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Vibration of Clamped Circular Symmetric Laminates

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Treated in this paper is the free-flexural vibration analysis of symmetrically laminated thin circular plates. The total energy functional for the laminated plates is formulated where the pb-*2 Ritz method is applied for the solution. The assumed displacement is defined as the product of (1) a two-dimensional complete polynomial function and (2) a basic boundary function. The simplicity and accuracy of the numerical procedure will be demonstrated by solving some plate examples. In the present study, the effects of material properties, number of layers and fiber stacking sequences upon the vibration frequency parameters are investigated. Selected mode shapes by means of contour plots for several 16-ply laminated plates with different fiber stacking sequences and composite materials are presented. This study may provide valuable information for researchers and engineers in design applications. In addition, the present solution plays an important role in increasing the existing data base for future references.*

1 Introduction

Laminated composites are important structural materials in weight sensitive aerospace applications where high strength-to-weight and stiffness-to-weight ratios are desired. These laminates are built up of plies having different orientations of fibers in which each individual lamina exhibits specially orthotropic properties with respect to the axes parallel and perpendicular to the fibers. They are often fabricated in a symmetric pattern with respect to the mid-plane so to avoid the bending-stretching coupling effects. From this symmetric arrangement, the resulting laminate exhibits bending-twisting coupling, thus the anisotropy must be included in analysis. However, the anisotropy has often been neglected in analysis so that the modelling process can be much simplified (Chamis, 1969; Nemeth, 1986). If this anisotropy has to be included in the analysis, the governing differential equation involves the odd derivatives of the lateral deflection which do not allow the harmonics to uncouple. Analytical solutions to this problem are, therefore, out of the question. Under these circumstances, approximate numerical methods must be employed for such analyses (Sivakumaran, 1987; Leissa and Narita, 1989; Narita, 1990).

Recently, Liew and his coworkers (Liew et al., 1992) have developed a numerical model using the so-called "*pb*-2 Ritz method" for predicting the deflections, bending moments, buckling loads, and vibration frequencies of plates. The method has been shown to be computationally efficient and numerically accurate for the analyses of plates having complex geometry and arbitrary support conditions. A set of *pb*-2 shape functions consists of the product of (*1*) a two-dimensional complete set of polynomial function and (*2*) a basic function defined by the product of equations of piecewise boundaries each raised to a power of 0, 1 or 2 corresponding to a free, simply supported or clamped edge, is used in the Ritz approach to formulate the governing eigenvalue equation.

In this paper, an extension of the *pb*-2 Ritz method to the study of free-flexural vibration of symmetrically laminated circular plates exhibiting bending-twisting coupling is attempted. The bending rigidities through the total thickness are obtained by superposing the contribution of each ply. The aim of the present work is to provide a set of accurate and reasonably comprehensive first known vibration frequencies for the symmetric angle-ply laminated circular composite plates for which no exact solutions are possible. This has been achieved by using sufficient numbers of displacement function terms to accurately approximate the mode shapes. The effects of different stacking sequences, fiber orientation angles, and material properties on the vibration behavior are investigated. Moreover mode shapes for several representative symmetrically laminated composite circular plates of clamped periphery are presented by means of contour plots.

2 Method of Solution

2.1 Problem Definition. Consider a thin, fiber reinforced composite laminated circular plate lying in the *xy*-plane of radius $r = a/2$, as shown in Fig. 1. The plate with thickness h in the z-direction consists of n-layer of orthotropic plies perfectly bounded together by a matrix material. The reference plane $z = 0$ is considered to be located at the undeformed middle plane as shown in Fig. 2. The fiber direction within a layer is indicated by angle $\beta°$. The moduli of elasticity for layers parallel to the fibers is E_1 and perpendicular to the fibers is E_2. The problem is to determine the vibration frequencies and mode shapes of the plate. In the present study, the layers are so arranged that a mid-plane symmetry exists.

2.2 Formulation of Energy Functional. The strain energy for a plate due to bending can be expressed as

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