



Introduction to
**Ordinary
Differential
Equations**

Shepley L. Ross

second edition

Contents

1

Differential Equations and Their Solutions 1

- 1.1 Classification of Differential Equations; Their Origin and Application 1
- 1.2 Solutions 5
- 1.3 Initial-Value Problems, Boundary-Value Problems, and Existence of Solutions 13

2

First-Order Equations for Which Exact Solutions are Obtainable 23

- 2.1 Exact Differential Equations and Integrating Factors 23
- 2.2 Separable Equations and Equations Reducible to This Form 36
- 2.3 Linear Equations and Bernoulli Equations 45
- 2.4 Special Integrating Factors and Transformations 57

3

Applications of First-Order Equations 66

- 3.1 Orthogonal and Oblique Trajectories 66
- 3.2 Problems in Mechanics 73
- 3.3 Rate Problems 84

4

**Explicit Methods of Solving
Higher-Order Linear Differential Equations 92**

| | | |
|-----|---|-----|
| 4.1 | Basic Theory of Linear Differential Equations | 92 |
| 4.2 | The Homogeneous Linear Equation with Constant Coefficients | 110 |
| 4.3 | The Method of Undetermined Coefficients | 121 |
| 4.4 | Variation of Parameters | 132 |
| 4.5 | The Cauchy-Euler Equation | 141 |
| 4.6 | Statements and Proofs of Theorems on the Second-Order Homogeneous Linear Equation | 146 |

5

**Applications of
Second-Order Linear Differential Equations
with Constant Coefficients 155**

| | | |
|-----|---|-----|
| 5.1 | The Differential Equation of the Vibrations of a Mass on a Spring | 155 |
| 5.2 | Free, Undamped Motion | 158 |
| 5.3 | Free, Damped Motion | 165 |
| 5.4 | Forced Motion | 176 |
| 5.5 | Resonance Phenomena | 183 |
| 5.6 | Electric Circuit Problems | 188 |

6

**Series Solutions of
Linear Differential Equations 199**

| | | |
|-----|--|-----|
| 6.1 | Power Series Solutions About an Ordinary Point | 199 |
| 6.2 | Solutions About Singular Points; the Method of Frobenius | 211 |
| 6.3 | Bessel's Equation and Bessel Functions | 231 |

7

**Systems of
Linear Differential Equations 243**

| | | |
|-----|---|-----|
| 7.1 | Differential Operators and an Operator Method | 243 |
| 7.2 | Applications | 257 |

| | | |
|-----|---|-----|
| 7.3 | Basic Theory of Linear Systems in Normal Form: Two Equations in Two Unknown Functions | 266 |
| 7.4 | Homogeneous Linear Systems with Constant Coefficients: Two Equations in Two Unknown Functions | 275 |
| 7.5 | Matrices and Vectors | 286 |
| 7.6 | Basic Theory of Linear Systems in Normal Form: n Equations in n Unknown Functions | 306 |
| 7.7 | Homogeneous Linear Systems with Constant Coefficients: n Equations in n Unknown Functions | 325 |

8

Approximate Methods of Solving First-Order Equations 338

| | | |
|-----|---|-----|
| 8.1 | Graphical Methods | 338 |
| 8.2 | Power Series Methods | 345 |
| 8.3 | The Method of Successive Approximations | 351 |
| 8.4 | Numerical Methods | 355 |

9

The Laplace Transform 373

| | | |
|-----|--|-----|
| 9.1 | Definition, Existence, and Basic Properties of the Laplace Transform | 373 |
| 9.2 | The Inverse Transform and the Convolution | 386 |
| 9.3 | Laplace Transform Solution of Linear Differential Equations with Constant Coefficients | 393 |
| 9.4 | Laplace Transform Solution of Linear Systems | 405 |

Appendix 411

Answers 413

Index 429