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Natural Vibrations of a Clamped-Clamped Arch With an Open Transverse Crack

The paper presents a finite element model of the arch with a transverse, one-edge crack. A part of the cracked arch is modelled by a curved beam finite element with the crack. Parts of the arch without the crack are modelled by noncracked curved beam finite elements. The crack occurring in the arch is nonpropagating and open. It is assumed that the crack changes only the stiffness of the arch, whereas the mass is unchanged. The method of the formation of the stiffness matrix of a curved beam finite element with the crack is presented. The effects of the crack location and its length on the changes of the in-plane natural frequencies and mode shapes of the clamped-clamped arch are studied.

1 Introduction

Fatigue cracking is undoubtedly the most important failure mode in engineering (Williams and Ellinger, 1953; Bishop, 1955; Bohnstedt and Leopold, 1985). From the point of view of optimum machine performance it is of great importance to detect cracks in the initial stages of growth. Since the crack influences the stiffness of the structure, and the stiffness in turn influences the dynamic behavior of such a system, vibration monitoring as a means of detecting crack initiation and growth should be a powerful tool (Cawley and Adams, 1979; Stubbs, 1985; Rizos et al., 1990). A detailed study of the vibrational behavior of cracked structures, therefore, is necessary.

To the best of the author's knowledge, the dynamic behavior of cracked arches has not been analyzed in the published literature. The review of the dynamics of cracked structures (Wauer, 1990) contains several papers in which the authors of which analyzed the influence of cracks on buckling and natural frequencies of rings (Dimarogonas, 1981; Yao and Dimarogonas, 1988; Hong et al., 1989). An essential idea, common to cited papers is to replace the crack by a spring with reduced stiffness and then to divide the analyzed structure into two undamaged parts. The reduced stiffness can be calculated by means of methods of fracture mechanics (Okamura et al., 1972). Next, the reduced stiffness quantity is incorporated into the equations of motion, and in this way the static and dynamic parameters of the cracked rings can be analyzed. The above method is restricted to rings with a constant cross-section.

The main objective of this paper is to use the curved beam finite element with a transverse, one-edged, nonpropagating, open crack and to present an analysis of the effects of the crack position, and of its location, on the changes of the in-plane natural frequencies and mode shapes of the clamped-clamped arch. It is assumed that the crack only changes the stiffness of the element, whereas the mass of the element remaining unchanged. The elaborated model of the cracked element is restricted to curved beams with rectangular cross-section. The validity of the obtained model is verified by numerical calculations.

2 The Stiffness Matrix of the Curved, Cracked Beam Finite Element

A curved cracked finite-beam element with two nodes and three degrees of freedom at the node is presented in Fig. 1. The transverse crack is open, nonpropagating and located in the middle of the element.

The stiffness matrix $\mathbb{K}_E$ of a finite element can be expressed by his flexibility matrix as (Przemieniecki, 1968)

$$\mathbb{K}_E = \mathbb{T}^t\mathbb{C}^{-1}\mathbb{T}, \tag{1}$$

where: $\mathbb{T}$ is the transformation matrix (see section 2.1), $\mathbb{C}^{-1}$ is the inversion of flexibility matrix of the element (see sections 2.2 and 2.3), upper index t denotes transposition of the matrix.

In the case of analyzed cracked finite element the flexibility matrix $\mathbb{C}$ is a sum of the flexibility matrix of the noncracked element $\mathbb{C}^0$ and the flexibility matrix due to the crack $\mathbb{C}^1$ (Gounaris and Dimarogonas, 1988). The elements of matrices $\mathbb{C}^0$ and $\mathbb{C}^1$ are calculated by using the well known relationships (Przemieniecki, 1968; Dimarogonas, 1983).

2.1 Matrix of Transformation $\mathbb{T}$. The elements of the matrix of transformation $\mathbb{T}$ can be calculated using the equations of overall equilibrium for element forces $F_1 - F_6$ and $S_1 - S_3$—Fig. 2. The final form of the matrix $\mathbb{T}$ is

$$\mathbb{T}^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\cos\beta & \sin\beta & 0 \\ -\sin\beta & -\cos\beta & 0 \\ -r(1-\cos\beta) & -r\sin\beta & -1 \end{bmatrix}, \tag{2}$$

2.2 The Flexibility Matrix of the Noncracked Element $\mathbb{C}^0$. The terms of the flexibility matrix of the noncracked element can be calculated by using the following relationship (Przemieniecki, 1968)

$$c^0_{ij} = \frac{\partial^2 U^0}{\partial S_i \partial S_j}, \quad (i = 1, 3;\ j = 1, 3) \tag{3}$$

where U^0 is the elastic strain energy of the noncracked curved beam.

The elastic strain energy of the curved finite-beam element presented in Fig. 1 is (Orłoś and Jakubowicz, 1966)

$$U^0 = \left(\frac{r}{2EJ} + \frac{1}{2EA}\right)\int_0^\beta M_g^2 d\alpha + \frac{r}{2EA}\int_0^\beta N^2 d\alpha + \frac{1}{EA}\int_0^\beta NM_g d\alpha, \tag{4}$$

where M_g is the bending moment, N is the axial force, E is Young's

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