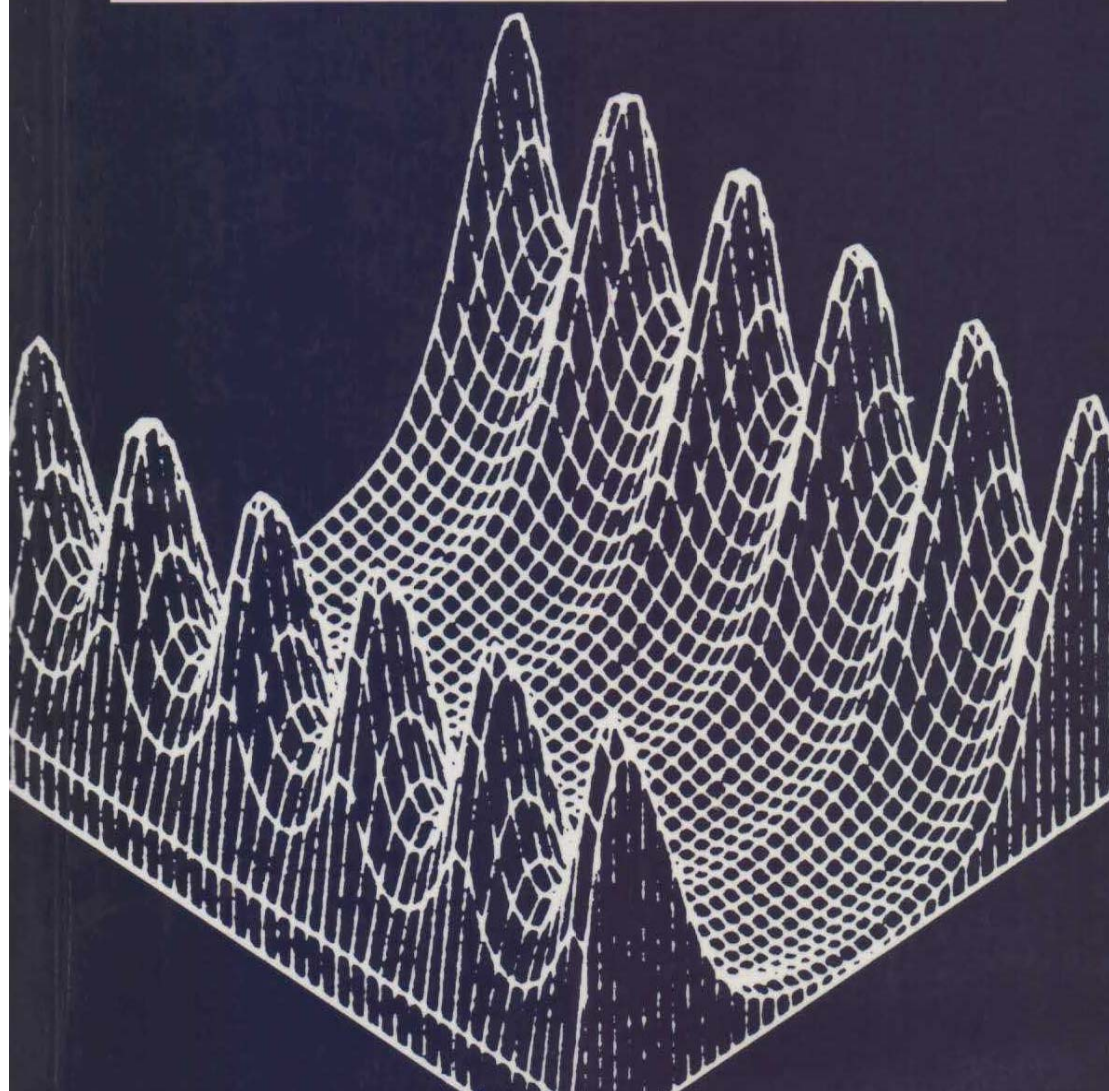


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CHEMICAL
OSCILLATIONS
AND INSTABILITIES
NON-LINEAR CHEMICAL KINETICS

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