by the reduction in the signal amplitude of the desired subcarrier and by the ICI from the neighboring subcarriers. The amplitude loss occurs because the desired subcarrier is no longer sampled at the peak of the equivalent function of the DFT. Adjacent subcarriers cause interference because they are not sampled at their zero crossings. The overall effect of subcarrier frequency offset affects the system performance [8].

The characteristics of ICI are similar to Gaussian noise; hence, it leads to degradation of the performance. The amount of degradation is proportional to the fractional of subcarrier frequency offset, which is equal to the ratio of subcarrier frequency offset to the subcarrier spacing. The diagram shown in 2.5 represents ICI
In order to reduce the effect of the ISI, each multicarrier (Multi-Carrier (MC)) symbol is extended with a guard interval. In MC systems, where the data symbols are transmitted in parallel on N different carriers, the length T of a symbol is extended with a certain factor. This extension of the symbol length causes the MC system to be less sensitive to channel dispersion than a single carrier system transmitting data symbols at the same data rate.

However, at the edges of a MC symbol, the channel dispersion still causes distortion,
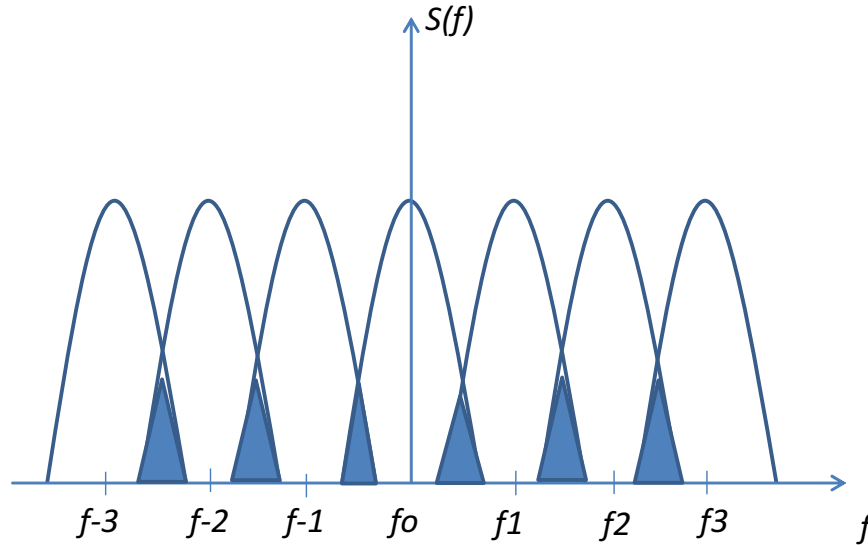
Figure 2.5: Typical representation of ICI.

and hence introduces interference between successive MC symbols (i.e. intersymbol interference, ISI) and interference between different carriers within the same MC symbol (i.e. intercarrier interference, ICI). As the transmission efficiency reduces with the insertion of the guard interval (during the guard interval, no new information can be transmitted), the guard interval must be chosen sufficiently small.

2.7 Cyclic Prefix (CP):

The basic idea is to replicate part of the OFDM time-domain waveform from the back to the front to create a guard period. The duration of the guard period should be longer than the worst-case delay spread of the target multi-path environment.

In this addition method, the last samples of each OFDM symbol of N samples are copied and added in front of the OFDM symbol, At the the receiving terminal, the samples in the cyclic prefix are discarded, as they are affected by interference, the

N samples outside the CP are kept for further processing. Because during the guard interval signal is transmitted, the CP-OFDM system suffers from a power efficiency loss with a factor N. The use of a cyclic prefix instead of a plain guard interval, simplifies the channel equalization in the demodulator [9]. The following figure 2.6, represents GI and CP :

TS= Extended Symbol Time

TGI= Guard Interval or Cyclic Prefix

TIFFT= Inverse FFT Time Period Or Useful Symbol Time Period
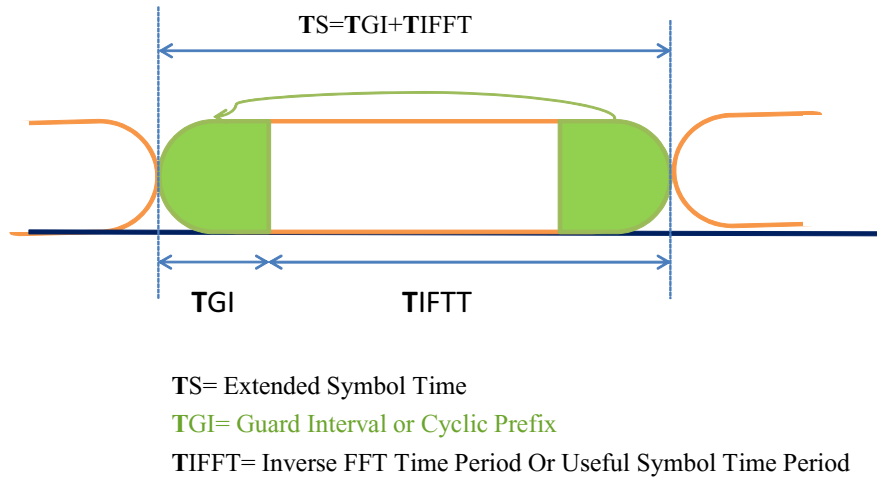
Figure 2.6: typical OFDM frame with cyclic extension

2.8   Advantages and disadvantages of OFDM:

The main advantage of OFDM modulation is the fact that is an efficient way to deal with multipath effects and to simplify the channel equalization over a frequency selective channel. There are of course other typical advantages of OFDM system :

- As shown before, OFDM makes efficient use of the spectrum by allowing overlap.

- By dividing the channel into narrow-band flat fading subchannels, OFDM is more resistant to frequency selective fading than single carrier systems are.

- Eliminates ISI and ICI through use of a cyclic prefix.

- Using adequate channel coding and interleaving one can recover symbols lost due to the frequency selectivity of the channel.

- Channel equalization becomes simpler than by using adaptive equalization techniques with single carrier systems.

- It is possible to use maximum likelihood decoding with reasonable complexity.

- OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions.

- Provides good protection against cochannel interference and impulsive parasitic noise.

OFDM communication systems suffer from some drawbacks. Since the transmit signals in an OFDM system can have high peak values in the time domain since many subcarrier components are added via an IFFT operation. As a result, OFDM systems are known to have a high peak-to-average power ratio (Peak-to-average power ratio (PAPR)) when compared to single-carrier systems. In fact, the high PAPR is one of the most detrimental aspects in an OFDM system as it decreases the signal-to-quantization noise ratio (Signal to Quantization Noise Ratio (SQNR)) of the analog-digital convertor (Analogue to Digital convertor (ADC)) and digital-analog convertor (Digital to Analogue convertor (DAC)) while degrading the efficiency of the power amplifier in the transmitter. As a side note, the PAPR problem is more of a concern in the uplink since the efficiency of the power amplifier is critical due to the limited battery power in a mobile terminal. As the result the OFDM signal has a noise like amplitude with a very large dynamic range, hence it requires RF power amplifiers with a high peak to average power ratio. Also OFDM is more sensitive to carrier frequency offset and drift than single carrier systems are due to leakage of the DFT. Due to these disadvantages, OFDM requires linear amplifiers to combat PAPR and it also needs very good carrier synchronization to maintain the orthogonality of carriers.

2.9 CONCLUSION

For communication over channels with frequency selective fading i.e. different frequency components of the signal experience different fading, it is very difficult to handle frequency selective fading at the receiver, in which case, the design of the receiver is hugely complex. Instead of trying to mitigate frequency selective fading as a whole, OFDM mitigates the problem by converting the entire frequency selective fading channel into small flat fading channels.

Nowadays, the need for high rate data transmission and the number of mobile device users increase very fast, high data rate and mobility are the important discussion points. To meet this need, some techniques such as reducing the symbol duration or using higher order modulation were introduced in the past. In the former method, the signals received through the multipath channel suffer severely from ISI because the delay spread becomes much larger than symbol period. In order to overcome this, the symbol duration must be larger than the channel delay spread. In OFDM system, cyclic prefix extension totally helps to eliminate the effect of ISI, when the guard interval is longer than the maximum length of delay spread.

Chapter 3

Channel Capacity and Channel State Information

3.1 Introduction

Channel State Infomation (CSI) is the information which is used to represent the actual state of a communication channel between the transmitter and the receiver. CSI provides the detail of signal propagation between transmitter and the receiver and tells about the effects such as scattering and fading. The CSI can incorporate current channel conditions with transmission data for achieving reliable communication with considering the famous Shannon's relation of channel capacity. This CSI is estimated at the receiver and fed back to the transmitter as shown in figure 3.1. The channel state information can be obtained through different types of channel estimation algorithms (e.g. Linear Square Estimator (LSE), Minimum Mean-Square Estimator (MMSE), Decision-feedback channel estimation (DFE), Maximum Likelihood (ML)). In channel estimation one is required to estimate the channel impulse response (or frequency response) for each burst separately from the well-known transmitted bits and corre-

sponding received samples. Depending on the available CSI (perfect or partial) at the transmitter, many strategies can be adopted in order to maximize the data rate according to the signal-to-noise ratio and the channel gain. Power loading and adaptive modulation adhering to a certain target Bit Error Rate (BER) per subcarrier are used to improve the performance. However, in practice, perfect channel state information (CSI) is rarely achieved. To avoid overloading the system and significantly increasing the BER of the system, knowledge of CSI through a quantitative and qualitative measures of its degradation at the receiver is necessary for the design of practical resource allocation schemes at the transmitter.

In this section, according the available CSI at the transmitter, we investigate the different schemes recommended to maximize the data rate (Shannon's capacity) under different scenarios and different constraints. The impact of imperfect channel information on OFDM systems is addressed.
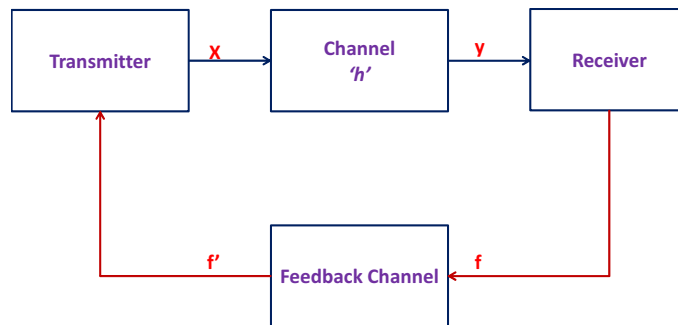
Figure 3.1: Feedback system model

3.2   Practical communication schemes improving the data rates

In the following subsections, some of the most important practical communication schemes are reviewed depending on the degree of CSI that is available at the transmitter and receiver. It is because it largely helps communication systems designers to practically define the objective function to measure the global performance of the system and to design of the system accordingly to that specific function definition.

3.2.1   Practical schemes with no CSI

When no CSI is available at the transmitter side, there exist two main philosophies to design communication architectures. The first techniques has the objective of increasing the transmission rate by exploiting the multiplexing gain, while the second technique takes advantage of the diversity gain provided by multi-antenna channels [10, 11].

However to enable a more reliable communication to systems with no CSI many studies were presented like the technique called Space-Time Code (Space-Time Code (STC)), This technique essentially consists in transmitting the same information stream through all the antennas, but applying a different delay at each antenna in such a way that different realizations of the same data are acquired at the receiver side.

3.2.2   Practical schemes with perfect CSI

The data rate and power allocation problem is commonly called the "bit-loading" problem. Several algorithms have been proposed in literature for maximizing the data

rates assuming a perfect knowledge of CSI and with different constraints [12, 13, 14]. Although the case of having a perfect CSI at both sides of the system has been studied deeply, the only work in which the transmitter is designed assuming that a perfect ML receiver shows the better results. The utilization of a ML receiver is motivated by recent efficient implementations which reduce the computational complexity by sacrificing optimality.

3.2.3 Practical schemes with imperfect CSI

In a practical scenario it may be too pessimistic that the transmitter is totally unaware of the CSI and assuming that both the transmitter and the receiver have full access to the CSI may not be too realistic. Consequently, usually only imperfect and/or incomplete CSI is available (specially at the transmitter side, since, in some scenarios, the receiver can accurately track the channel variations).

3.3 AWGN channel capacity

Information theory was invented by Claude Shannon in 1948 to characterize the limits of reliable communication. Before Shannon, it was widely believed that the only way to achieve reliable communication over a noisy channel, such as, to make the error probability as small as desired, was to reduce the data rate. Shannon disapproved this by his information theory, he showed that, by more intelligent coding of the information, one can in fact communicate at a strictly positive rate but at the same time with as small an error probability as desired. However, there is a maximal rate, called the capacity of the channel, for which this can be done: if one attempts to communicate at rates above the channel capacity, then it is impossible to drive the error probability to zero. Shannon's information theory tells us the amount of information a channel

can carry. In other words it specifies the capacity of the channel. The theorem can be stated in simple terms as follows :

- A given communication system has a maximum rate of information C known as the channel capacity.

- If the transmission information rate R is less than C, then the data transmission in the presence of noise can be made to happen with arbitrarily small error probabilities by using intelligent coding techniques.

- To get lower error probabilities, the encoder has to work on longer blocks of signal data. This entails longer delays and higher computational requirements.

The Shannon-Hartley theorem indicates that with sufficiently advanced coding techniques, transmission that nears the maximum channel capacity is possible with arbitrarily small errors. One can intuitively reason that, for a given communication system, as the information rate increases, the number of errors per second will also increase [15]. Shannon-Hartley equation relates the maximum capacity (transmission bit rate) that can be achieved over a given channel with certain noise characteristics and bandwidth. For an AWGN the maximum capacity is given by [16]

$$C = B \times \log_2(1 + SNR) \tag{3.1}$$

Here C is the maximum capacity of the channel in bits/second otherwise called Shannon's capacity limit for the given channel, B is the bandwidth of the channel in Hertz, S is the signal power in Watts and N is the noise power, also in Watts. The ratio S/N is called Signal to Noise Ratio (SNR), it can be ascertained that the maximum rate at which we can transmit the information in reliable communication, is limited by the bandwidth, the signal level, and the noise level.

Shannon's capacity limit is defined for the given channel. It is the fundamental maximum transmission capacity that can be achieved on a channel given any combination

of any coding scheme, transmission or decoding scheme [15]. It is the best performance limit that we hope to achieve for that channel.

The above expression for the channel capacity makes intuitive sense: To increase the information rate, the signal-to-noise ratio and the allocated bandwidth have to be traded against each other. For no noise, the signal to noise ratio becomes infinite and so an infinite information rate is possible at a very small bandwidth. Thus we may trade off bandwidth for SNR. However, as the bandwidth B tends to infinity, the channel capacity does not become infinite since with an increase in bandwidth, the noise power also increases. The above Shannon's equation relies on two important concepts [15] :

1. That, in principle, a trade-off between SNR and bandwidth is possible.

2. That, the information capacity depends on both SNR and bandwidth.

Remarks :

- When the SNR is large ($SNR \gg 1$ dB), the capacity is logarithmic in power and approximately linear in bandwidth. This is called the bandwidth-limited regime.

- When the SNR is small ($SNR \ll 1$ dB), the capacity is linear in power but insensitive to bandwidth. This is called the power-limited regime.

The bandwidth-limited regime and power-limited regime are illustrated in the figure 3.2
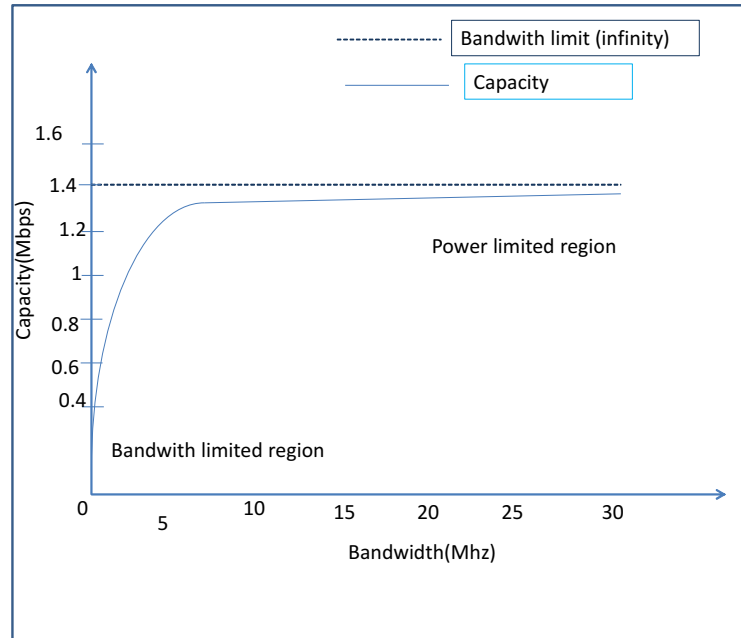
Figure 3.2: Bandwidth limited regime and power limited regime

3.4   Capacity of flat fading channels

Capacity is very important to measure the channel capacity limit especially due to the fact that, there is a disruptive growth in demand for wireless communication. When we ignore the constraints in delay or complexity of encoder and decoder, we can transmit higher data rates in wireless channel with minimum error probability if these capacity limits are not exceeded by data rates. In 1948 Claude Shannon pioneered channel capacity by using mathematical theory of communication by the notion of mutual information between the channel input and output. Shannon defined the capacity as maximum mutual information between the input and output of the channel. Shannon coding theorem and converse proved that if the data rate is inferior to the channel capacity then it exists a code that can transmit the information with almost negligible error probability but if the rate is superior to the channel capacity there will be almost impossible to transmit the information without error probability. Here, we will consider the different capacities for single-user wireless channel where the transmitter and

receiver have single antenna.

The capacity of fading channel, depends on the channel characteristics and also on the knowledge available at transmitter and/or receiver. Capacity is not always easy to determine and there are different definitions of channel capacity. We distinguish time-invariant fading channel or time-variant channel and flat-fading channel or frequency-selective fading channel.

As discussed in chapter 1, fastness of channel variations is characterized by coherence time T_c and flat/frequency selective fading is characterized by coherence bandwidth B_c. We consider a discrete-time channel with stationary and ergodic time-varying gain h_n and its channel power gain $|h|^2$.

3.4.1   When the CSI is known at the receiver.

When the fading statistics are known but the channel gains are known at the receiver only. In this case, two relevant definitions of capacity are derived : **ergodic capacity**, and **capacity with outage**

A. Ergodic capacity :

The ergodic capacity C_{erg} represents the average capacity among all channel states, and is mainly used for **fast fading channels** when coding is performed over many channel states. In a fast-fading channel, where the latency requirement is greater than the coherence time and the codeword length spans many coherence periods, one can average over many independent channel fades by coding over a large number of coherence time intervals. Thus, it is possible to achieve a reliable rate of communication of [bits/s/Hz] and it is meaningful to speak of this value as the capacity of the fast-fading

channel. Since no CSI is available at the transmitter, similarly as Shannon capacity formula, the information transmission rate is constant. Data transmission takes place over all fading states including deep fades where the data is lost and hence the effective capacity is significantly reduced. The mathematical formula of ergodic capacity is as follows.

$$C_{erg} = E\left\{ B\log_2\left(1 + \frac{E_s}{N_0}|h|^2\right)\right\} \texttt{ (bits/s)} \tag{3.2}$$

B. Capacity with outage :

Outage capacity is used for slow fading channels. Here the instantaneous SNR is constant over a number of transmissions and it goes to another value according to fading distribution. Since the transmitter does not know the instantaneous value of SNR, it must fix the transmission rate which is independent to received SNR. The transmitter must fix minimum received SNR ie SNR_{min} such that if the received SNR is greater than or equal to SNR_{min} then the data will be received correctly and if the received SNR is less than SNR_{min} then the received bursts will be decoded with probability approaching to 1 then the receiver declares outage.

$$P_{out} = \texttt{Prob}\left(\log_2\left(1 + \frac{E_s}{N_0}|h|^2\right) < R\right) \tag{3.3}$$

3.4.2 When the CSI is known to both transmitter and receiver

When CSI h_n and the pdf $p(h)$ are known at the transmitter and receiver at time n, the transmitter can adapt its transmission strategy and optimally exploiting the CSI by varying the transmit power P_n and rate R as a function of SNR. The fading channel capacity subject to an average power constraint $\bar{P}$ is defined by [15]

$$C = \max_{P_n} \sum_n B\log_2\left(1 + \frac{P_n|h_n|^2}{N_0}\right)p(h) \tag{3.4}$$

subject to

$$\sum_n P_n = \bar{P}, \ P_n \geq 0 \ \forall n \tag{3.5}$$

where $p(h)$ is the pdf of h_n.

Knowing that the capacity of a channel is a fundamental limit in the maximum rates at which information can be transmitted with arbitrarily low probability of error. Furthermore, when the CSI is available in both ends, we optimize and adapt the transmission rate by using **waterfilling algorithm** which is basically the name given to the ideas in communication systems design and practice for equalization strategies on communications channels. To find the optimal power allocation we use Lagrange method such that:

$$L = \sum_n \log\left(1 + \frac{P_n}{N_0}|h_n|^2\right)p(h) - \lambda\sum_n P_n \tag{3.6}$$

where λ is the Lagrange multiplier. Next we differentiate lagrangian an set it equals to zero

$$\frac{\delta L}{\delta P_n} = 0 \tag{3.7}$$

The optimal power allocation is

$$P_n^{opt} = \left(\frac{1}{\lambda} - \frac{N_0}{|h_n|^2}\right)^+ \tag{3.8}$$

3.5   Frequency-selective channel capacity:

Frequency-selective channels generally employ an equalizer to recover the transmitted sequence altered by ISI. Most practical systems use a training sequence to learn the channel impulse response and thereby design the equalizer. An important issue is determining the optimal amount of training. Assume that the channel is Time-Invariant with discrete frequency response $H(k)$. The capacity of frequency-selective channel is obtained by dividing the the whole frequency band into N small bands of bandwidth

$B_c = B/N$, in which the subchannel $H(k)$ can be considered frequency-flat. This is of course the principle of OFDM modulation presented in chapter 2. The received signal with an OFDM communication system is

$$Y(k) = H(k)X(k) + W(k), k = 1, 2, 3, \cdots N \tag{3.9}$$

where $Y(k)$, $X(k)$ and $W(k)$ are the input, output and noise signals of *kth* tone, respectively. $H(k)$ denote the channel frequency response. The capacity of each subchannel is given by

$$C_k = \log_2\left(1 + \frac{P_k}{B_c N_0}|H(k)|^2\right) bit/symbol \tag{3.10}$$

3.5.1 Optimal power allocation

In order to maximize channel capacity in a frequency selective channel, water-pouring distribution is a well known technique used. However, this distribution has some shortcomings in practical data transmission because it assumes non-integer-bit constellations, does not obey a given probability of error, and is difficult to compute. Instead, the data throughput optimization problem is of more practical importance[17]. considering;

$$\max_{P_k} \sum_{k=1}^{N} C_k \tag{3.11}$$

is subject to

$$\sum_{n=1}^{N} P_k = \bar{P} \tag{3.12}$$

and where C_k, P_k and $\bar{P}$ are the rate (in bits/symbol), allocated power and a total power constraint, respectively. Knowing that the capacity of a channel is a fundamental limit in the maximum rates at which information can be transmitted with arbitrarily low probability of error In this work, error probability will be taken into consideration when estimating the capacity of the channel. Different cases will be studied and hence the comparison will be given with respect to better capacity optimizing technique.

3.5.2 Water filling

The process of water filling algorithm is similar to pouring the water in the vessel. The dark brown shaded portion of graph 3.3 represents the inverse of the power gain of a specific channel. The blue portion represents the power allocated or the water. The total amount on water filled (power allocated) is proportional to the Signal to Noise Ratio of channel.

The power allocation is thus the solution to the optimization problem.

$$C = \max_{P_k} \sum_{k=1}^{N} \log_2 \left(1 + \frac{P_k}{N_0} |H(k)|^2 \right) bit/symbol \tag{3.13}$$

Now, the optimization problem can be solved by the Lagrangian method and the modified equation is:

$$L = \sum_{k=1}^{N} \log_2 \left(1 + \frac{P_k}{N_0} |H(k)|^2 \right) - \lambda \sum_{k=1}^{N} P_k \tag{3.14}$$

where λ is the Lagrange multiplier. The optimal power allocation is

$$P_k^{opt} = \left(\frac{1}{\lambda} - \frac{N_0}{|H(k)|^2} \right)^{+} \tag{3.15}$$

Generally, the inverse of the Lagrange multiplier can be regarded as the water level as shown in figure 3.3 We have to maximize the total number of bits to be transported
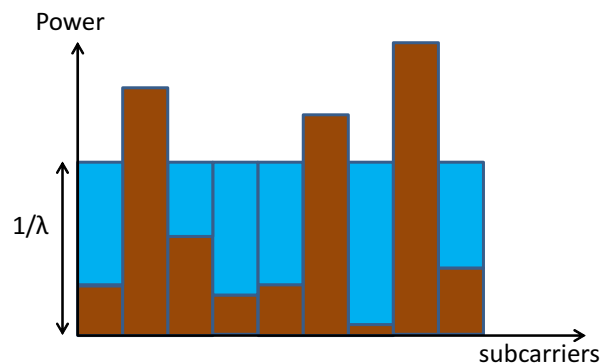
Figure 3.3: Water-filling power allocation

hence the following steps are followed to carry out the water filling algorithm

- Take the inverse of the channel gains.

- Water filling has non uniform step structure due to the inverse of the channel gain.

- Initially take the sum of the total power P_t and the inverse of the channel gain. It gives the complete area in the water filling and inverse power gain.

- Decide the initial water level by the formula given below by taking the average power allocated

- The power values of each subchannel are calculated by substracting the inverse channel gain of each channel.

remark: In case the power allocated value become negative stop iteration.

3.6   Channel estimation

From the characteristics of a wireless channel, the factors such as mobility of users, multipath propagation, frequency and time selectivity make channel estimation one of the main technical challenges in advanced wireless communications systems. Channel estimation is essential for achieving reliable information transmission for practical wireless communication applications.

Channel estimation techniques can be divided into two categories: data aided (using training symbols) and non-data aided (blind channel estimation).

For OFDM systems, the non-data aided method usually makes use of the presence of CP or finite alphabet property of the input data. Since it does not require any preamble, the non-data aided channel estimation enjoys high spectrum efficiency. However, it

usually requires many signal samples before reaching convergence, which results in long estimation latency. The data-aided channel estimation, on the other hand, is widely used because of its reliable estimation performance [18]. There are many data-aided channel estimation techniques, for example: least square (LS), minimum mean-square error (MMSE), and maximum likelihood (ML) algorithms.

3.6.1   Pilot signal

A pilot signal is usually a single frequency signal. This signal is transmitted over a communication system for control, equalization, continuity, synchronization,supervisory or reference purposes. Pilot aided channel is the most suitable for mobile radio channel, furthermore in LTE it is the proposed solution.This technique consists of transmitting Pilot symbols which are known by both transmitter and receiver in order to estimate symbols at the receiver. Basically, there are two types of pilot insertion which are bloc insertion and comb type insertion.

3.6.2   Bloc insertion

In block type pilot placement, all pilot tones are inserted into each OFDM symbol with a specific period of time. If the channel is constant during the bloc there will be no channel estimation error. the estimation can be done by using either LS or MMSE. The figure 3.4 represents bloc type pilot placement.
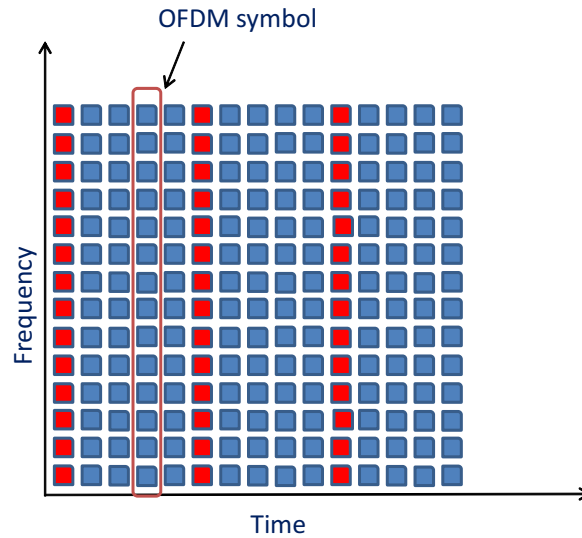
Figure 3.4: Bloc type insertion

3.6.3   Comb insertion

In this type, the pilots are transmitted at all times but with an even spacing on the sub carriers. A comb type pilot placement is shown in 3.5. MIMO-OFDM system performance is evaluated by means of the plot of Mean Square Error (Mean Square Error (MSE)) and Bit Error Rate (BER).Block-type and comb type pilot based channel estimation using LS and MMSE algorithms are used to model Rayleigh fading channel of Multiple Input Multiple Output (MIMO)-OFDM system and Mean square error is estimated. Comb type insertion is used when the channel is rapidly changing over time. The first step in determining the LS estimate is to extract the pilot symbols from their known location within the received blocs. Because the value of these pilot symbols is known, the channel response at these locations can be determined using the LS estimate. The LS estimate is obtained by dividing the received pilot symbols by their expected value. For the OFDM symbol, the received complex value is given by the formula:

$$Y(k) = H(k)X(k) + N(k) \qquad (3.16)$$

OFDM symbol

Frequency

Time
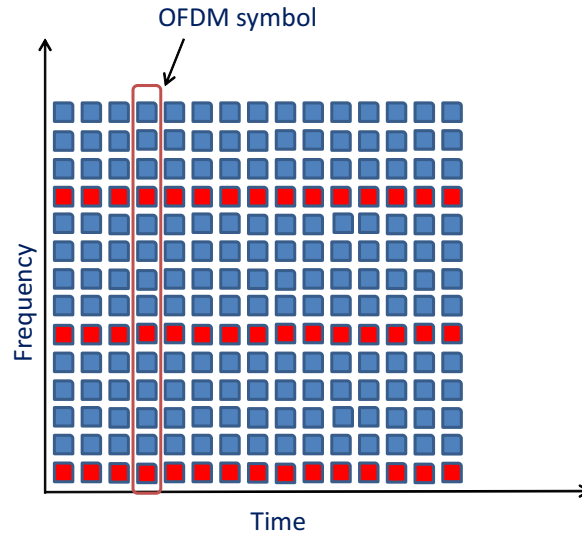
Figure 3.5: Comb type insertion

where $Y(k)$ is a received complex symbol value, $N(k)$ is the noise, $X(k)$ is a transmitted complex symbol value and $H(k)$ is a complex channel gain experienced by a symbol. Therefore the LS estimate value is given by:

$$H_p(k) = \frac{Y_p(k)}{X_p(k)} \tag{3.17}$$

$Y_p(k)$ represents the received pilot symbol values, $X_p(k)$ represents the known transmitted pilot symbol values and $H_p(k)$ is the true channel response for the RE occupied by the pilot symbol.

However, since pilot symbols carry no data information, the time and the power spent on pilot symbols degrades the efficiency and the throughput of the system. Therefore, it is necessary to minimize the pilot insertion ratio without degrading the error performance.

3.7 Acquisition of Partial CSI at the Transmitter

When the uplink and downlink wireless channels cannot be inferred from each other, the receiver needs to feedback the downlink channel to the transmitter. Knowing that the feedback channel is a scarce resource, the receiving end is required to find a better and efficient way to represent the channel. Basically, this is a quantization problem. The difference from traditional quantization problems is that the distortion measure here is not mean-square error but the capacity penalty resulting from using non-perfect CSI instead of perfect CSI at the transmitter.In practical adaptive OFDM systems, only imperfect CSI is available at the transmitter. Furthermore, imperfect channel knowledge will be taken into consideration in order to efficiently maximize the expected transmission rate given the fixed total transmission power [17].

As shown in the figure 3.1, the downlink transmitter receives partial CSI feedback f' from the receiver. The true channel matrix, which the transmitter does not fully know, can be modeled as a Gaussian random matrix or vector whose mean and covariance is given in the feedback.

Channel estimation methods also show some difference between systems with a single-transmit antenna and systems with transmitter diversity. While the complexity of the estimators is low for systems with a single-transmit antenna, the estimators for systems with multiple transmitters are very complex.

3.8 Adaptive OFDM Systems with BER-Constraint

In this section, lets consider a selective-frequency and time-varying channel. The impulse response of the channel is given by

$$h(t, \tau) = \sum_{\ell=1}^{L} \alpha_\ell(t) \delta(\tau - \tau_\ell) \tag{3.18}$$

where L is the total number of paths, τ_ℓ is the delay of the ℓth path, and $\alpha_\ell(t)$ is the corresponding complex amplitude. The $\alpha_\ell(t)$'s are Wide Sense Stationary (WSS) complex Gaussian processes and independent for different paths with $E|\alpha_\ell(t)|^2 = q_\ell^2$ which is the PDP's channel. The correlation function is given by :

$$E\{\alpha_\ell(t + \Delta t)\alpha_\ell^*(t)\} = q_\ell^2 r_t(\Delta t). \tag{3.19}$$

and

$$r_t(\Delta t) = J_0(2\pi f_D \tau \Delta t) \tag{3.20}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function, and f_D is the maximum Doppler frequency. Let B be the total bandwidth of the system and N the total number of sub-channels. Let $H[n, k]$ denote the frequency response of the kth tone in the nth OFDM bloc. According to [19], $H[n, k]$ is a complex Gaussian random variable with zero mean and unit variance for any n and k.

In adaptive OFDM systems, we can either modulate differently each subchannels in different times or modulate the same way to all subchannels at given times. Considering the first adaptation, Assume QAM is employed for each subchannel, and $\beta[n, k]$ bits/symbol are sent for the kth tone in the nth block. We investigate the spectral efficiency under a constrained BER requirement. Given the channel frequency response $H[n, k]$, the instantaneous BER for the kth tone in the nth block can be approximated by [20]

$$P_e[n, k] \approx c_1 \exp\left(-\frac{c_2 \frac{E_s}{N_0} |H[n, k]|^2}{2^{\beta[n,k]} - 1}\right) \tag{3.21}$$

where $c_1 = 0.2$, $c_2 = 1.6$. where c_1 and c_2 are constants.

3.8.1 Ideal Adaptive OFDM with prefect CSI

In this part, we consider the perfect knowledge of CSI at the transmitter hence, different subchannels are assigned with different modulation schemes and the assignments vary over time.

One way to choose the modulation schemes to achieve the target BER is to set instantaneous BER $P_e[n, k]$ equal to P_{tar}. Using this approach, the number bits transmitted in each subchannel can be derived by inverting (3.21) and we obtain

$$\beta_{ideal}[n, k] = \log_2 \left[\frac{c_2 \frac{E_s}{N_0} |H[n, k]|^2}{\ln \frac{c_1}{P_{tar}}} + 1 \right] \tag{3.22}$$

It is assumed that the identical statistics of $H[n, k]$ for all k are available at the transmitter and Therefore, the average spectral efficiency does not depend on the system parameters or the specific power delay profile of the WSSUS mobile radio channel. Therefore, the average spectral efficiency

$$R_{ideal} = E_{H[n,1],\cdots,H[n,N]} \left\{ \frac{1}{N} \sum_{k=1}^{N} \beta_{ideal}[n, k] \right\} \tag{3.23}$$

3.8.2 Adaptive OFDM with imperfect CSI

In practice, two types of errors when measuring CSI are inevitable, first, errors due to noisy channel estimation and second, the unavoidable delay between when channel estimation is performed and when the estimation result is used for actual transmission.

A. New loading Algorithm:

The imperfections in CSI knowledge due to channel estimation error or outdated CSI affect the BER. It is necessary to take theses imperfections in consideration in order to keep a given instantaneous BER for all subchannels. The approach proposed by [19], uses statistical information about the CSI errors to exactly maintain the required average BER level and gives the resulting performance in terms of spectral efficiency.

Suppose the estimated channel gain $H'[n,k]$ is the only known information about the current CSI for the kth tone in the nth block, and $\beta[n,k]$ is computed based on this value of $H'[n,k]$. Since the instantaneous BER, $P_e[n,k]$, depends on the value of the true channel $H[n,k]$, which is assumed unknown, it is not possible to fix $P_e[n,k]$ to be the target value. However, we can define the average BER given $H'[n,k]$ for the kth tone in the nth block as

$$\bar{P}_e[n,k] = E_{H[n,k]|H'[n,k]}\left\{P_e[n,k]\right\} \tag{3.24}$$

where the expectation is evaluated over $H[n,k]$.

The proposed algorithm employs the $\beta[n,k]$ which sets $\bar{P}_e[n,k]$ to P_{tar} for the known $H'[n,k]$, thus satisfying the final average BER requirement. Intuitively, this loading algorithm tends to underload each subchannel to account for the statistical properties of the errors in CSI. The system performance is then measured by the average spectral efficiency, which is

$$R_{imp} = E_{H'[n,1],...,H'[n,K]}\left\{\frac{1}{N}\sum_{k=1}^{N}\beta[n,k]\right\} \tag{3.25}$$

If the $H'[n,k]$'s have the same distribution, this reduces to

$$R_{imp} = E_{H'[n,k]}\left\{\beta[n,k]\right\} \tag{3.26}$$

In this case, as in Ideal Adaptive OFDM, the performance does not depend on the system parameters or the power delay profile of the WSSUS channel when the guard interval is ignored.

Consider the special case when $H[n,k]$ given $H'[n,k]$ is complex Gaussian with mean s and variance σ^2. It follows that $r = |H[n,k]|$ conditioned on $H'[n,k]$ is Ricean distributed. Using (3.22) and the conditional pdf of $|H[n,k]|$ to calculate the expectation in (3.24), the average BER becomes

$$
\begin{aligned}
\bar{P}_e[n,k] &\approx \int_0^\infty c_1 \exp\left(-\frac{c_2 \frac{E_s}{N_0} r^2}{2^{\beta[n,k]} - 1}\right) . \frac{2r}{\sigma^2} \exp\left(-\frac{r^2 + |s|}{\sigma^2}\right) I_0\left(\frac{2r|s|}{\sigma^2}\right) dr \\
&= c_1 \frac{2^{\beta[n,k]} - 1}{a + (2^{\beta[n,k]} - 1)} \exp\left(-\frac{b}{a + (2^{\beta[n,k]} - 1)}\right)
\end{aligned}
\tag{3.27}
$$

where

$$
a = c_2 \sigma^2 \frac{E_s}{N_0}
\tag{3.28}
$$

and

$$
b = c_2 |s|^2 \frac{E_s}{N_0}
\tag{3.29}
$$

Here $I_0(.)$ is the zeroth-order modified Bessel function of the first kind. By taking the derivative of (3.27) with respect to $\beta[n,k]$, we have

$$
\begin{aligned}
\frac{d\bar{P}_e}{d\beta[n,k]} = \frac{c_1 \ln(2) . 2^{\beta[n,k]}}{[a + (2^{\beta[n,k]} - 1)]^2} &\left[a + \frac{b(2^{\beta[n,k]} - 1)}{a + (2^{\beta[n,k]} - 1)}\right] \\
&. \exp\left(-\frac{b}{a + (2^{\beta[n,k]} - 1)}\right) > 0 \qquad \texttt{for } \beta[n,k] > 0
\end{aligned}
$$

$$
\tag{3.30}
$$

Therefore, $\bar{P}_e[n,k]$ is a monotonically increasing function of $\beta[n,k]$, with $\bar{P}_e[n,k] = 0$ for $\beta[n,k] = 0$. This means that there is a unique solution for $\beta[n,k]$ when we use the proposed loading algorithm. Although it is not easy to find a closed-form solution for $\beta[n,k]$, a numerical approach can easily be used to find the solution. Brent's method [21] in annex; is used for approximately solving the non linear problem given by (3.27). For example, in our simulations, we use the function $fzero$ provided by Matlab to solve this nonlinear equation. The two different sources of imperfect CSI, noisy estimation and delay, will be considered separately in the following subsections.

B. Errors due to noisy channel estimation

Since the statistics of the channel estimation error is very complicated, and it highly depends on the estimation approach used and the system details, we choose to characterize the estimation error as additive Gaussian noise and hence we assume that $H[n,k] = H[n,k] + e[n,k]$, where $e[n,k]$ is complex Gaussian with zero mean and variance σ_e^2. The $e[n,k]$'s are assumed to be independent from each subchannel and $H'[n,k]$ are received channel impulse responses. The mean square error (MSE) is then given as:

$$MSE = E\left\{ \frac{1}{N} \sum_{k=1}^{N} (|H'[n,k] - H[n,k]|^2) \right\} \tag{3.31}$$

So the MSE here indicates how channel estimation error compares to the true channel energy.

Using the Gaussian error assumption, it can be shown that $H[n,k]$ given $H'[n,k]$ is complex Gaussian with mean

$$s_1 = \frac{1}{1+\sigma_e^2} H'[n,k] \tag{3.32}$$

and variance

$$\sigma_1^2 = \frac{\sigma_e^2}{1+\sigma_e^2} \tag{3.33}$$

Replacing s and σ^2 in (3.28) and (3.29) with s_1 and σ_1^2, respectively, yields the average BER $\bar{P}_e[n,k]$ for the case under consideration. Note that for ideal CSI ($\sigma_e = 0$), this $\bar{P}_e[n,k]$ reduces to (3.21), as expected.

C. Errors due to delay in CSI

Consider that we have perfect CSI but the information is feed-back to the transmitter with a certain delay. the new channel estimate will be given as:

$$H'[n,k] = H[n-\Delta n, K] \tag{3.34}$$

where $\tau_D = \Delta n T$ is the delay time between the channel estimation and the actual transmission. The correlation function of the frequency response at different times and for $\Delta k = 0$ is given by

$$E\left\{H[n + \Delta n, k + \Delta k]H^*[n, k]\right\} = r_t(\Delta n T) \sum_{\ell=1}^{L} q_\ell^2 \qquad (3.35)$$

In practical systems, if we can estimate the maximum Doppler frequency, then this correlation coefficient can be calculated.

According to (3.20) and (3.35), the correlation coefficient between $H[n, k]$ and $H'[n, k]$ is $\rho = r_t(\tau_D) \sum_{\ell=1}^{L} q_\ell^2 = J_0(2\pi f_D \tau_D) \sum_{\ell=1}^{L} q_\ell^2$. Considering the fact that both $H[n, k]$ and $H'[n, k]$ are complex Gaussian with zero mean and unit variance, $H[n, k]$ given $H'[n, k]$ is complex Gaussian with mean

$$s_2 = \rho H'[n, k] \qquad (3.36)$$

and variance

$$\sigma_2^2 = 1 - \rho^2 \qquad (3.37)$$

The average BER $\bar{P}_e[n, k]$ is given by (3.15) with $s = s_2$ and $\sigma^2 = \sigma_2^2$. This $\bar{P}_e[n, k]$ reduces to (3.8) when $f_D \tau_D = 0$ (ideal CSI).

3.9 Mathematical Characterization of systems

Despite the interest of the theoretical study of the capacity limits in multicarrier systems, the system designer has different constraints which turn the capacity limits into an unreachable objective in a practical approach. Rather, one of the primary concerns of communication systems designers is the definition of a practical objective function to measure the global performance of the system. In the following some of the most widely utilized performance measures are reviewed.

- **MSE** which refers to the expected value of the squared error committed by an estimator of a random scalar quantity is given by formula (3.31). Obviously, when the MSE refers to the error committed at the receiver when estimating the information that is sent from the transmitter, it is desired that the MSE be as small as possible, since it means that the estimation matches closely the desired information. Hence, any reasonable system has to be designed to have a low MSE.

- **SNR:** As it is intuitive from its definition, the higher the SNR the better the system. Hence, any reasonable system has to be designed to have a SNR as high as possible. The definition of SNR can be further generalized to include the effects of interferences in the undesired signal.

- **Symbol and bit error rates:** The symbol error rate (SER) of a communication system is empirically defined as the quotient between the number of symbols received in error and the total number of received symbols. Similarly, the BER is defined as the fraction of bits in error.

3.10   Conclusion

As discussed in above chapters, for wireless communications, the channel is time-varying and, in that case, channel capacity admits multiple definitions, depending on the degree of knowledge about the channel state that is available at the transmit and receive ends. Moreover, capacity can be measured by averaging over all possible channel states or by maintaining a fixed minimum rate. If the channel varies continuously the capacity is usually referred to as ergodic capacity, which is the natural extension of the Shannon capacity for time-varying channels as shown above. However, if the unknown channel variations are slow compared to the symbol length, such that the channel can be considered fixed but unknown during the transmission, then the outage capacity formulation is more relevant.

In mobile environments, the fading coefficients can change quite rapidly and the estimation of channel parameters becomes much more difficult. Therefore it is very important to know the channel capacity with respect to different degrees of knowledge of the CSI at both the transmitter and the receiver so as to better apply the relevant techniques so as to optimize the transmission with minimum errors. As seen in **Case 1**, the kind of communication that can be established is usually referred to as coherent communication. In this particular case of communication, the channel capacity depends on the CSI that is made available at the transmitter side. For the case where the transmitter has perfect CSI, it is also well known that the capacity achieving transmit strategy consists in splitting the signal among a set of beam-formers giving each one a fraction of transmission power according to a **water-filling algorithm** as discussed above. But it is very optimistic to say that we have perfect knowledge of CSI and it is very pessimistic to say that we have completely no knowledge of CSI that is why it is desirable to consider the real practical example where the partial knowledge of CSI (Imperfect CSI) is available i.e some channel parameters are known and some are not and we will discuss the optimal power allocation with this case because it plays a large part in the designing of equalizers for the frequency selective channels.

Chapter 4

Simulations, results and discussions

In this chapter, we present and we discuss the results of achieved simulations . The objective here is to analyse the performance of the new loading algorithm and the system spectral efficiency subject to a bit error rate (BER) constraint for each subchannel with partial CSI knowledge.

The proposed adaptive modulation algorithm for the OFDM system has been implemented and the results are validated using Matlab. An OFDM system of $N = 1024$ subcarriers modulated in MQAM is considered. The bandwidth is $B = 10MHz$, then the subcarrier frequency spacing is 9.765 KHz. The different parameters are summarized in table 4.1. The simulation results are averaged over 1000 channel realizations. Several scenarios are analyzed :

- Using an optimal power allocation across all subchannels with or without BER constraint.

- Using a uniform power allocation across all subchannels with or without BER constraint.

OFDM system	
FFT size	$N = 1024$
Bandwidth	$B = 10$ Mhz
Subcarrier spacing	$\Delta f = B/N = 9.765$ kHz
Frequency carrier	$f_c = 2$ GHz
The sampling interval	$T_s = 1/B = 100$ ns
OFDM block length	$T = 1/\Delta f = N/B = 102.4$ μs
Time-varying fading channel	Jakes' model
Power delay profile	$[0, -3, -6, -8, -17.6]$ (in dB)
Delay	$[0,\ 10,\ 20,\ 30,\ 100]$ (in samples)
Maximum delay spreads	10μs
The spaced-time correlation function	$r_t\left(\Delta t\right) = J_0\left(2\pi f_D \tau_D\right)$
Doppler spread $f_D \tau_D$	$[0,\ 0.02,\ 0.03,\ 0.05,\ 0.08,\ 0.1]$

Table 4.1: OFDM system parameters

- Using a uniform power allocation across all subchannels with BER constraint and a perfect CSI knowledge.

- Using a uniform power allocation across all subchannels with BER constraint and a partial CSI knowledge due to noisy channel.

- Using a uniform power allocation across all subchannels with BER constraint and a partial CSI knowledge due to delay in CSI.

The channel is considered to be Rayleigh fading channel (where the number of channel paths is $L = 4$) and the noise is assumed to be AWGN. The power delay profile (PDP) used for our simulation is similar to EVA Delay Profile of an Long Term Evolution (LTE) system and it is given in Table 4.2. The excess tap delay is given in samples and its relative power in dB.

Relative power in dB (in dB)	Excess tap delay (in samples)
0	0
-3	10
-6	20
-8	30
-17.2	100

Table 4.2: Multipath Fading channel parameters

4.1   Optimal and uniform power allocation :

Our simulations begin by an analyze of the performance in average spectral efficiency by using an optimal or uniform power allocation over a time-variant selective frequency channel. The first two curves in figure 4.1 are obtained with no BER constraint and it is a comparison between the average spectral efficiency vs SNR obtained with uniform power allocation and the one with optimal power allocation using (3.13), (3.14) and (3.15).

We observe a better performance with optimal power allocation but mainly for low SNR values (< 10 dB). When the SNR ≥ 10 the average spectral efficiency is the same for either optimal or uniform power allocation. The other pairwise of curves are obtained with supplementary constraint for (BER= 10^{-3}, 10^{-5} or 10^{-7}). We observe that more is the BER requirement, less is the the average spectral efficiency.

The figure 4.2, shows the loading bits per subchannel obtained by uniform power allocation at SNR=10 dB and BER $=10^-5$.
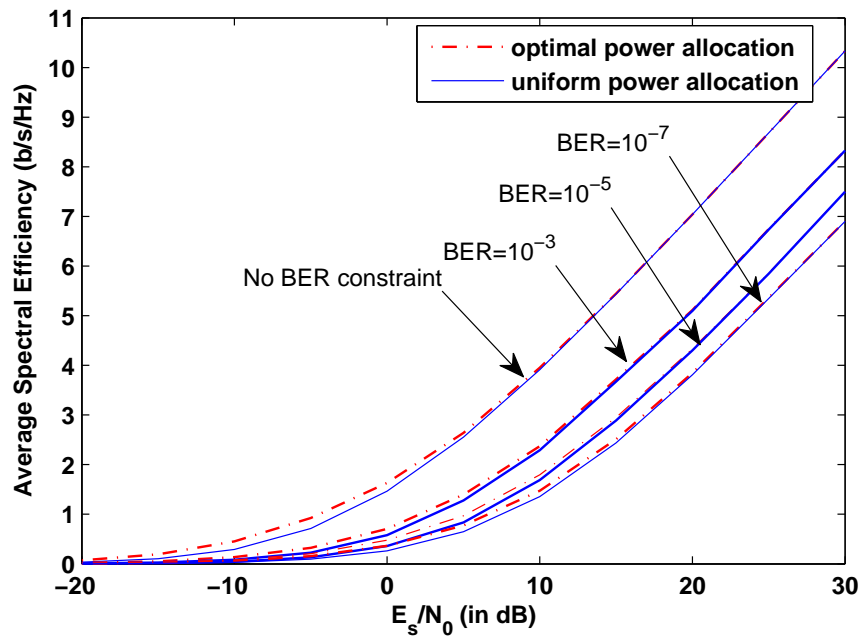
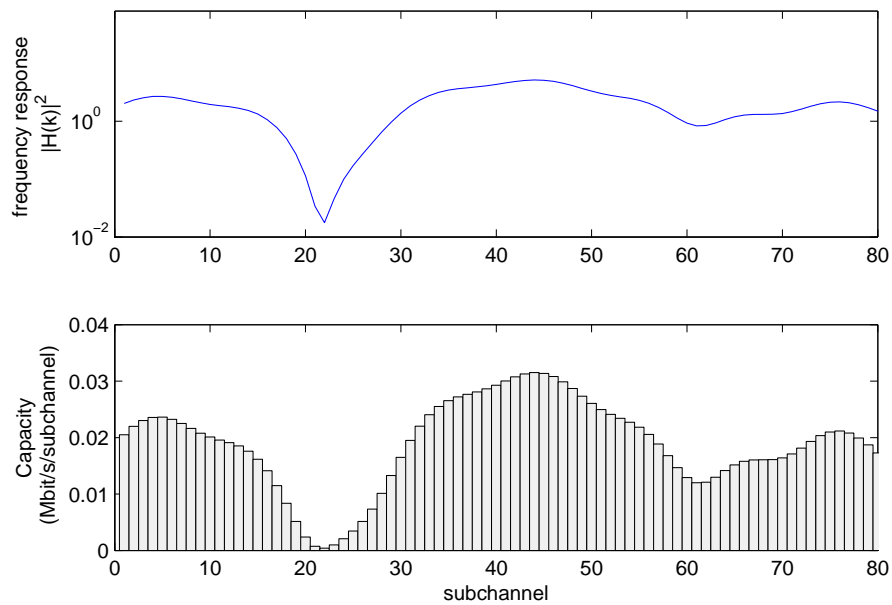Figure 4.1: Uniform and Optimal power allocation with BER constraint.

Figure 4.2: Bit loading with SNR= 10dB, BER $=10^-5$ and uniform power allocation and corresponding frequency channel response.

4.2 Non-adaptive Modulation

When no CSI knowledge is available, the same modulation size is used for all the sub-channels k and each instant n. For this case, $\beta(n, k)$ is constant for all values of n and k, the error probability is targeted (fixed), $H[n, k]$ is complex Gaussian random variable and it has uniform distribution over all sub-channels, therefore the resulted formulas are as shown below:

$$\bar{P}_e = E_{H[n,k]} P_e[n, k] \approx \frac{c_1}{\frac{c_2 E_s/N_0}{2^\beta - 1} + 1} \tag{4.1}$$
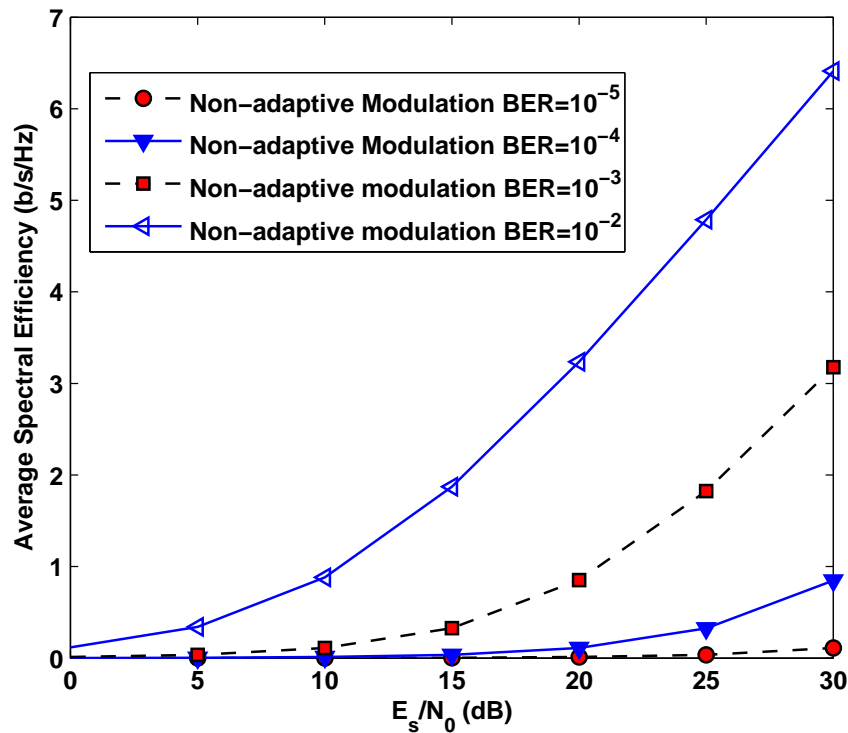

Figure 4.3: The average spectral efficiency vs SNR for non adaptive modulation with different desired BER.

Assume P_{tar} is the target average BER, then, by inverting (3.21) with $\bar{P}_e = P_{tar}$, the

maximum number of bits that can be transmitted given the average BER constraint is

$$\beta = \log_2 \left[\frac{c_2 \frac{E_s}{N_0}}{\frac{c_1}{P_{tar}} - 1} + 1 \right] \tag{4.2}$$

whereas the spectral efficiency is equal to β, under the assumption that the symbol interval is the reciprocal of the subchannel bandwidth. Its graphs under a targeted error probability of $10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$ are shown in figure 4.3.

4.3 Noise channel errors

OFDMA technologies are attractive because they achieve higher spectral efficiency with lower overall complexity, especially in larger bandwidths. In the following, some simulation results are provided in order to give insight into the benefits of the proposed techniques for the power allocation. Using the proposed loading algorithm, the average spectral efficiency curves for different cases obtained from simulations are shown in figures 4.4 and 4.5.
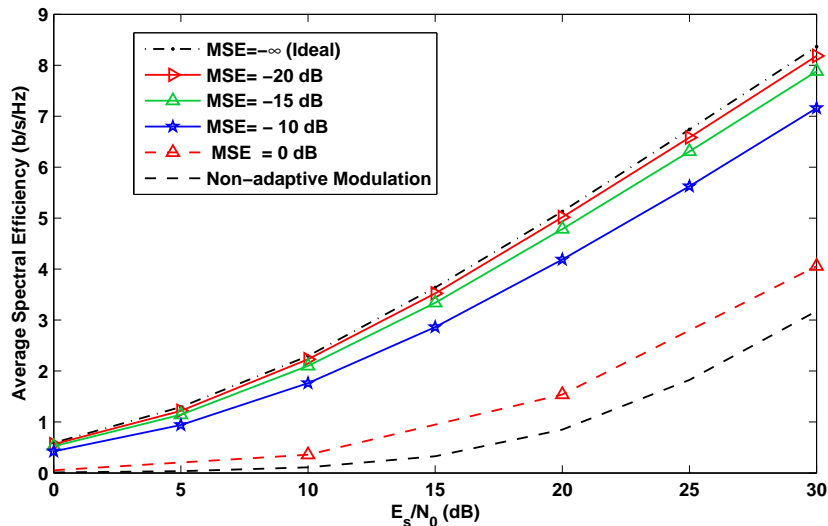

Figure 4.4: Average spectral efficiency for adaptive OFDM with Gaussian noise in CSI for $P_{tar} = 10^-3$

The noisy channel due to channel estimation errors, increase the BER. Intuitively, to maintain the BER requirement for each subchannel, the used loading algorithm tends to underload each subchannel by taking in consideration the imperfections in channel estimation. Otherwise the BER will be greater than expected.

- Ideal case:

In practice, the channel varies, so the modulation parameters need to vary correspondingly. Therefore, we have to adapt the modulation parameters, and that is why we need the adaptive modulation algorithm which indicates how to adapt the parameters with respect to channel parameters. For ideal OFDM adaptive transmissions, if we assume that each subcarrier can get an independent adaptive mode according to the CSI of the corresponding subcarrier then it would be possible to reach the highest spectral efficiency as shown in figure 4.4. Here, the perfect knowledge of the receiver channel information is assumed to be available at the transmitter, but this is not the case in practical. This technique is meant to change the modulation parameters dynamically according to the instantaneous channel situation in order to satisfy the system transmission requirements and to get maximum throughput.

- non adaptive modulation:

Non-adaptive modulation systems use fixed modulation parameters to get a certain throughput and to guarantee transmission quality at the same time. Obviously this strategy is not optimal since the channel varies most of the time. When the channel is very good, spectrum and power will be wasted as parameters for a worse situation are used; and when it is very bad, maybe the compromised parameters could not guarantee the transmission quality.

The average spectral efficiency for adaptive OFDM with Gaussian noise in CSI, having

the same channel parameters as as the previous simulation but the targeted BER is now fixed to 10^{-5} is shown by the figure 4.5 below
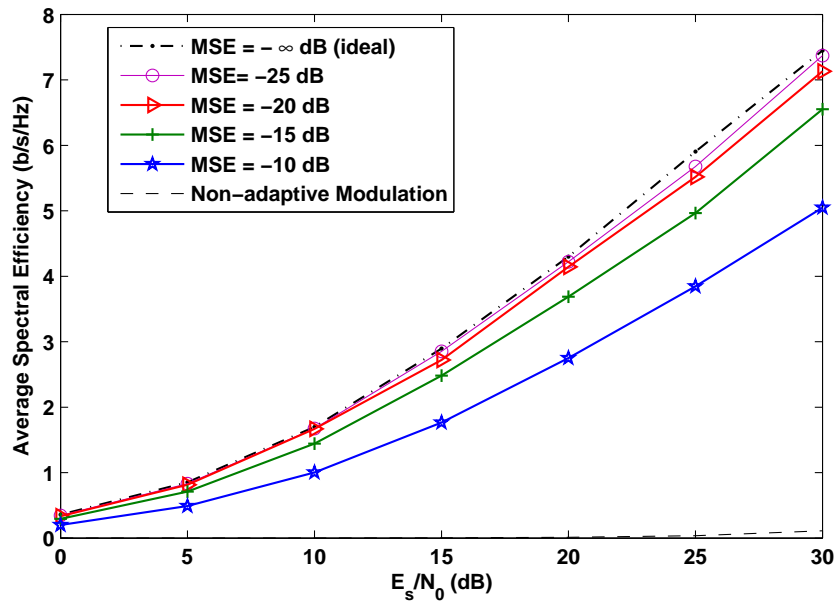

Figure 4.5: Average spectral efficiency for adaptive OFDM with Gaussian noise in CSI for $P_{tar} = 10^{-5}$

When compared with other modulation techniques (as shown in Fig 4.4), this technique has the lowest spectral efficiency under the same fixed P_{tar} this is because we have to assume the number of bits per second per Hertz is constant for all values of n and k. When we change P_{tar} to 10^{-5} we notice that the performance in spectral efficiency varies with P_{tar} i.e. decreasing the P_{tar} will result in small reduction in spectral efficiency as shown in Figure 4.5. The two reference points show that even tough the ideal modulation scheme is impossible to implement in practice, it is still important to know it since it gives us the highest bound through which we can have better throughput if the channel conditions are favorable. Therefore a higher rate is chosen only if the BER is lower than the specified target for the given channel realization.

4.3.1 Delay in CSI

The general effect of the transmitter having delayed channel information is a mismatch between the actual channel characteristics and the modulation schemes used. Figure 4.6 shows how different delays can influence the channel spectral efficiency.

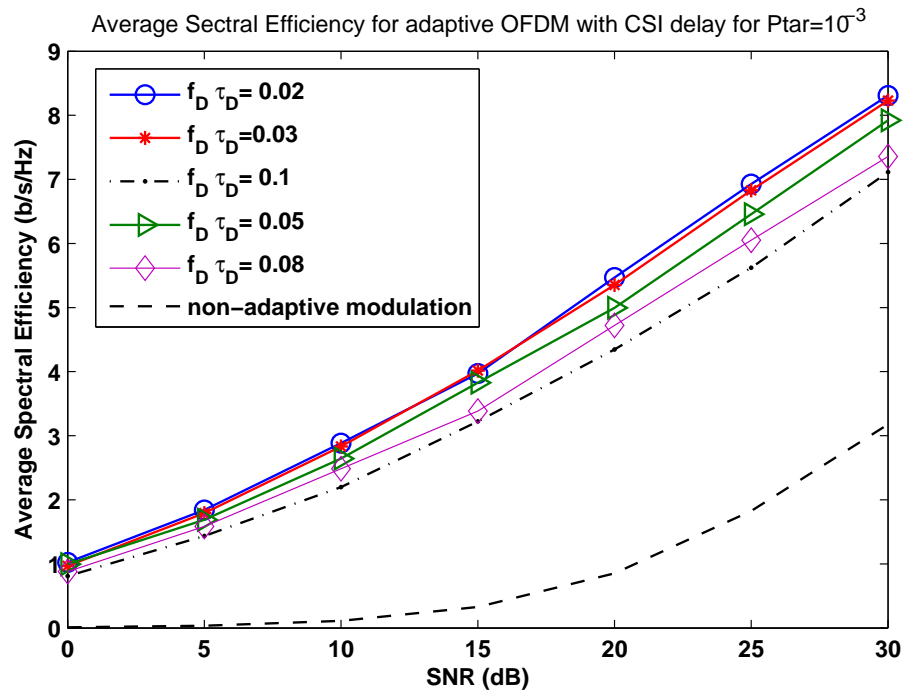Figure 4.6: Average spectral efficiency for adaptive OFDM with Delay in CSI for $P_{tar} = 10^-3$

Assuming that we have different delays between the transmission time and the actual CSI estimation time, figure 4.6 represents the effects of these delays in the form of the channel spectral efficiency. The scenarios are observed and their description is as follows:

- For ideal adaptation (zero delay):

We consider this as an ideal case,note that the system does not depend on channel parameters and hence these parameters are available at the transmitting end without any delay.From the figure above, it is shown that if there are no outdated CSI at the transmitter i.e $f_D \tau_D = 0$ we can achieve maximum spectral efficiency as shown by the ideal curve. This is again the reference through which we can model different transmission schemes after knowing the highest throughput.

- Non adaptive:

in this case, the transmitter has to wait for maximum delay so as to allow the transmission of all the information, this is done in order to avoid ISI, in consequence, If the average delay between the information transmission and the feeding up of CSI at the transmitter is not well known it would be hard to maximize the spectral efficiency. This is why we have to know the delay spread variation with respect to the speed of the mobile station.

- Delay Variation:

In the simulations, after changing to different delays, as shown if fig 4.6 we observe that, when the delay spread is minimum we obtain better spectral performances and this can help us in adapting the OFDM modulation schemes in relation to the movement of the mobile station.

Remarque: When taking into consideration the channel capacity, we don't only have to take SNR into account, we have to consider the error probability too for each subchannel. This is because for adaptive OFDM systems each subchannel has its own channel gain, so taking into consideration the error probability allow the systems to
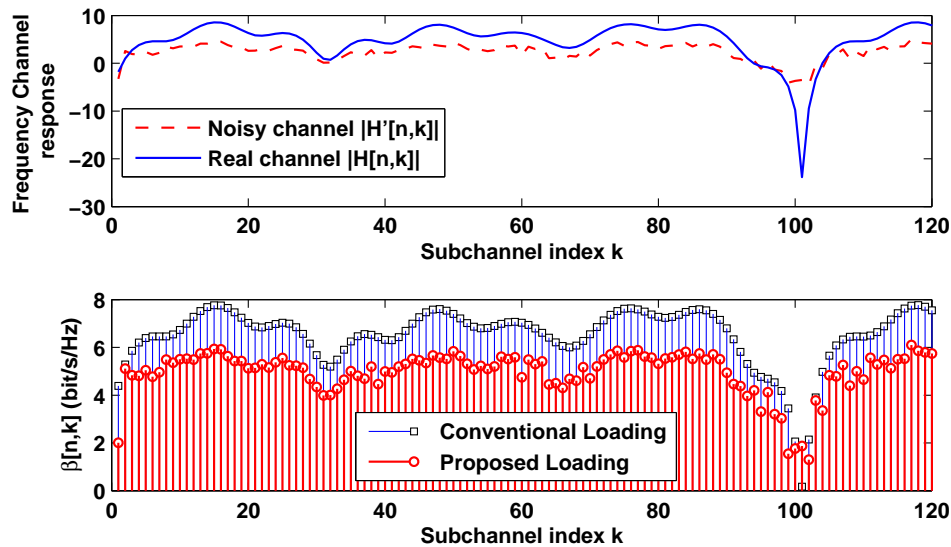
transmit information with minimum error.

Figure 4.7: Comparisons between the estimated channel and the results of the bit allocation

From the snapshot taken using $P_{tar} = 10^-3$, $MSE = -10dB$ and $SNR = 20dB$, we can remark that, at points where the channel gains are maximum the bit allocation is of maximum value too and where there are least gains the bits allocation is of low value as well.

4.3.2   Conclusion:

Taking into consideration the figure 4.7 we can conclude that knowing the channel impulse response variation will lead us to better bit allocation of every sub-channel. This is because the channel variations give us the idea on how to effectively use the bit allocation algorithm with respect to the channel gains. Furthermore, the conventional bit loading (the ideal case) using equation 3.22 gives us the maximum bits per second

per Hertz that we can transmit to every sub-channel and it is of higher spectral efficiency as compared to the proposed loading. However this case is practically impossible but it shows us the limit above which we can not transmit without bigger error probability We also notice that, for partial CSI knowledge, the uniform power distribution adapts well the modulation techniques. We therefore have to apply a modulation scheme that provides minimum error probability to every sub-channel. Furthermore, in this chapter, the simulation results show us that it is necessary to know how the CSI noise and delay errors and the different channel parameters can influence the estimation of channel. We have also seen how the capacity can be adapted while taking into consideration the error probability for every sub-channel. It is therefore necessary to take into consideration all these factors because they have direct impact in the designing on different telecommunication equipments.

General conclusion

As we discussed in chapter one, different phenomena such as coding, different modulation techniques, pathloss and delay spread are some of the factors that characterize wireless channel. The frequency selective-fading channel is the most complex channel especially due to multipath propagation, we proposed the technique that is used to combat it. Until now, the best known technique is OFDM and it was discussed in detail.

In second chapter, we discussed OFDM as a technique to combat multipath fading. OFDM history, basic principle, implementation, advantages and disadvantages were discussed. We also discussed that guard interval/cyclic prefix insertion is the appropriate solution to combat ISI and we saw the orthogonality principle with an adequate spacing that helps in avoiding ICI. However PARP is the problem associated with OFDM systems and the use of linear amplifiers is the best fit to the mitigation of this problem

In chapter three, we discussed that the knowledge of different factors such as the influence of channel capacity over the error probability of the communication link is very important. This is because for frequency selective channel, different changes over channel gains cause significant distortions on the transmitted signal, Hence in order to load the bits efficiently knowledge of the channel variations knowledge are considerably important. Furthermore, we discussed different degrees of CSI knowledge over the channel, power constraints and error probability and hence we saw how these factors influence the channel spectral efficiency. As we have seen from the previous discussion,

efficient bit allocation for OFDM systems is necessary for all the telecommunication systems. The actual, relevant and updated CSI is important for simulations of the channel parameters variations and hence better characterization of variations with better modulation techniques are therefore possible in practical applications.

In last chapter we discussed the simulation results that took into consideration the power budget and error probability constraints and these results were proving the theory discussed above. Moreover, we remarked that the knowledge of channel gives us the maximum upper bound and this bound limits our modulation and power allocation so as to assure minimum error probabilities.

Annex

Brent's method

Brent's method for approximately solving $f(x) = 0$, where $f : R \to R$, is a hybrid method that combines aspects of the bisection and secant methods with some additional features that make it completely robust and usually very efficient. Like bisection, it is an enclosure method that begins with an initial interval across which f changes sign and, as the iterations proceed,determines a sequence of nested intervals that share this property and decrease in length. Convergence of the iterates is guaranteed, even in floating-point arithmetic.

If f is continuous on the initial interval, then each of the decreasing intervals determined by the method contains a solution, and the limit of the iterates is a solution. Like the bisection and secant methods, the method requires only one evaluation of f at each iteration; in particular, f' is not required.

The following provides a rough outline of how the method works. The method builds upon an earlier method of T. J. Dekker and is the basis of MATLAB's $fzero$ routine.[21] At each iteration, Brent's method first tries a step of the secant method or something better. If this step is unsatisfactory, which usually means too long, too short, or too close to an endpoint of the current interval, then the step reverts to a bisection step. There is also a feature that occasionally forces a bisection step to guard against too little

progress for too many iterations. In the details of the method, a great deal of attention has been paid to considerations of floating-point arithmetic (overflow and underflow, accuracy of computed expressions, etc.).

An overview of the operation of the method is as follows:

- The method begins with

    - a stopping tolerance $\delta > 0$.

    - points a and b such that $f(a).f(b) < 0$.

    If necessary, a and b are exchanged so that $|f(b)| \leq |f(a)|$, thus b is regarded as the better approximate solution. A third point c is initialized by setting $c = a$.

- At each iteration, the method maintains a, b, and c such that $b \neq c$ and
    (i) $f(b).f(c) < 0$, so that a solution lies between b and c if f is continuous.
    (ii) $|f(b)| \leq |f(c)|$, so that b can be regarded as the current approximate solution.
    (iii) either a is distinct from b and c, or $a = c$ and is the immediate past value of b.

Each iteration proceeds as follows:

1. If $|b - c| \leq \delta$, then the method returns b as the approximate solution.

2. Otherwise, the method determines a trial point $\hat{b}$ as follows:[21]
    (i) If $a = c$, then $\hat{b}$ is determined by linear (secant) interpolation: $\hat{b} = \frac{af(b) - bf(a)}{f(b) - f(a)}$

    (ii) Otherwise a, b, and c are distinct, and $\hat{b}$ is determined using inverse quadratic interpolation:

- Determine α, β, and γ such that $p(y) = \alpha y^2 + \beta y + \gamma$ satisfies $p(f(a)) = a$, $p(f(b)) = b$, and $p(f(c)) = c$.

- Set $\hat{b} = \gamma$.

3. If necessary, $\hat{b}$ is adjusted or replaced with the bisection point.

4. Once $\hat{b}$ has been finalized, a, b, c, and $\hat{b}$ are used to determine new values of a, b, and c.

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