



Statistical Analysis in Climate Research

Hans von Storch and Francis W. Zwiers

CAMBRIDGE

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Climatology is, to a large degree, the study of the statistics of our climate. The powerful tools of mathematical statistics therefore find wide application in climatological research.

The purpose of this book is to help the climatologist understand the basic precepts of the statistician's art and to provide some of the background needed to apply statistical methodology correctly and usefully. The book is self contained: introductory material, standard advanced techniques, and the specialised techniques used specifically by climatologists are all contained within this one source. There are a wealth of real-world examples drawn from the climate literature to demonstrate the need, power and pitfalls of statistical analysis in climate research.

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Computers and Geosciences

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