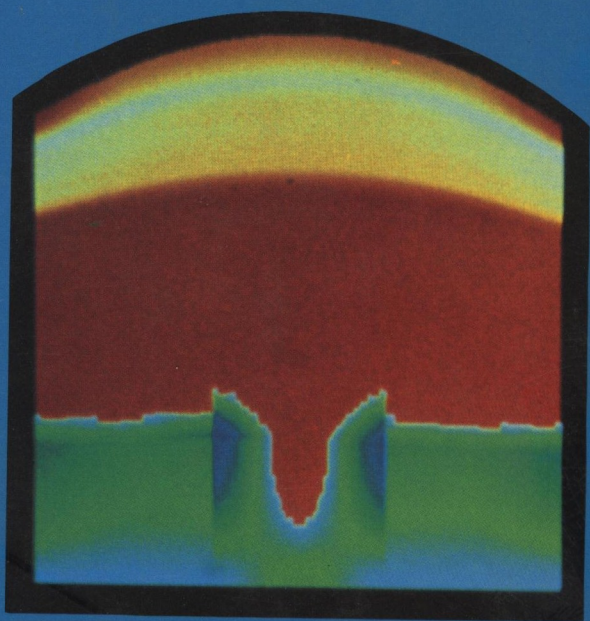


# Curves and Surfaces for Computer Aided Geometric Design

*A Practical Guide*

**Second Edition**



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