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Influence of the number of vehicles on the dynamic bridge-convoy behavior

Promoter:

Mr. MEGHATRIA M’hamed

Directed by:

HADJ MEHDI ABDERRAHIM

EZZRAIMI SOFIANE

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Abstract:

The dynamic loads acting on bridges result in the induction of deformations in their structure. These deformations, should they surpass the allowable threshold, could lead to significant structural deterioration or even complete collapse. It is imperative to accurately assess these deformations. Our research is focused on this task, aiming to calculate the deflection in the flooring of a bridge under isotropic and orthotropic conditions. The formulation of mathematical expressions that depict the behavior of anisotropic beams under dynamic loads, specifically a moving mass, has been undertaken. MATLAB scripts have been created based on these equations, enabling the determination of deflection at various locations on the two types of flooring being analyzed.

Résumé :

Les charges dynamiques agissant sur les ponts entraînent l'induction de déformations dans leur structure. Ces déformations, si elles dépassaient le seuil admissible, pourraient entraîner une détérioration structurelle importante ou même un effondrement complet. Il est impératif d'évaluer précisément ces déformations. Notre recherche se concentre sur cette tâche, visant à calculer la déviation dans le plancher d'un pont dans des conditions isotropes et orthotropes. La formulation d'expressions mathématiques décrivant le comportement des faisceaux anisotropes sous des charges dynamiques, en particulier une masse en mouvement, a été entreprise. Des scripts MATLAB ont été créés sur la base de ces équations, permettant la détermination de la déflexion à divers endroits sur les deux types de revêtements de sol analysés.

ملخص:

تؤدي الأحمال الديناميكية التي تعمل على الجسور إلى تحريض التشوهات في هيكلها. هذه التشوهات ، إذا تجاوزت العتبة المسموح بها ، يمكن أن تؤدي إلى تدهور هيكلي كبير أو حتى انهيار كامل. من الضروري تقييم هذه التشوهات بدقة. يركز بحثنا على هذه المهمة بهدف حساب الانحراف في أرضية الجسر في ظل ظروف الخواص والتقويم. تم صياغة التعبيرات الرياضية التي تصور سلوك الحزم متباينة الخواص تحت الأحمال الديناميكية ، وتحديدا الكتلة المتحركة. تم إنشاء نصوص ماتلاب بناء على هذه المعادلات ، مما يتيح تحديد الانحراف في مواقع مختلفة على نوعين من الأرضيات التي يتم تحليلها.

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List of notations and symbols

c_{cr}: Critical damping of the system

m : Mass of the structure

k: Rigidity of the structure

a_0, a_1 : Modification coefficients

ζ : Depreciation rate

ω_n: Circular frequency

AD: Dynamic Amplification

R_{dyn}: Dynamic response

R_{static}: Static response

I_{mp}: Impact factor

FAD: dynamic amplification factor

t_{static}: Instant of maximum static response

t_{dyn}: Instant of maximum dynamic response

$u(x, y, t)$,$v(x, y, t)$,$w(x, y)$: Displacement field

x, y: Coordinate of a point

t: Time

D: Modulus of rigidity

h: Thickness of the plate

ρ: Density of the material of the plate

f: Imposed force

∇: Operator

∂: Partial derivative

E: Young's modulus

I: Quadratic moment

S: Section of the beam

ω: Pulsation

ω_0: Propre pulsation

k_x, k_y : Wave numbers associated with the ox and oy directions

l : Length

N: Simple intermediate supports

M_{v1}: Mass of the vehicle's wheel

M_{v2}: Mass of the vehicle

k_v: Spring stiffness

c_v: Damping constant damper

$x_v(t)$: Vehicle's position

$x(t)$: The spatial position of a point on the bridge

$r(x)$: Imperfections of the rolling track

$y_{v1}(t)$, $y_{v2}(t)$: Vertical movements of masses

T: kinetic energy

U: Potential energy

D: Dissipation

y_i , $\dot{y}_i$, F_i : Generalized displacement, generalized velocity and generalized force

F_v : Dynamic interaction force

α : Amplitude

ω: Angular frequency

θ: Phase

θ_n: Phase angle

$S(\omega)$: Power spectral density function

A: Coefficient of roughness of the roadway

$\overline{m}$: Mass per unit length

δ : Dirac operator

n: Number of modes necessary for the convergence of the modal series.

ϕ_i : Natural modes of free vibration of the bridge.

q_i : Generalized coordinates

Q_i: Generalized force

T_p: Kinetic deformation energy

V_p: Elastic deformation energy

V_s: Potential energy due to the intermediate supports

$F_v(T)$: Virtual work of interaction forces

$\phi_i(x)$: Clean deformations of the bridge

$\underline{v}$: Dimensionless speed

$\underline{a}$: Dimensionless acceleration

m_{ij}: Mass

c_{ij}: Damping

k_{ij}: Stiffness

P_i: Generalized forces

DOF: Degrees of freedom

γ, β: Stability parameters of the Newmark method

Δt: Time step

$\{q\}_{t+\Delta t}$,$\{\dot{q}\}_{t+\Delta t}$: Generalized displacements and velocities respectively

GENERAL INTRODUCTION

Since the inception of the earliest bridges, whether shaped by natural forces or crafted by early civilizations, they have endured the effects of degradation and eventual collapse. These phenomena are immutable laws of nature. Consequently, the dynamic nature of external forces acting on bridges over time, including periodic stresses from moving vehicles, inevitably leads to fatigue-induced damage.

Over the past century and a half, extensive theoretical and experimental research has been conducted on the interaction between bridges and vehicles. This research is driven not only by the practical significance of such interactions in everyday life but also by the inherent complexity of the problem.

Central to this endeavor is the primary objective of conducting a thorough examination of bridge decks. This entails a meticulous consideration of both their structural dimensions and their dynamic response to the passage of vehicles. The primary objective of this study is to analyze the effects of different factors that impact the dynamic behavior of bridges. Understanding these dynamics is crucial for ensuring the safety, reliability, and longevity of bridge structures. The interaction between moving vehicles and bridges leads to various forces and vibrations that can affect the structural integrity of the bridge over time.

When vehicles travel over a bridge, they generate dynamic loads that cause deformations and vibrations in the bridge structure. These vibrations are influenced by several factors, including vehicle speed, road conditions, vehicle mass, and the bridge's own physical properties like mass, stiffness, and damping. The study examines these interactions in detail to model and predict the dynamic effects accurately.

Understanding the dynamic behavior of bridges under vehicle loads is vital for designing safe and durable structures. This study provides insights into the factors that contribute to these dynamic effects and presents methods for accurately modeling and analyzing them. The ultimate goal is to enhance the safety and longevity of bridges in the face of increasing traffic loads.

CHAPTER I

State of knowledge

1 Introduction:

The objective of this chapter is to present the phenomenon studied and to provide an overview of the methods used to analyze the dynamic behavior of bridges loaded with moving traffic loads.

The study of the literature makes it possible to identify the important factors that play a role and to determine the requirements for modeling the dynamic effects in the load-bearing slabs of concrete bridges as realistically as possible.

The nature of vibrations caused by vehicles on bridges is also presented. A detailed description of the dynamic interaction phenomenon that develops when a vehicle crosses a bridge is provided, complemented by a review of the main factors that contribute to this phenomenon.

The concept of dynamic amplification factor is explained as well as the different definitions commonly used. The methods used to determine these factors are discussed.

2 Nature of vibrations generated on bridges by vehicles:

2.1 Explanation of the phenomenon:

The phenomenon of vibrations generated on bridges by vehicles is explained by a complex interaction between vehicle movements and the structure of the bridge. When a vehicle travels on a bridge, it exerts forces that cause deformations and vibrations in the structure. These vibrations are influenced by various factors such as vehicle speed, road condition, vehicle and bridge mass, as well as inertia and damping forces.

The relative displacements of the ends of the springs that model the vehicle's suspension change, which alters the interaction forces under the tires. In addition to elastic forces, the bridge is subject to inertia and damping forces, which contribute to vibrations. The balance configuration of the vehicle changes the intensity of the applied loads, thereby disrupting the balance of the bridge throughout the vehicle's passage.

Factors such as variation in the vehicle-to-bridge mass ratio, vehicle rolling speed, axle spacing, as well as the vertical movement of the vehicle and the profile of the roadway affect these vibrations. These complex interactions between the different elements contribute to the generation of vibrations on bridges when vehicles pass by.

This explanation highlights the importance of understanding these phenomena to design structures capable of effectively resisting the dynamic effects induced by the passage of vehicles on bridges [1] [2] [3] [4]

2.2 The bridge:

The geometry and the static system make each bridge a particular structure with specific static and dynamic behavior. The main characteristics which dictate the behavior of the structure are the span length, the mass, [5] stiffness and damping **(Figure 1)**

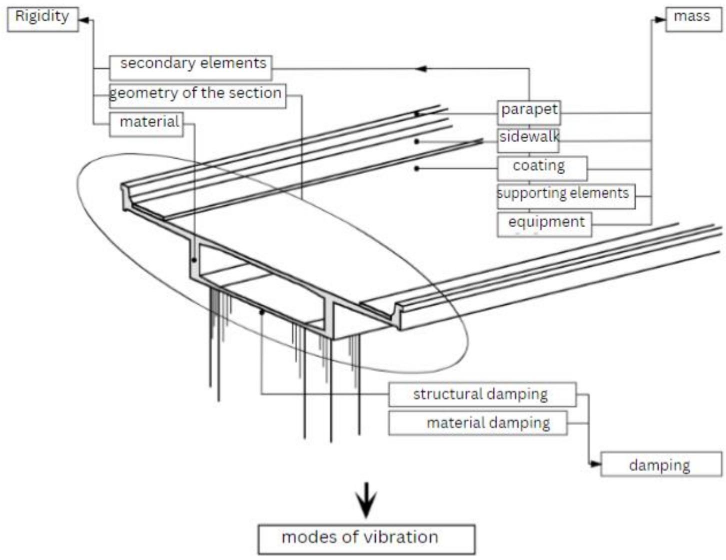

Figure I-1: Bridge elements

Effects of span length: The length of the span is a parameter having a major impact on the dynamic behavior of road bridges. Indeed, in the literature correlations are proposed between the span of the bridge and its fundamental frequencies. The graph presented in **(Figure 1)** shows that as the length of a bridge span increases, the proper frequency of Deck clean decreases [6]Furthermore, there are national standards for the design of road bridges which provide formulas for calculating the FAD based on the length of the bridge span.

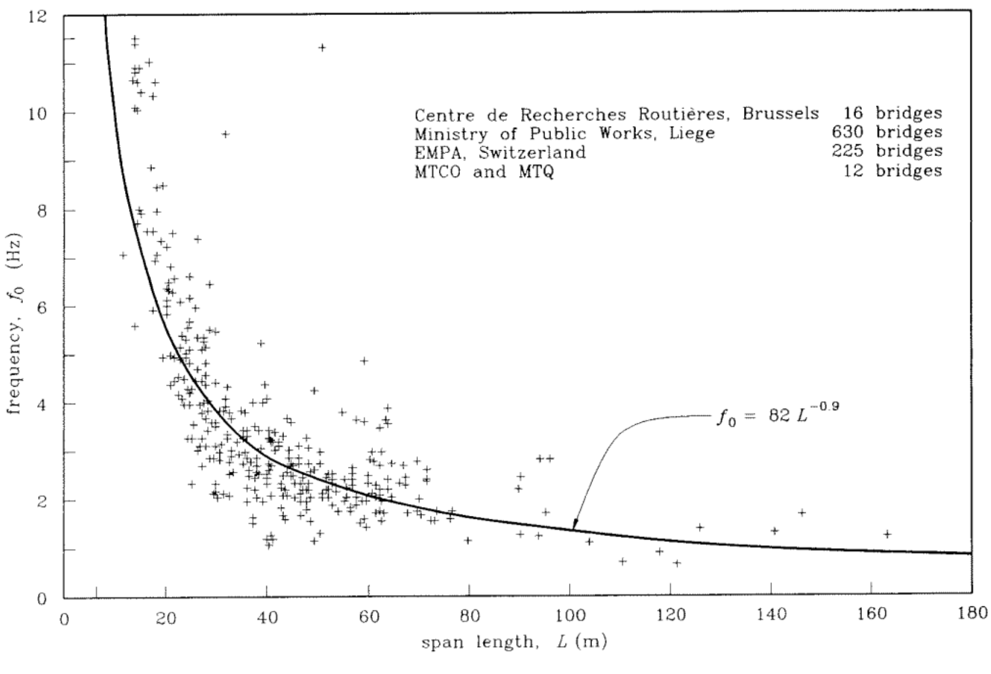

Figure I-2: Fundamental frequencies versus span length for 898 highway bridges

The mass: The mass of the superstructure includes the mass of the supporting structure, the covering, the parapets, the sidewalks and all the bridge equipment. Mass is a dominant factor for bridge vibration frequencies [5].

Rigidity: The rigidity of a bridge is determined mainly by the material, by the dimensions and by the type of section. Bridges whose section is formed by a box section provide greater rigidity with respect to transverse bending and torsion than bridges composed of independent beams. The latter type is very influenced by the spacers or diaphragms which transversely connect the beams together. Secondary elements such as cladding, sidewalks and parapets increase the overall rigidity of the structure. High stiffness helps increase bridge vibration frequencies [5].

Depreciation: Damping is a physical phenomenon responsible for the dissipation of energy during structure vibrations. Indeed, damping can be defined by a phenomenon Thermodynamic since it involves a transfer of mechanical energy to thermal energy. There are two types of depreciation: **material depreciation** and **structural damping**. Material depreciation corresponds to energy losses in the molecular structure of the material. It depends on multiple factors including the geometry of the structure, the natural frequency, the temperature, as well as the type of deformation. Structural damping is of fundamentally frictional origin. In other words, it is explained by the energy dissipated at the interfaces between the distinct elements of a structure, such as joints, connections, supports [5].

Generally, it is very difficult to estimate the actual damping value of a bridge structure. In order to simplify the problem, the approach generally used in the literature is based on the Rayleigh method [7]. which employs equivalent viscous damping. This method is given by the formula presented by the equation (1)

$$c_{cr} = a_0 m + a_1 k \tag{1}$$

c_{cr} : Critical damping of the system

m : Mass of the structure

k : Rigidity of the structure

The coefficients a_0 And a_1 of the equation (1) are called modification coefficients and they are calculated and chosen so as to estimate the overall damping of the model so that it is as close as possible to the real damping of the structure. As the equation (1),shows, damping can be identified by only knowing the masses and stiffnesses. However, in order to determine these coefficients, it is important to have an idea of the depreciation rate ζ of the structure studied. Therefore, for a mode i, the values of the constants a_0 and a_1 can be determined using the equation (2)

$$\zeta = \frac{a_0}{2\omega_{ni}} + \frac{a_1 \omega_{ni}}{2} \tag{2}$$

ω_n : Circular frequency $\left[{rad}/{s}\right]$

In order to find the values of these modification coefficients, it is enough to use the first two modes as the number of unknowns is equal to 2.

2.3 The roadway profile:

The roadway profile is an element of great interest for the study of the dynamic behavior of a bridge. A distinction is made between the static profile which represents the irregularities of the roadway depending on the position and the dynamic profile which corresponds to the deflections caused by the loads applied to the bridge. The static profile is determined by the profile along the length of the road on which is superimposed the differences in level at the supports, the differential settlements of the foundations or the misalignments of the spans, the counter deflections, the deflections due to delayed effects and the roughness of the coating $(Figure\ 2)$. These different components come from construction tolerances, operating conditions or the effects of time. Irregularities in the static profile of the roadway have the effect of inducing and maintaining oscillations of the load on the vehicle suspension system [5].

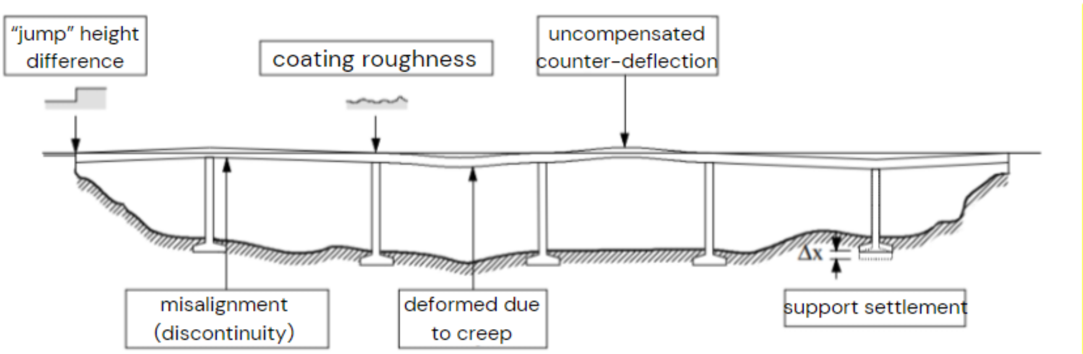

Figure I-3: Static profile of a bridge

2.4 Vehicles:

When examining the dynamic responses of highway bridges, heavy vehicles, commonly referred to as trucks, are frequently used due to their large mass. There are different models of trucks, each model having unique geometric characteristics.

The representation of the vehicle requires specifying certain properties which can be classified either as static properties or as dynamic properties. Static properties include geometry, i.e., axle spacing, number of axles and static vehicle weight distribution across the axles. Dynamic properties include, the natural frequencies of the axles and the damping characteristics of the suspension system. These quantities must be obtained from analytical estimates and the study of field measurements on real vehicles. There **Figure 3** summarizes the majority of the static and dynamic properties of the vehicle.

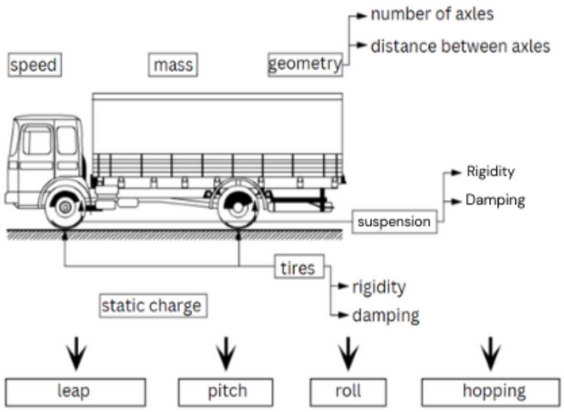

Figure I-4: Vehicle elements

The movement of a vehicle is mainly characterized by four oscillations: vertical oscillations (or bounding) and 3 possible rotations which are pitching, rolling and yaw. These 3 rotations are illustrated in **Figure 3**. In most numerical studies only vertical oscillations, which are sometimes coupled with pitching, are considered [8], the effect of these oscillations on the dynamic behavior of bridges is mainly influenced by the roughness of the surface of the roadway of the bridge as well as the suspension system of the vehicle considered, including the distributed masses. In order to take into account all possible oscillations, the vehicle must be modeled three-dimensionally, that is to say in its longitudinal plane as well as in its transverse plane.

2.4.1 Suspension system:

The suspensions on a vehicle are the elements making it possible to connect the unsprung masses (typically the wheel, the braking systems, the wheel drive, etc.) to the sprung masses (typically the chassis, the engine and all the components of the vehicle). vehicle attached to the chassis).

The use of suspension is imposed by the irregularities of the surface on which the vehicle is moving. It reduces the impact on the machine, avoiding breakages and excessive wear, improving driving comfort and maintaining contact between the wheels and the ground despite its irregularities: an essential condition for road holding. Furthermore, the fact that a vehicle has mass requires the use of a return mechanism to prevent the system from sagging indefinitely as the terrain becomes uneven.

Thus, the suspension consists of a connecting device between the “unsprung masses” and the “sprung masses”, a spring and possibly a shock absorber. In some cases, the suspension arm is also called a “suspension triangle”, a name due to its shape. We also distinguish between “independent” suspensions, on the same axle the left part is separated from the right part, and “rigid axle” suspensions where the left and right parts are linked. Some explanations may require prior reading of the suspension geometry article, particularly for land vehicles [9].

2.4.2 Tires:

Tires also influence the vibration movements of road vehicles. To account for vibrations caused by tire flexibility, the suspension system and tire must be separated. The structure of the tire, characterized by the number and arrangement of the rubber strips that constitute it, as well as the inflation air pressure, influence the rigidity of the tire and its damping.

2.5 Interaction of the elements contributing to the phenomenon:

Several parameters described in the previous paragraphs not only have a direct influence on the dynamic behavior of bridges crossed by vehicles, but, moreover, they intervene in interaction with other factors.

Depending on the value of **mass ratio** defined by the total mass of the vehicle divided by the total mass of the superstructure, the frequency content of the bridge vibrations changes over time, because the mobility of the loads varies the mass distribution of the bridge + vehicle system. If the amplitude of the oscillations of the vehicle mass is large, the sensitivity of the response of the bridge to the variation of the mass ratio also depends on the **frequency ratio**. This ratio is expressed by the frequency of the vehicle's vertical oscillations divided by the fundamental frequency of the bridge. Large amplitude vibrations are observed in the event of resonance, when a disturbing force stresses a vibratory system with a frequency equal to that of the system. In the case of bridges, it is more appropriate to speak of a quasi-resonance phenomenon, because the amount of energy transmitted to the bridge is not infinite and the forced vibration does not last long enough for resonance to develop.

The influence of **speed** is inseparable from **axle spacing** of the vehicle and **pavement profile**. Speed represents the means by which the roadway becomes the disruptive force of the vehicle. The speed and roughness of the roadway are the parameters which favor the movements of vehicles in one mode rather than another. Vehicle speed associated with axle spacing determines the frequency of application of loads to a structural element. Different values of this frequency can be determined for a vehicle made up of several axles. For low speeds the tandem axle spacing must be considered, while for higher speeds the basic spacing, considering the tandem as a single axle, becomes more important [5]

3.Definitions of dynamic amplification factor:

The notion of the dynamic increase coefficient (CMD) or the dynamic amplification factor (FAD) is a parameter of crucial importance for engineers responsible for the design and evaluation of bridges. But as it is currently formulated by the standards, it does not reflect the reality of the bridge-vehicle coupling because of the complex interaction between the movements of the bridge and the vehicle. The dynamic behavior of bridges when vehicles pass depends on: the dynamic characteristics of the bridge (mass, rigidity, damping, etc.), the dynamic characteristics of the vehicles (mass, suspension system, etc.), the progression history of the vehicles on the bridge (initial state of vibration, speed, etc.), and the roughness of the roadway (bridge deck). The dynamic increase coefficient (CMD), also called dynamic amplification (AD), caused by the passage of a vehicle on a specific bridge is given by:

$$\text{Dynamic Amplification}(AD) = \frac{R_{dyn} - R_{static}}{R_{static}} \tag{3}$$

Or R_{dym} And R_{static} are the absolute maximum responses obtained for the dynamic and static cases respectively and AD is the dynamic amplification. we find in the literature the notion of the impact factor expressed as a percentage. The impact factor (I_{mp}) is defined as follows [10]:

$$(I_{mp}) = \left(\frac{R_{dyn}}{R_{static}} - 1\right) \times 100\% \tag{4}$$

From the equation (3)we can write:

$$R_{dyn} \;= R_{static}(1 + AD) \tag{5}$$

The dynamic amplification factor (FAD) is expressed by the term $(1 \;+\; AD)$.

More marked divergences exist in the definition of FADs calculated experimentally by recording static and dynamic responses using gauges. The static response used to define the FAD is, in certain cases, that obtained at the instant t_{dyn} where the maximum dynamic response is obtained or even at the instant t_{static} where the maximum static response is obtained. Maximum static and dynamic responses do not generally occur for the same load position **(Figure 5)**

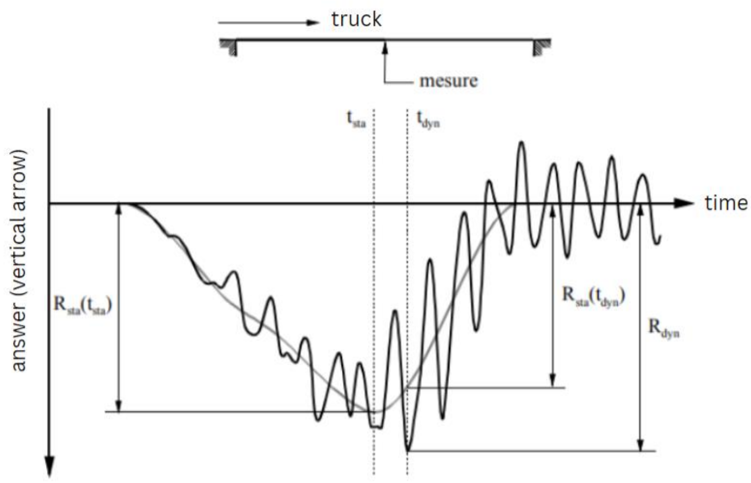

Figure I-5: Static and dynamic response

Importance of Dynamic Amplification Factor:

The Dynamic Amplification Factor is a very important parameter for the analysis and design of road bridge bearing slabs. Their importance can be summarized in the following points:

- Present the dynamic effects in the rolling slabs which are caused by the passage of vehicles. These effects are determined in relation to a static reference
- Determine the sensitivity of bearing slabs to dynamic stresses caused by road traffic
- Define the locations considered important on the bearing slabs according to a given loading
- Determine which types of bearing slabs are most sensitive to dynamic stresses.

Conclusion:

A study which involves the dynamic study of bridges under moving traffic loads requires firstly to present the phenomenon at hand, in this case the dynamic response from the structure comes in form of vibrations, these vibrations are the collective response from the different elements which constitute the structure, these different elements such as the bridge and its parameters (mass, rigidity, depreciation, roadway profile), the vehicles (suspension system, tires etc....). add on that an important factor (dynamic amplification factor) and its importance.

CHAPTER II

The modelisation of a bridge

1 Introduction:

Bridges are more than just static structures connecting two points; they are dynamic systems constantly interacting with the environment. When a vehicle crosses a bridge, it sets off a chain reaction of forces and vibrations that ripple throughout the structure. Understanding and predicting how a bridge dynamically responds to these loads is crucial to ensuring its safety, reliability and longevity.

One of the main concerns of engineers is the dynamic response of bridges to automobile traffic. Unlike static loads, dynamic loads are transient and can induce vibrations, oscillations and resonance within the bridge structure. These dynamic effects can lead to fatigue, excessive deflection and even structural failure if not properly considered during the design and maintenance phases.

There are several reasons why studying the dynamic response of the bridge is essential:

- **Security:** The safety of motorists, pedestrians and goods relies on the structural integrity of bridges. Dynamic loads, such as those generated by moving vehicles, can induce sudden and unexpected reactions in a bridge. Understanding these dynamic effects allows engineers to design bridges that can safely support traffic loads without compromising structural stability.
- **Ease of maintenance:** Bridges must not only support heavy loads, but also provide a smooth and comfortable ride for users. Dynamic response analysis helps engineers optimize bridge designs to minimize vibration, reduce vehicle-induced discomfort, and ensure a satisfactory level of serviceability throughout the service life. life of the structure.
- **Fatigue assessment:** Bridges are subjected to millions of load cycles throughout their service life. Dynamic loading, particularly in heavy traffic conditions, can accelerate fatigue and lead to cracking and structural deterioration. By simulating the dynamic response of bridges, engineers can assess fatigue damage and develop maintenance strategies to extend the life of the structure.
- **Resonance avoidance:** Resonance occurs when the frequency of external loads matches the natural frequency of the bridge, leading to amplified vibrations and potential structural failure. Dynamic analysis helps identify critical frequencies and vibration modes, allowing engineers to mitigate resonance effects through design modifications or implementation of damping mechanisms.
- **Environmental factors:** External factors such as wind, seismic activity and temperature variations can also induce dynamic responses in bridges. Understanding how these environmental loads interact with vehicular traffic is essential to designing resilient bridges capable of withstanding a wide range of operating conditions.

Understanding how bridges dynamically respond to vehicle traffic is vital for engineers for two main reasons: First, the dynamic effects of vehicles induce greater stresses than static loads alone, while excessive vibrations can lead to fatigue, shortening the lifespan of the bridge. Despite the use of a dynamic increase coefficient, factors such as traffic conditions and vehicle characteristics are not fully taken into account. Various methods, including simulations and experiments, help model bridge-vehicle interaction, but challenges include complexity and measurement accuracy. Nevertheless, capturing the dynamic response of the bridge is crucial to ensure safety, reliability and longevity.

in the design and maintenance of bridges, meeting modern transportation needs while preserving public safety.

2 Modeling a bridge:

A **bridge** is a structure built to span a physical obstacle without blocking the path underneath, It is constructed for the purpose of providing passage over the obstacle [11], one of the ways that a mechanical structure such as a bridge can be represented by is through using beams and plates, which will simplify our theoretical understanding into a more practical one making it easier to integrate theoretical solutions to realistic ones

2-1 Modeling by a beam:

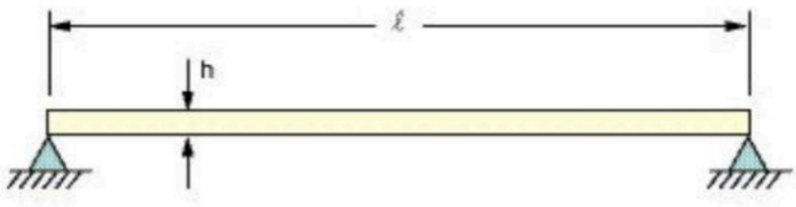

Figure II-1: Modeling a bridge using a beam

A beam is a structural element that primarily resists loads applied laterally on the axis [12]of the beam. One of the commonly used and simple methods for modeling bridges involves the application of a Euler-Bernoulli beam, usually supported at both ends. However, this conventional approach oversimplifies the bridge behavior, focusing only on one-dimensional aspects while neglecting the influence of transverse bending. This limitation becomes obvious for bridges whose spans resemble length and width proportions, such as highway bridges built with slabs. In such scenarios, the shortcomings of beam theory become apparent, particularly when the bridge encounters off-center loads. As vehicles move away from the centerline bridge, bending and twisting effects become significant, posing challenges to the accuracy of beam-based models [13]

2-2 Modeling using a plate:

To model the bridge using a plate simply supported on two of its ends, there are several choices:

- ➢ Thin plates in the Love-Kirchhoff theory
- ➢ Thick plates in Mindlin's theory

The bending of a plate is modeled by essentially two theories (apart from the theory of continuous media): that of a thin plate presented in this chapter and that of a thick plate presented in the appendix.

The first approach to the problem of a homogeneous and elastic plate was formulated by Love-Kirchhoff. This theory contains a certain number of hypotheses which do not allow a certain number of mechanical phenomena such as shear force or rotational inertia to be taken into account. R. D. Mindlin proposed a new theory to take these phenomena into account. In the literature, there are a multitude of other theories that have been carried out by other researchers who have followed Mindlin. These theories will not be cited in this work, because they will not be used [13].

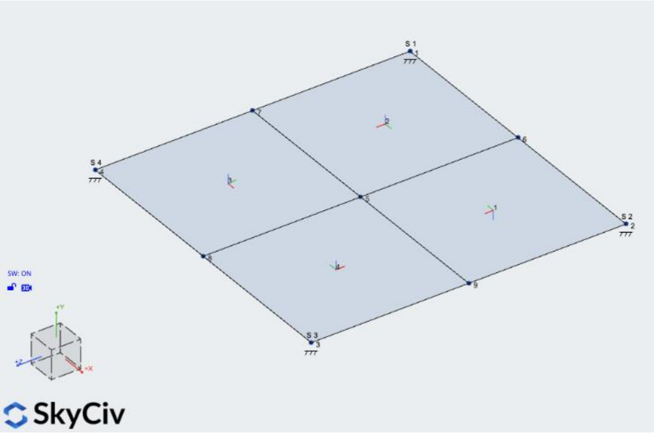

Figure II-2: Modeling a bridge using a plate [14]

2-2-1 The love-Kirchhoff theory:

The Love-Kirchhoff theory is an extension of the Euler–Bernoulli beam theory to thin plates. The theory was developed in 1888 by Love using hypotheses proposed by Kirchhoff. It is assumed that a surface midplane can be used to represent the three-dimensional plate in two-dimensional form [15].

The hypotheses:

The Love-Kirchhoff theory is used to study thin plates. he states the following hypothesis:

- The average plane is initially flat, that is, it has no curvature.

- The average sheet does not undergo deformation in its plane, that is to say that we only consider the transverse displacement (noted w) of the points of the average sheet [16].
- The sections normal to the average sheet remain normal during deformation, that is to say that shear can be neglected.
- The thickness of the plate is low, that is to say only in the direction of thickness.
- The deformation is zero which implies that the constraints in this direction can be neglected, and we place ourselves in small deformations.

2-2-2 The displacement field:

These hypotheses lead to the following displacement field:

$$u(x,y,t) = u_0(x,y,t) - z\frac{\partial w}{\partial x} \tag{1}$$

$$v(x,y,t) = v_0(x,y,t) - z\frac{\partial w}{\partial x} \tag{2}$$

$$w(x,y) = w(x,y) \tag{3}$$

Or x, y And $With$ are the coordinates of a point on the plate in a Cartesian and Galilean reference plane, t is the time variable write the equation of the bending motion of the plate, in the linear approximation the elasticity translates into:

$$D\nabla^4 w + \rho h\frac{\partial^2 y}{\partial x^2} = 0 \tag{4}$$

Or D represents the modulus of rigidity in bending, h the thickness of the plate, ρ the density of the material constitutes the plate, f the imposed force and finally the operator ∇ :

$$\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \tag{5}$$

This equation can be compared to that obtained for the bending movement of Euler-Bernoulli beams:

$$EI\nabla^4 w + \rho S\frac{\partial^2 w}{\partial t^2} = f \tag{6}$$

where E is the Young's modulus of the material used, I the quadratic moment of the sections of the beam considered.

Due to the fourth order operator and the sign $+$ between the terms on the left side, it's not a wave equation. Therefore, bending vibrations transmitted in a beam or plate will be dispersive in nature (propagation speed is a function of frequency).

In the case of a plate simply supported on all these edges, in harmonic time regime, the transverse displacement $w\ (x, y, t)$ is of the form:

$$w\ (x, y, t) = w \sin(k_x x) \sin(k_y y) \sin(wt) \tag{7}$$

Where w is the pulse k_x , k_y , are the wavenumbers associated with the directions (ox) and (oy), related to the wavenumber k by the relation:

$$k^2 = k_x^2 + k_y^2 \tag{8}$$

using the equation (2.2), without the force term, and using the previous displacement expression $w\ (x, y, t)$, we can write the dispersion equation for this simply supported thin plate as follows:

$$w^2 = k^2 \frac{D}{\rho h} \tag{9}$$

Determining the own pulsations allowed us to determine the deviation of the plate and therefore to study its behavior [13].

3 Load modeling:

There are many ways to model the load which depend on the solution method and required accuracy, such as moving load, train signature method, lumped mass with spring-dash unit, model $2D$ complete including bodywork with bogies and two layers of suspension and model $3D$ complete for the bodywork. Figure 3 shows the evolution of the $2D$ train/vehicle modeling [17].

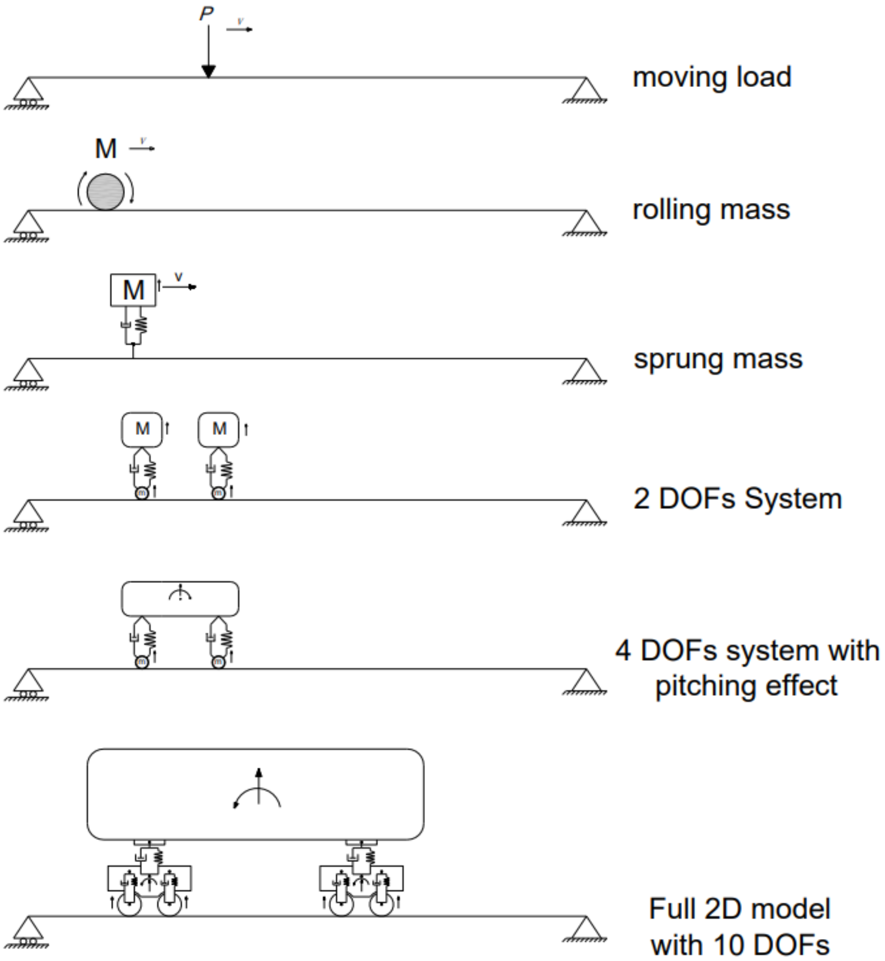

Figure II-3: Evolution of 2D load modeling

4 Modeling a vehicle:

Weighing in motion is a technique aimed at estimating the static weight of a heavy vehicle from pressure measurements or deformations of the structure on which this vehicle is moving.

Estimating static weight in this way is expected to improve the use and efficiency of existing weighing systems, and overloading heavy vehicles contributes significantly to damage to roads and bridges. To remedy this, manufacturers and maintenance services want to have reliable tools allowing them to estimate in real time the loads and weights coming from heavy vehicles traveling on bridges.

Vehicles are generally described as static loads. However, they drive across and across the bridge at high speed. In addition, the repeated passage of axles at constant gauge can cause significant excitation of the bridge. The description of vehicles in the form of mobile loads is then necessary.

The choice of vehicle plays an important role in the response of the bridge. From the simplest model which consists of a constant force moving to that of three-dimensional modeling, we have chosen the simplest case of a mass mounted on a spring.

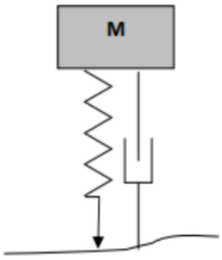

Figure II-4: model of a simple vehicle

Indeed, the use of a more sophisticated vehicle model, Figure 4 (comprising several degrees of freedom) could mask the essential aspect of the two-dimensional behavior of the bridge [13]

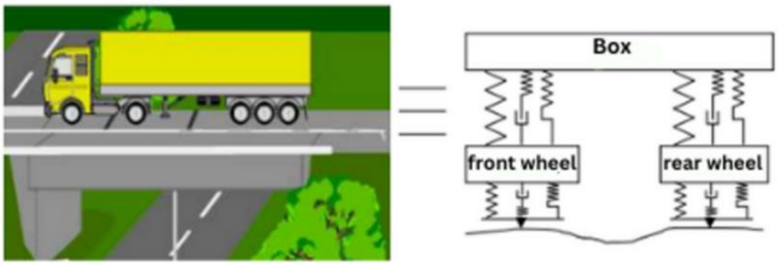

Figure II-5: Sophisticated vehicle model

Conclusion:

Bridges are dynamic systems that constantly interact with their environment. When vehicles cross a bridge, they induce a series of forces and vibrations throughout the structure. Understanding and predicting these dynamic responses is crucial to ensuring bridge safety, reliability and longevity.

Bridges can be modeled using beams and plates to simplify theoretical understanding into practical applications.

Various methods are used to model loads on bridges, depending on the accuracy required. These range from simple moving loads to detailed two- and three-dimensional models.

Bridge-structural dynamics is a vital area that ensures the safety and functionality of bridges. By comprehensively understanding the behavior of structural components subjected to dynamic loads, engineers can design more robust bridges and develop effective bridge maintenance strategies. This not only improves the lifespan of these critical infrastructures, but also ensures their safe operation under varying conditions.

CHAPTER III

Vibration behavior of the bridge during the passage of the convoy

1 Introduction:

In this chapter, a vibration model of a multi-span bridge during the passage of a convoy will be presented. Vibration modeling takes into account the interaction between the bridge and the convoy as well as track irregularities. The design of the bridge consists of a continuous beam, its intermediate supports being designed by high rigidity linear springs.

On the other hand, the convoy is designed with a series of moving vehicles. is represented by a series of vehicle models with two degrees of freedom of movement. Using the modal superposition method and the Lagrange of the system, two coupled equations of motion for the bridge and the convoy are determined. These equations are then solved using a classic Newmark scheme with iterative calculation.

2 Vibration modeling of a road bridge during the passage of a convoy:

2-1 Modeling:

The bridge is modeled by an equivalent continuous beam, of length l, of linear mass m, moment of inertia I and young's modulus E, simply supported at the ends. The N simple intermediate supports (Figure 2), The convoy is modeled by nv single-speed vehicles IN, The mass of the vehicle in is noted $M_{v2}$and the mass of its wheel M_{v1}. These two masses are linked by a stiffness spring k_v and a damping constant damper c_v . Vehicle position v is noted $x_v(t)$and the vertical displacement of the bridge $w(x,t)$. Intermediate abscissa supports $x = s_p(p = 1,2,\ldots,N)$are modeled by linear springs of high rigidity and the same stiffness k. The spatial position of a point on the bridge is noted $x(t)$and the imperfections of the rolling track are represented by the function $r(x)$. Vertical movements of masses M_{v1} And M_{v2} are respectively $y_{v1}(t)$ And $y_{v2}(t)$.

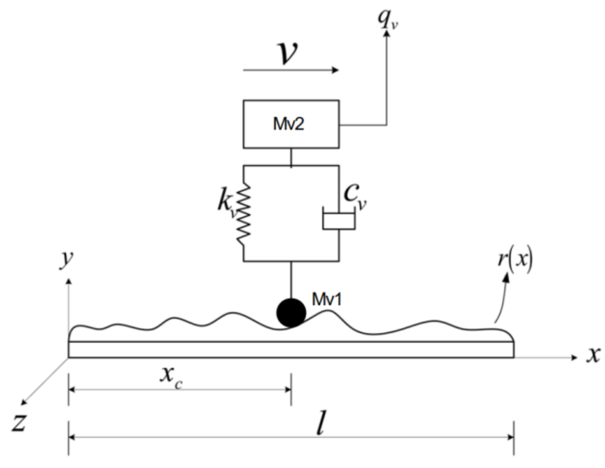

Figure III-1: Vehicle-bridge interaction element[18]

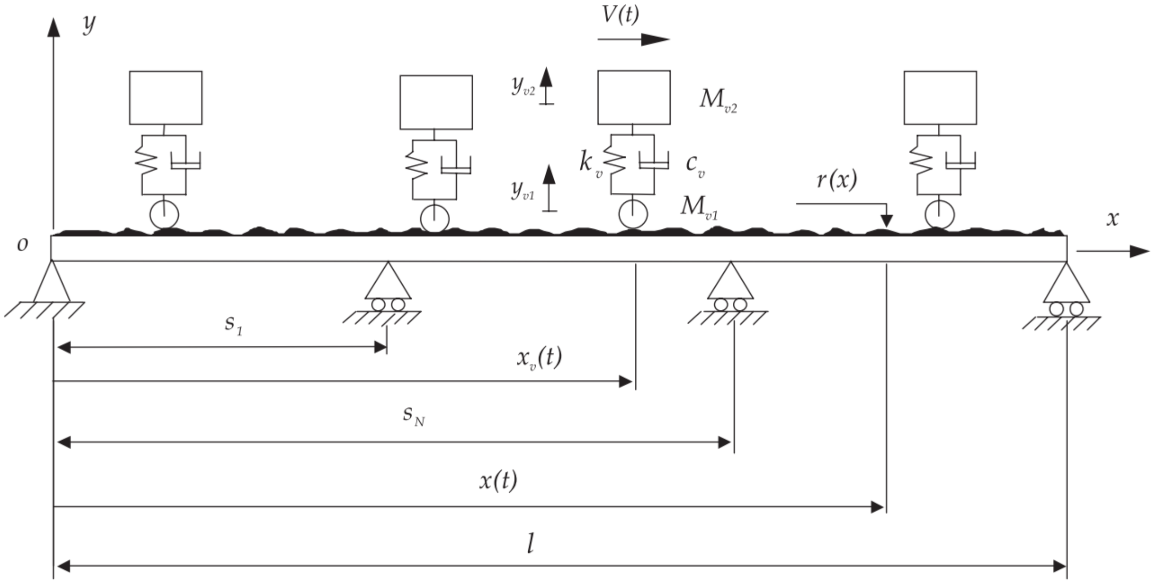

Figure III-2: Beam continues with N simple intermediate supports, excited by nv mobile vehicles [19]

2-2 Equations of movement of the vehicle model:

The equations of motion of the vehicle model with two degrees of freedom are given in the following derivation. The static equilibrium position is chosen as the reference position for the vehicle movement. The kinetic energy T and potential energy U of the vehicle are described respectively by the equations (1) And (2).

$$T = \frac{1}{2} M_{v1} \dot{y}_{v1}^2 + \frac{1}{2} M_{v2} \dot{y}_{v2}^2 \tag{1}$$

$$U = \frac{1}{2} K_v (y_2 - y_1)^2 \tag{2}$$

The dissipation function D is expressed by:

$$D = \frac{1}{2} C_v (\dot{y}_2 - \dot{y}_1)^2 \tag{3}$$

The Lagrange equation is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_i}\right) - \frac{\partial L}{\partial y_i} + \frac{\partial D}{\partial \dot{y}_i} = F_i \qquad i = 1,2 \ldots ni \tag{4}$$

Or $L = T - U$ is Lagrange, and y_i , $\dot{y}_i$ *And* F_i are generalized displacement, generalized velocity and generalized force, respectively. t : the weather.

The total number of degrees of freedom is two. By replacing the equations (1) has (3) in the equation (4), the equations of motion of the vehicle can be obtained as follows:

$$M_{v2}\ddot{y}_{v2}(t) + c_v(\dot{y}_{v2}(t) - \dot{y}_{v1}(t)) + k_v\left(y_{v_2}(t) - y_{v_1}(t)\right) = 0 \quad (5)$$

$$M_{v1}\ddot{y}_{v1}(t) + c_v(\dot{y}_{v1}(t) - \dot{y}_{v2}(t)) + k_v\left(y_{v_1}(t) - y_{v_2}(t)\right) = F_v \quad (6)$$

The resulting equations in matrix form are written:

$$\begin{bmatrix} M_{v1} & 0 \\ 0 & M_{v2} \end{bmatrix} \begin{Bmatrix} \ddot{y}_{v1} \\ \ddot{y}_{v2} \end{Bmatrix} + \begin{bmatrix} c_v & -c_v \\ -c_v & c_v \end{bmatrix} \begin{Bmatrix} \dot{y}_{v1} \\ \dot{y}_{v2} \end{Bmatrix} + \begin{bmatrix} k_v & -k_v \\ -k_v & k_v \end{bmatrix} \begin{Bmatrix} y_{v1} \\ y_{v2} \end{Bmatrix} = \begin{Bmatrix} F_v \\ 0 \end{Bmatrix} \quad (7)$$

F_v : the dynamic interaction force between the mass M_{v1} and the beam [20]

2-3 Bridge-convoy interaction forces:

From the equation (7), by adding the static contribution, we can determine the interaction force at each bridge–vehicle contact point v :

$$F_v(t) = (M_{v1} + M_{v2})g + M_{v1}\ddot{y}_{v1} + M_{v2}\ddot{y}_{v2} \qquad v = 1,2,\dots,nv \quad (8)$$

The vertical displacement, velocity and acceleration of mass M_{v1} are respectively are:

$$y_{v1}(t) = w(x_v(t), t) + r\left(x_v(t)\right) \quad (9)$$

$$\dot{y}_{v1}(t) = \left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x} + V\frac{\partial r}{\partial x}\right)\Bigg|_{x=x_v(t)} \quad (10)$$

$$\ddot{y}_{v1}(t) = \left(\frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x \partial t} + V^2\frac{\partial^2 w}{\partial x^2} + V^2\frac{d^2 r}{dx^2}\right)\Bigg|_{x=x_v(t)} \quad (11)$$

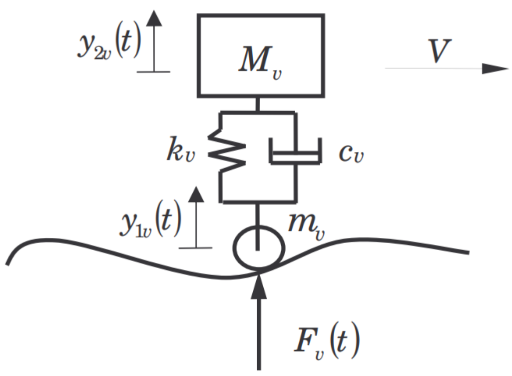

Figure III-3: Interaction force representation

2-4 Modeling of the roadway profile [21]:

Given the wide variety of phenomena involved in shaping the profile of a roadway, this profile should be assimilated to a stationary Gaussian random process with zero mean. This random process, $r(x)$, can be generated by summing n cosine functions of amplitude α, of angular frequency ω and phase θ. On a:

$$r(x) = \sum_{k=1}^{N} a_n \cos(\omega_n x - \theta_n) \tag{12}$$

Amplitude a_n is the main parameter which determines the importance of road irregularities. Assuming that the phase angle θ_n is an independent random variable having a uniform probability density in the interval 0 has 2π, we can demonstrate that the amplitude of the cosine functions is given by the following equation:

$$a_n = \sqrt{4S(\omega_n)\Delta\omega} \tag{13}$$

Or $S(\omega)$ is the power spectral density function representative of the irregularities. The exponential function describing the following power spectral density was proposed by:

$$S(\omega) = A\omega^{-t} \tag{14}$$

with A the coefficient of roughness of the roadway and t taken equal to 2. The expression describing the profile of the roadway becomes [1]:

$$r(x) = \sum_{k=1}^{N} \left[\sqrt{4S(\omega_n)\Delta\omega}\cos(\omega_n x + \theta_n)\right] \tag{15}$$

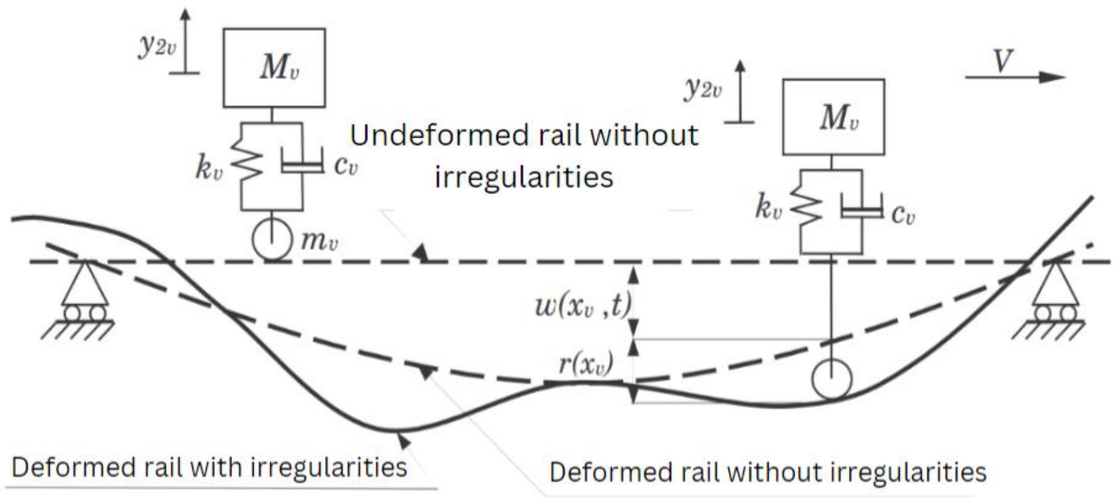

Figure III -4: Modeling of bridge-vehicle interaction and evaluation of movement

2-5 Equations of the movement of the vehicle model in the modal base [21]:

Considering the expressions (9)-(10) , the equation (5) takes the following form:

$$M_{v2}\ddot{y}_{v2}(t) + c_v\dot{y}_{v2}(t) + k_v y_{v_2}(t) - \sum_{j=1}^{n} c_v\phi_j\big(x_v(t)\big)\dot{q}_i(t) - \sum_{j=1}^{n}\big(k_v\phi_j(x_v(t)\big) + c_v V\phi_j{}'\big(x_v(t)\big)q_i(t)$$
$$= k_v r\big(x_v(t)\big) + c_v V r'\big(x_v(t)\big)\,, \qquad v = 1,2,\dots,nv \qquad (16)$$

The equation (18), in matrix form is written:

$$[M]\{\ddot{y}_2\} + [C]\{\dot{y}_2\} + [K]\{y_2\} - [C][\phi]^T\{\dot{q}\} - ([K][\phi]^T + V[C][\phi']^T)\{q\}$$
$$= [K]\{r\} + V[C]\{r'\} \qquad (17)$$

with

$[M] = diag[M_{v2}]$;

$[C] = diag[c_v]$; $\qquad v = 1,2,\dots,nv \qquad (18)$

$[K] = diag[K_v]$;

$\{r\} = \{r(x_v)\}$,

2-6 Bridge motion equation [21]:

The equation of transverse movement in pure bending of the bridge (continuous beam) is written:

$$\bar{m}\frac{\partial^2 w(x,t)}{\partial t^2}+c\frac{\partial w(x,t)}{\partial t}+EI\frac{\partial^4 w(x,t)}{\partial x^4}+k\sum_{p=1}^{na} w(x,t)\delta\left(x-x_p\right)=-\sum_{v=1}^{nv} F_v(t)\delta(x-x_v) \qquad (19)$$

$\bar{m}$: mass per unit length

c : the damping constant of the bridge

E : Young's module

I : the moment of inertia of the bridge

δ : the Dirac operator

The mass $\bar{m}$ and the moment of inertia I are considered independent entities because it is assumed that the irregularities present in the raceway are of negligible magnitude. Consequently, these irregularities do not have a significant influence on the linear mass and inertia of the bridge.

To determine the equations of motion of the bridge, we used the modal method and the Lagrange equations. The vertical displacement of the bridge can be expressed by:

$$w(x,t)=\sum_{i=1}^{n}\phi_i(x)q_i(t) \qquad (20)$$

n : is the number of modes necessary for the convergence of the modal series.

ϕ_i : the natural modes of free vibration of the bridge.

q_i : generalized coordinates.

2-7 Determination of the equation of movement of the bridge projected into the modal base

The equation of motion of the bridge projected into the modal base is obtained using the Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i \qquad (21)$$

Or q_i is the generalized coordinate and Q_i the corresponding generalized force.

The kinetic energy and the elastic deformation energy of the beam in pure bending are given by:

$$T_p = \frac{1}{2} m \int_0^l \left(\frac{\partial w}{\partial T}\right)^2 dX \tag{22}$$

$$V_p = \frac{1}{2} EI \int_0^l \left(\frac{\partial^2 w}{\partial X^2}\right)^2 dX \tag{23}$$

Intermediate abscissa supports $X = Sp(p = 1,2,\ldots,N)$ are modeled by linear springs of high rigidity and the same stiffness k.

The potential energy due to the intermediate supports is given by:

$$V_s = \frac{1}{2} k\left(w^2(S_1) + \cdots + w^2(S_N)\right) \tag{24}$$

Or $w(S_p)$ is the transverse displacement of the beam evaluated at $X = Sp$. The virtual work of interaction forces $F_v(T)$ applied to the beam at the abscissa sections $F_{in}(T)$ for a virtual trip In δw est:

$$\delta\Im = -\sum_{v=1}^{nv} F_v(T)\delta w\left(x_v(T)\right) \tag{25}$$

For convenience, we use the following dimensionless quantities:

$t = T\sqrt{EI/ml^4}$; $x_v = X_v/l$;

$w = W/l$; $s_p = S_p/l$;

$\bar{k} = kl^3/EI$; $\bar{g} = gml^3/EI$; $\bar{M}_{v1} = M_{v1}/ml$;

$\bar{M}_{v2} = M_{v2}/ml$; $\bar{k}_v = k_v l^3/EI$;

$y_{v_1} = Y_{v_1}/l$; $y_{v_2} = Y_{v_2}/l$;

$\xi_v = c_v/2M_{v2}\omega_v$; $\bar{\omega}_v = \omega_{p,1}/\omega_v$

$r = R/l; \bar{v} = V\sqrt{ml^2/EI}$; $\bar{F}_v = F_v l^2/EI$; $x = X/l$ (26)

Or $\omega_{P,1} = \pi^2\sqrt{\left(\frac{EI}{ml_1^4}\right)}$ is the first proper pulsation of the bridge,

$\omega_v = \sqrt{\left(\frac{k_v}{M_{v2}}\right)}$ the vehicle's own pulsation v

l : the length of the bridge

m : the mass of the bridge per unit of length

ξ_v : the damping factor in the vehicle suspension v

l_1 : the length of the span.

The Lagrange of the continuous beam is written:

$$L = T_P - V_P - V_S \tag{27}$$

Considering the expressions (2) has (4) and dimensionless quantities (6), l'expression (7) becomes:

$$L = \frac{EI}{2l}\left(\int_0^l \left(\frac{\partial w}{\partial t}\right)^2 dx - \int_0^l \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx - \bar{k}\left(w^2(S_1) + \cdots + w^2(S_N)\right)\right) \tag{28}$$

Using the modal method, the dimensionless transverse displacement of the bridge is given by:

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t) \tag{29}$$

Where the $q_i(t)$ are the generalized coordinates to be determined, and $\phi_i(x)$ the clean deformations of the bridge. Taking into account dimensionless quantities (6) and after projection of the expression (5) in the modal base, the generalized force Q_i has the expression:

$$\begin{aligned} Q_i = -\frac{EI}{l}\sum_{v=1}^{nv} \Big[&(\bar{M}_{v1} + \bar{M}_{v2})\bar{g}\phi_i\big(x_v(t)\big) + \bar{M}_{v2}\phi_i\big(x_v(t)\big) + \ddot{y}_{v2}(t) \\ &+ \bar{M}_{v1}\phi_i\big(x_v(t)\big)\phi_j\big(x_v(t)\big)\ddot{q}_j(t) + 2\bar{v}\bar{M}_{v1}\phi_i\big(x_v(t)\big)\phi'_j\big(x_v(t)\big)\dot{q}_j(t) \\ &+ (\bar{v}^2\bar{M}_{v1}\phi_i\big(x_v(t)\big)\phi''_j\big(x_v(t)\big) + \bar{a}\bar{M}_{v1}\phi_i\big(x_v(t)\big)\phi'_j\big(x_v(t)\big)q_j(t) \\ &+ \bar{M}_{v1}\phi_i\big(x_v(t)\big)(\bar{v}^2 r''\big(x_v(t)\big)\bar{a}r'\big(x_v(t)\big))\Big] \end{aligned} \tag{30}$$

Or $\bar{v}$ And $\bar{a}$ are respectively the dimensionless speed and acceleration of the convoy.

By projecting (8) in the modal base (9), using the Lagrange equations (1), and after factorization, we obtain the equation of motion of the bridge:

$$\sum_{j=1}^{n} m_{ij}\ddot{q}_j(t) + \sum_{j=1}^{n} c_{ij}\dot{q}_j(t) + \sum_{j=1}^{n} k_{ij}q_j(t) + \sum_{v=1}^{nv} M_{v2}\phi_i\big(x_v(t)\big)\ddot{y}_{2v}(t) = P_i(t) \quad i = 1,2,\dots,n \tag{31}$$

With m_{ij} , c_{ij} , k_{ij} and P_i respectively masses, damping, stiffness and generalized forces.

with

$$m_{ij} = \delta_{ij} + \sum_{v=1}^{nv} M_{v1}\phi_i(x_v)\phi_j(x_v) \tag{32}$$

$$c_{ij} = 2\xi_j\omega_j\delta_{ij} + 2V\sum_{j=1}^{n} M_{v1}\phi_i(x_v)\phi_j'(x_v) \tag{33}$$

$$k_{ij} = \omega_j^2\delta_{ij} + k\sum_{p=1}^{na}\phi_i(s_p)\phi_j(s_p) + V^2\sum_{v=1}^{nv} M_{v1}\phi_i(x_v)\phi_j''(x_v) \tag{34}$$

$$P_i = -\sum_{v=1}^{nv}(M_{v1}+M_{v2})g\phi_i(x_v) - V^2\sum_{v=1}^{nv} M_{v1}r''(x_v)\phi_i(x_v) \tag{35}$$

The equation (31) , in matrix form is written:

$$[M^*]\{\ddot{q}\} + [C^*]\{\dot{q}\} + [K^*]\{q\} + [\phi][M]\{\ddot{y}_2\} = \{P^*\} \tag{36}$$

With:

$$[M^*] = [m_{ij}]\ ;[C^*] = [c_{ij}]\ ;[K^*] = [k_{ij}]\ ;\{P^*\} = \{P_i(t)\}\ ; \qquad i,j\ 1,2,\ldots,n \tag{37}$$

$$[\phi] = [\phi_i(x_v)];\ [M] = diag[M_{v2}];\ \{\ddot{y}_2\}\{\ddot{y}_{v2}\} \qquad i = 1,2,\ldots,n, v = 1,2,\ldots,nv \tag{38}$$

m_{ij} : the masses

c_{ij} : The shock absorbers

k_{ij} : stiffness

P_i : generalized forces

Let's put the equations of motion together (17) And (36) in the following matrix form:

$$\begin{bmatrix}[M^*] & [\phi][M]\\ [0] & [M]\end{bmatrix}\begin{Bmatrix}\ddot{q}\\ \ddot{y}_2\end{Bmatrix} + \begin{bmatrix}[C^*] & [0]\\ -[C][\phi]^T & [C]\end{bmatrix}\begin{Bmatrix}\dot{q}\\ \dot{y}_2\end{Bmatrix}$$

$$+\begin{bmatrix}[K^*] & [0]\\ -[K][\phi]^T - V[C][\phi']^T & [K]\end{bmatrix}\begin{Bmatrix}q\\ y_2\end{Bmatrix} = \begin{Bmatrix}\{P^*\}\\ [K]\{r\} + V[C]\{r'\}\end{Bmatrix} \tag{39}$$

To solve the coupled equations of bridge-convoy movement, there are two methods:

- The coupled method which consists of coupling the physical DOF of the vehicles with the modal variables of the bridge (39) using the modal method and direct integration

- The decoupled method which consists of solving the two coupled systems of equations (17)And (36)in a decoupled manner. In this case, an iterative calculation process seeks the dynamic balance of the interaction forces between the bridge and the vehicles at each moment. We solve the system of coupled equations (17)And (36), in a way decoupled by the Newmark method whose unknowns are the $q_j(j = 1, \dots, n)$, and y_{2v} $(v = 1, \dots, nv)$.

2-8 Numerical resolution of the equations of motion [21]:

To solve the coupled bridge-vehicle equations of motion (17) And (36), we use the Newmark method. The equation of motion of the bridge (36) has $t + \Delta t$ is written:

$$[M^*]\{\ddot{q}\}_{t+\Delta t} + [C^*]\{\dot{q}\}_{t+\Delta t} + [K^*]\{q\}_{t+\Delta t} + [\phi][M]\{\ddot{y}_2\}_{t+\Delta t} = \{P^*\}_{t+\Delta t} \qquad (40)$$

Using Newmark's method, the generalized displacements and velocities respectively are:

$$\{q\}_{t+\Delta t} = \{q\}_t + \Delta t\{\dot{q}\}_t + \Delta t^2(0{,}5 - \beta)\{\ddot{q}\}_t + \beta\Delta t^2\{\ddot{q}\}_{t+\Delta t} \qquad (41)$$

$$\{\dot{q}\}_{t+\Delta t} = \{\dot{q}\}_t + (1-\gamma)\Delta t\{\ddot{q}\}_t + \gamma\Delta t\{\ddot{q}\}_{t+\Delta t} \qquad (42)$$

Or γ And β are the stability parameters of the Newmark method, Δt and the time step of integration.

Let's replace the expressions (41) And (42) In (40), after factorization we obtain:

$$[S]\{\ddot{q}\}_{t+\Delta t} + [C^*]\{\dot{q}^*\}_{t+\Delta t} + [K^*]\{q^*\}_{t+\Delta t} + [M_2^*]\{\ddot{y}_2\}_{t+\Delta t} = \{P^*\}_{t+\Delta t} \qquad (43)$$

With

$$[S] = [M^*] + \gamma\Delta t[C^*] + \beta\Delta t^2[K^*]; [M_2^*] = [\phi][M] \qquad (44)$$

$$\{\dot{q}^*\}_{t+\Delta t} = \{\dot{q}\}_t + (1-\gamma)\Delta t\{\ddot{q}\}_t \qquad (45)$$

$$\{q^*\}_{t+\Delta t} = \{q\}_t + \Delta t\{\dot{q}\}_t + (0{,}5 - \beta)\Delta t^2\{\ddot{q}\}_t \qquad (46)$$

Multipions (43) about $[S]^{-1}$, we obtain:

$$\{\ddot{q}\}_{t+\Delta t} = \{P\}_{t+\Delta t} - [U]\{\dot{q}^*\}_{t+\Delta t} - [V]\{q^*\}_{t+\Delta t} - [Z]\{\ddot{y}\}_{t+\Delta t} \qquad (47)$$

With

$$\{P\}_{t+\Delta t} = [S]^{-1}\{P^*\}_{t+\Delta t};\ [U] = [S]^{-1}[C^*]; [V] = [S]^{-1}[K^*]; [Z] = [S]^{-1}[M_2^*] \qquad (48)$$

The equation of motion of mass M_v has $t + \Delta t$ is written:

$$[M]\{\ddot{y}_2\}_{t+\Delta t} + [C]\{\dot{y}_2\}_{t+\Delta t} + [K]\{y_2\}_{t+\Delta t} - [C][\phi]^T\{\dot{q}\}_{t+\Delta t} - ([K][\phi]^T + V[C][\phi']^T)\{q\}_{t+\Delta t} = [K]\{r\}_{t+\Delta t} + V[C]\{r'\}_{t+\Delta t} \qquad (49)$$

Using Newmark's method, the vehicle speeds and movements respectively are:

$$\{\dot{y}_2\}_{t+\Delta t} = \{\dot{y}_2\}_t + (1-\gamma)\Delta t\{\ddot{y}_2\}_t + \gamma\Delta t\{\ddot{y}_2\}_{t+\Delta t} \qquad (50)$$

$$\{y_2\}_{t+\Delta t} = \{y_2\}_t + \Delta t\{\dot{y}_2\}_t + (0{,}5-\beta)\Delta t^2\{\ddot{y}_2\}_t + \beta\Delta t^2\{\ddot{y}_2\}_{t+\Delta t} \quad (51)$$

Let's replace the expressions (50) and (51) in equation (49), after factorization, we obtain

$$[T]\{\ddot{y}_2\}_{t+\Delta t} + [C]\{\dot{y}_2^*\}_{t+\Delta t} + [K]\{y_2^*\}_{t+\Delta t} - [A]\{\dot{q}\}_{t+\Delta t} - [G]\{q\}_{t+\Delta t} = \{R\}_{t+\Delta t} \quad (52)$$

With

$$[T] = [M] + \gamma\Delta t[C] + \beta\Delta t^2[K];\ [A] = [C][\phi]^T;\ [G] = [K][\phi]^T + v[C][\phi'] \quad (53)$$

$$\{R\}_{t+\Delta t} = [K]\{r\}_{t+\Delta t} + V[C]\{r'\}_{t+\Delta t} \quad (54)$$

$$\{\dot{y}_2^*\}_{t+\Delta t} = \{\dot{y}_2\}_t + (1-\gamma)\Delta t\{\ddot{y}_2\}_t \quad (55)$$

$$\{y_2^*\}_{t+\Delta t} = \{y_2\}_t + \Delta t\{\dot{y}_2\}_t + (0{,}5-\beta)\Delta t^2\{\ddot{y}_2\}_t \quad (56)$$

Let's multiply the equation (52) about$[T]^{-1}$, we obtain:

$$\{\ddot{y}_2\}_{t+\Delta t} = \{R^*\}_{t+\Delta t} - [T^*]\{\dot{y}_2^*\}_{t+\Delta t} - [U^*]\{y_2^*\}_{t+\Delta t} + [A^*]\{\dot{q}\}_{t+\Delta t} + [G^*]\{q\}_{t+\Delta t} \quad (57)$$

With

$$\{R^*\}_{t+\Delta t} = [T]^{-1}\{R\}_{t+\Delta t};\ [T^*] = [T]^{-1}[C];\ [U^*] = [T]^{-1}[K]$$

$$[A^*] = [T]^{-1}[A];\ [G^*] = [T]^{-1}[G] \quad (58)$$

Conclusion

The vibration model presented in this chapter provides a comprehensive framework for analyzing the dynamic behavior of a multi-span bridge during convoy passage, we have gone through the different elements to establish the modeling of the essential parts of this study, a road bridge during a passage of a convoy, roadway profile, vehicle, and the equations necessary, equations of movement of the vehicle, the bridge motion equation, and lastly the method used to perform the numerical resolution of the equations of motion, by precising and defining the mathematical tools, it will enable us to run the simulation and analyze the results.

CHAPTER IV

Results and discussions

1-INTRODUCTION

The bridge-convoy interaction is an important phenomenon, generates several undesirable effects due to the vibrations of the structure and of the vehicle. The dynamic behavior of road bridges has become a necessity in the field of engineering of engineering structures either to take measures during the design to minimize these effects or to maintain these structures.

In this chapter, we present a study of the dynamic behavior of a road bridge modeled by a continuous equivalent multi-span beam simply supported, excited by the passage of vehicles. Several physical parameters can be studied: the displacement, the vertical acceleration and speed, the bridge-vehicle interaction forces and the damping factor.

Due to the difficulties of experimental validation of the results, the numerical simulations are carried out under MATLAB.

2-Numerical example of validation

In this example, we study the dynamic behavior of a beam on two simple supports under the effect of a mobile vehicle with two degree of freedom **Figure1**

The vehicle speed is assumed to be constant $V = 80\ km/h$. The equivalent beam of length $L = 60\ m$, mass per unit length $m = 230000\ kg/m$, quadratic moment $I = 2.9\ m^4$ and a Young's modulus $E = 2.87 \times 10\ ^9 N/m^2$ The modeling of the bridge is general enough to be able to introduce the number of intermediate supports and vehicles **Table 1**

The vehicles of the convoy studied are of mass $Mv_2 = 6000\ kg$ and of mass $Mv_1 = 200\ kg$ (wheel mass). The stiffness and the damping of the viscoelastic suspension are respectively, $\mathrm{k_v} = 1.595 \times 10^6\ \mathrm{N/m}$ and $\mathrm{c_v} = 0$. For a bridge simply supported at its ends, the convergence takes place for a number of modes $\mathrm{n} = 2, 6, 8$ and 12,for a number of intermediate supports $\mathrm{N} = 0$. **Table 2**

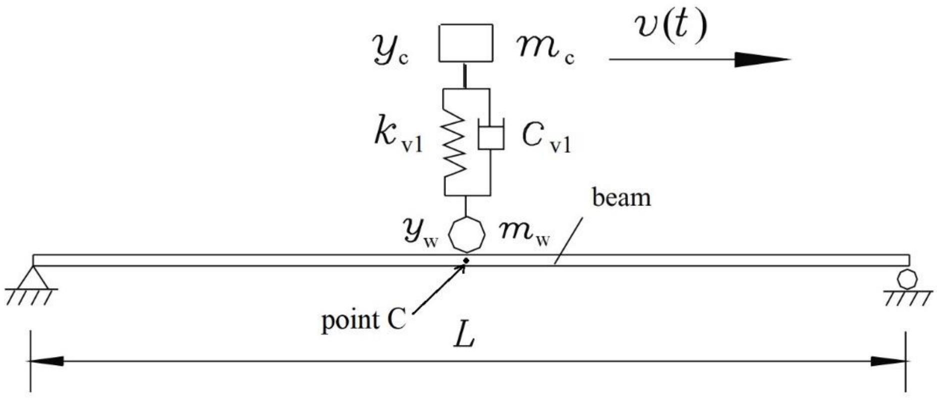

Figure IV -1: A simply supported beam subjected to the displacement of a vehicle with one axle

Table 1. Parameters of the bridge model (continuous beam)

Data	Values	Unit
Length L	60	m
Mass m	230000	kg/m
Quadratic moment I	2.9	m^4
Young's modulus E	2.87×10^9	$N/m2$

Table 2. Parameters of the vehicle model

Data	Values	Unit
Mass Mv_2	6000	kg
Mass Mv_1	200	kg
The stiffness k_v	1.595×10^6	N/m
The damping c_v	0	Ns/m

Figs 2.3 and 4 respectively represent the displacement, speed and the vertical acceleration in the middle of the bridge **(Point C)** of the beam as a function of the travel time.

Figs. 5.6 and 7 show the vertical displacement speed and acceleration of the vehicle **(Masse $\boldsymbol{Mv_2}$)** as a function of time. It is noted that the results obtained from the present study are in excellent agreement

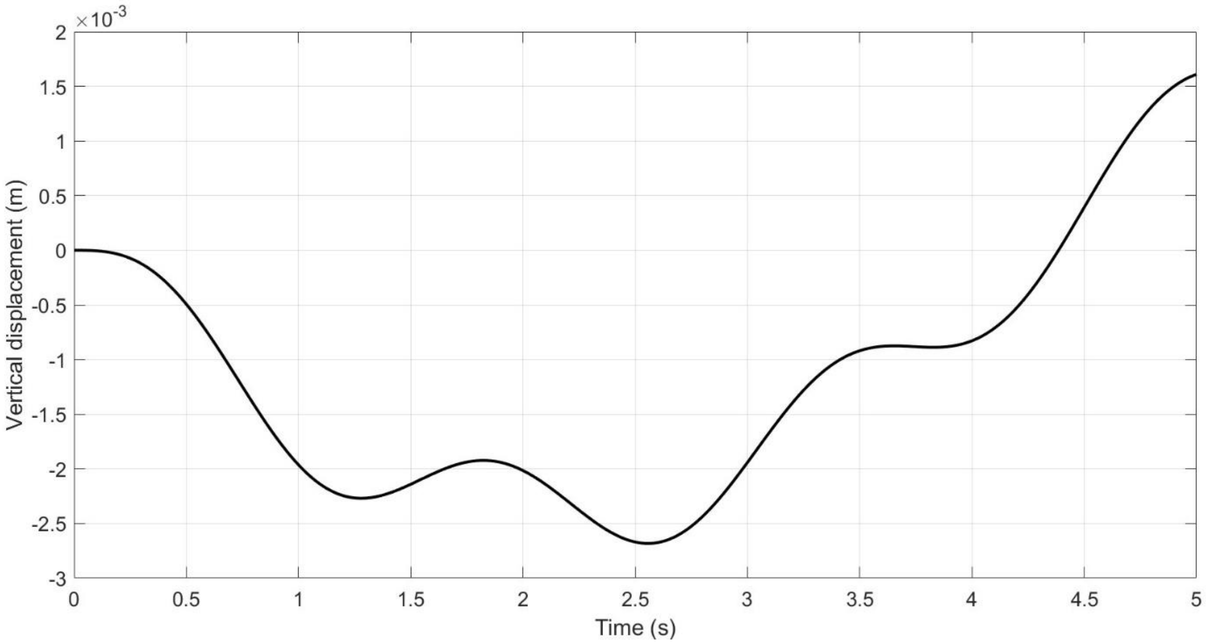

Figure IV-2: Vertical displacement in the middle of the bridge **(Point C)** as a function of time

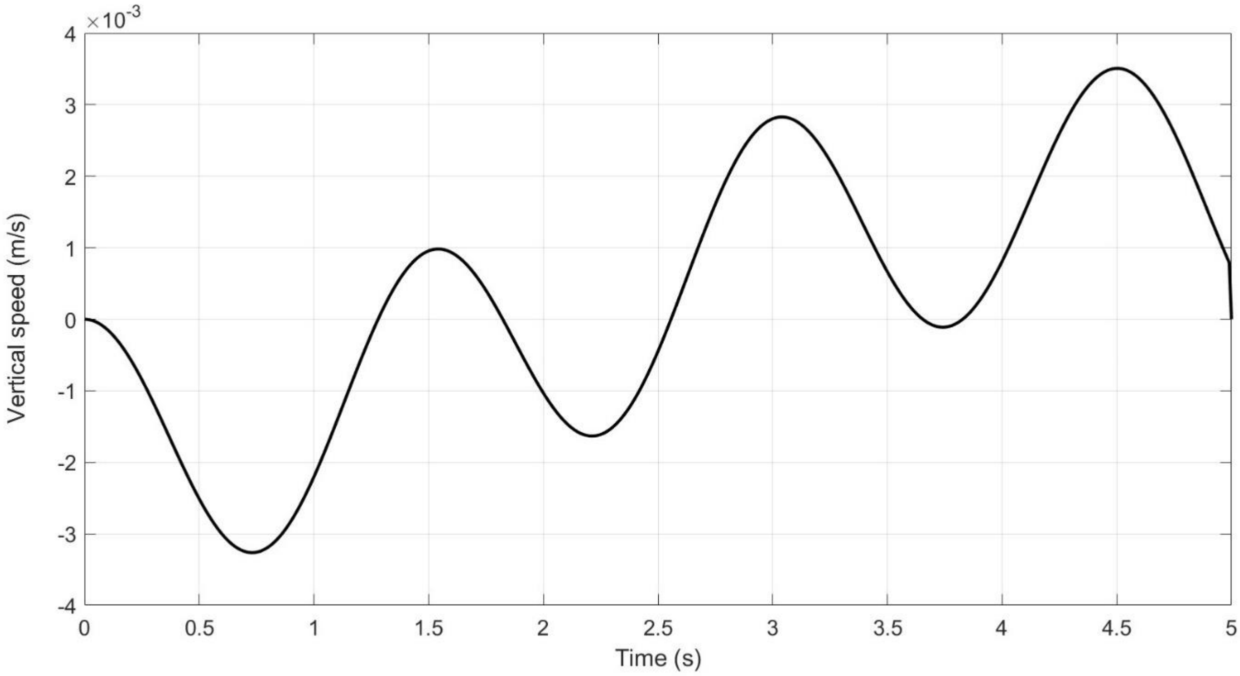

Figure IV-3: Vertical speed from the bridge to the middle **(Point C)** as a function of time

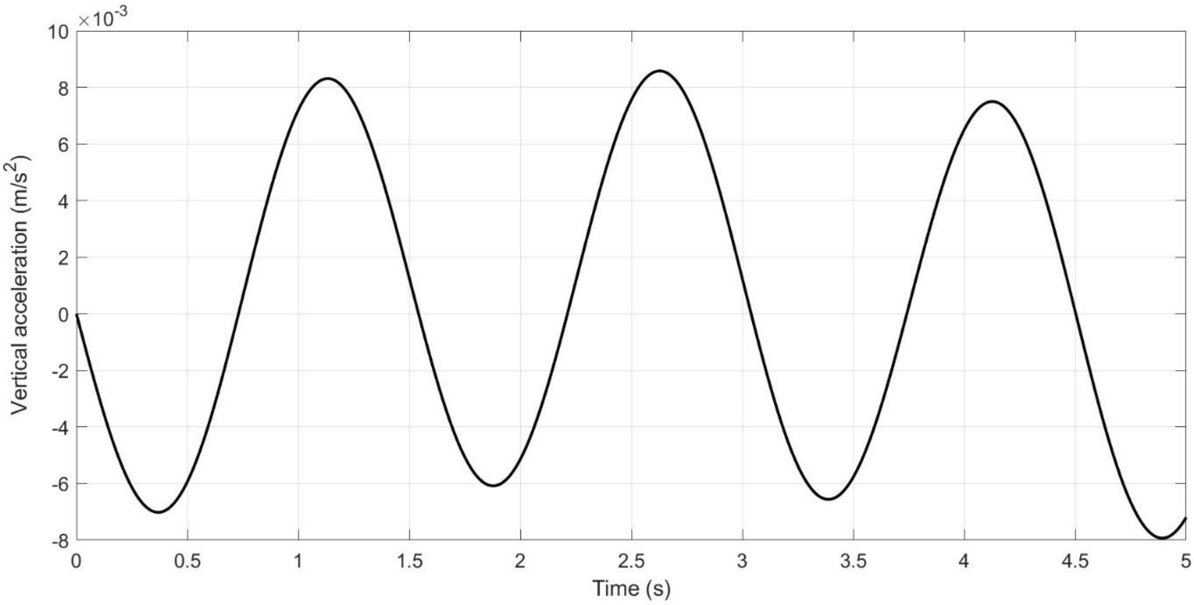

Figure IV-4: Vertical acceleration in the middle **(Point C)** of the bridge as a function

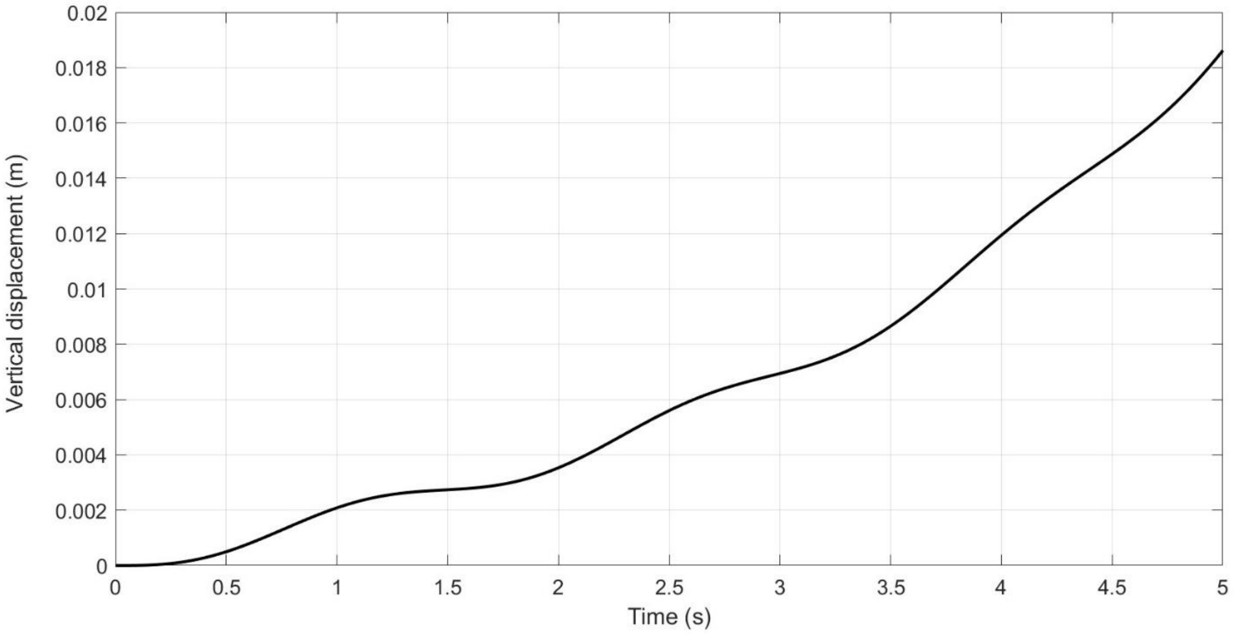

Figure IV-5: Vertical displacement of **the vehicle body Mv_2** as a function of time

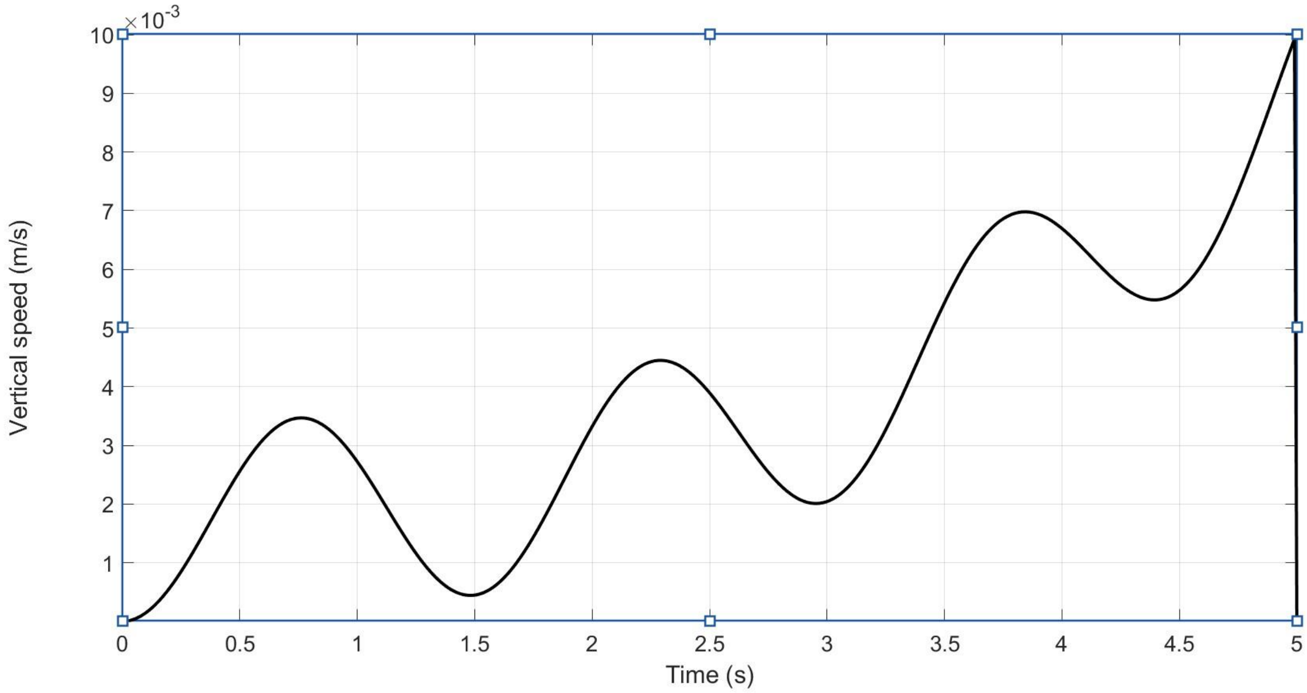

Figure IV-6: Vertical speed of **the vehicle body Mv_2** as a function of time

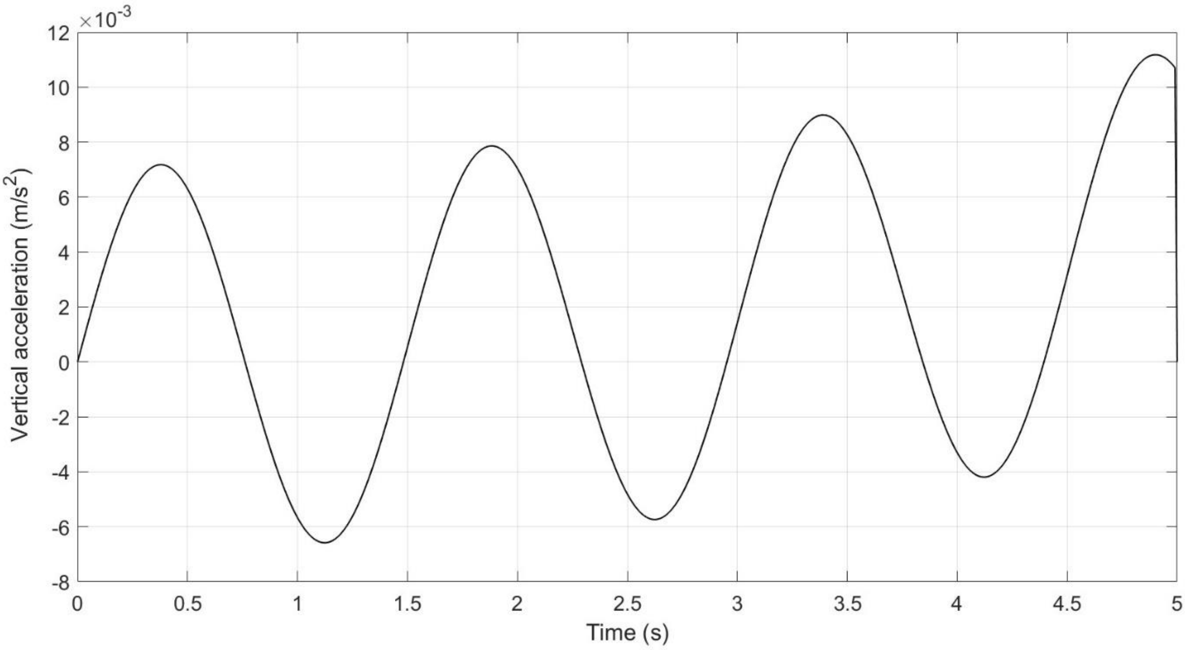

Figure IV-7: Vertical acceleration of **the vehicle body Mv_2** as a function of time

3-Influence of the damping factor on the dynamic behavior of the convoy

The numerical resolution of the system of equations of coupled motion is carried out using the Newmark average acceleration algorithm with an iterative calculation, without taking into account the defects in the shape of the running track and keeping the speed of the rolling of the convoy constant. As described in the previous chapter.

Based on previous mathematical developments, we have developed a software in MATLAB language, allowing the numerical resolution of the two coupled equations of motion, using the Newmark average acceleration algorithm $(\gamma = 0{,}5)$ et $(\beta = 0{,}25)$ with an iterative calculation. This scheme is unconditionally stable, of maximum precision; however, the choice of the time step is not limited. The results of this article do not take into account the shape defects of the raceway, given the complexity of defining the profile $R(X)$

In this case, we have modified the parameters of the bridge-convoy system.

The vehicle speed is assumed to be constant $V = 20\ m/s$. The structure of the bridge tested is modeled by an equivalent beam of length $L = 60\ m$, mass per unit length $m = 35000\ kg/m$, quadratic moment $I = 3.81\ m^4$ and a Young's modulus $E = 29 \times 10^9$ N/m^2 The modeling of the bridge is fairly general to be able to introduce the number of intermediate supports and vehicles, the number of intermediate supports $N = 0$

The vehicles of the studied convoy are of mass $Mv_2 = 2000\ kg$ kg and of mass $Mv_1 = 200\ kg$ (masse de roue). The stiffness and the damping of the viscoelastic suspension are respectively $k_v = 9.10^6\ N/m$ and $c_v = 1.8 \times 10^5\ Ns/m$. For a bridge simply supported at its ends, the convergence takes place for a number of modes $n = 2, 6, 8\ et\ 12$

Table 3. Parameters of the bridge model (continuous beam)

Data	Values	Unit
Length L	60	m
Mass m	35000	kg/m
Quadratic moment I	3.81	m^4
Young's modulus E	29×10^9	$N/m2$

Table 4. Parameters of the vehicle model

Data	Values	Unit
Mass Mv_2	2000	kg
Mass Mv_1	200	kg
The stiffness k_v	9×10^6	N/m
The damping c_v	1.8×10^5	Ns/m

The bridge displacement is being calculated at each time step for along the bridge as the vehicle move across it. This means the bridge displacement is being updated for vehicle position as it traverses the bridge (The point of contact between the bridge and the vehicle)

Figs 8 to 11 show the influence of the viscous damping factor of the suspensions on the displacements, the speeds, the accelerations and the interaction force of the vehicle, for a vehicle speed $V = 20\ m/s$

The essential remarks that can be drawn from these figures are:

- The significant decrease in the amplitudes for damping coefficients of ξv = 0,15 and 0,2 in comparison with the case without damping.

- The disappearance with the damping of the dynamic amplification of the vertical displacements, speeds and accelerations of the vehicles which manifests itself for which ξv =0.

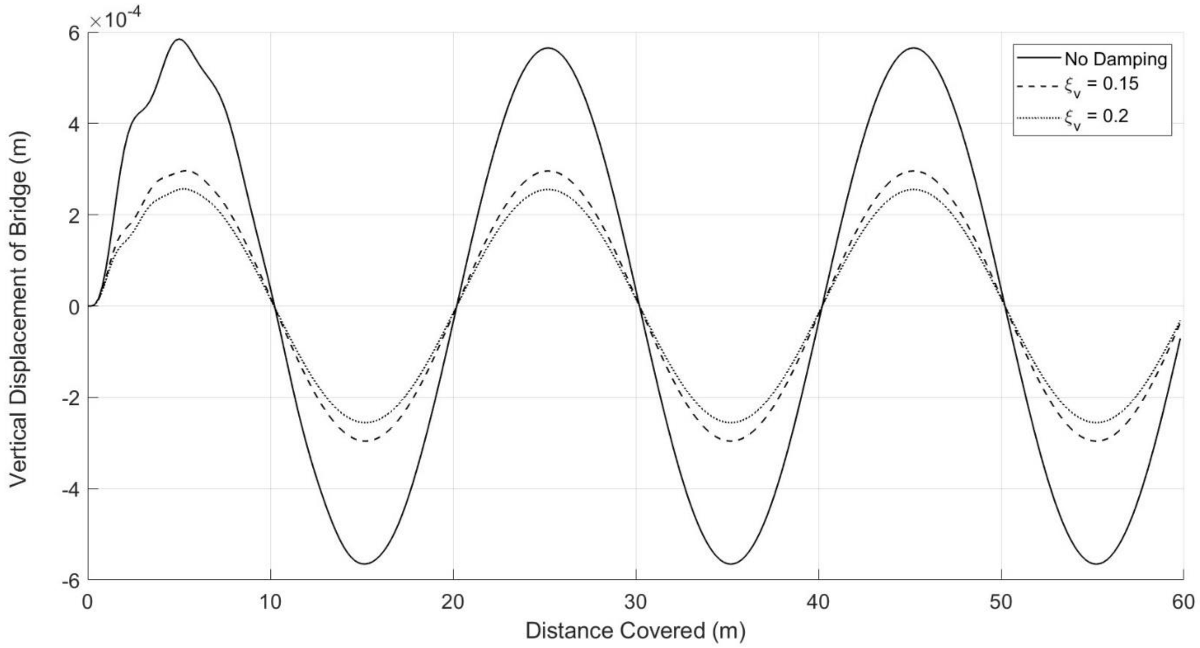

Figure IV-8: Influence of Damping Factor on Bridge Displacement

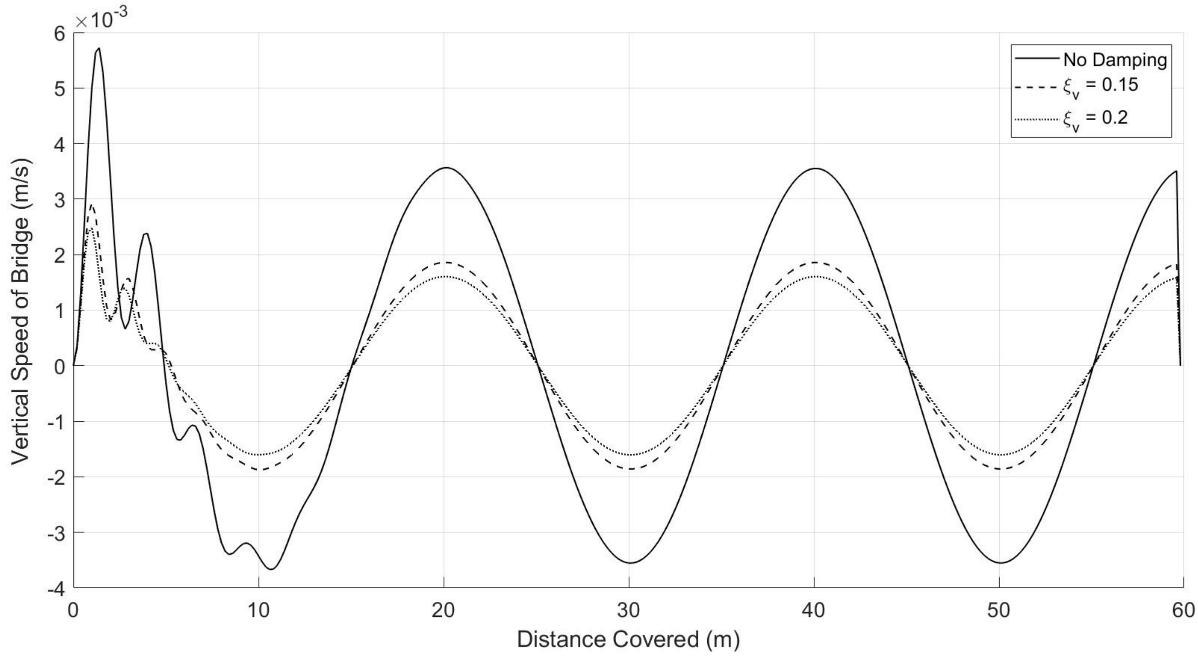

Figure IV-9: Influence of Damping Factor on Bridge Vertical Speed

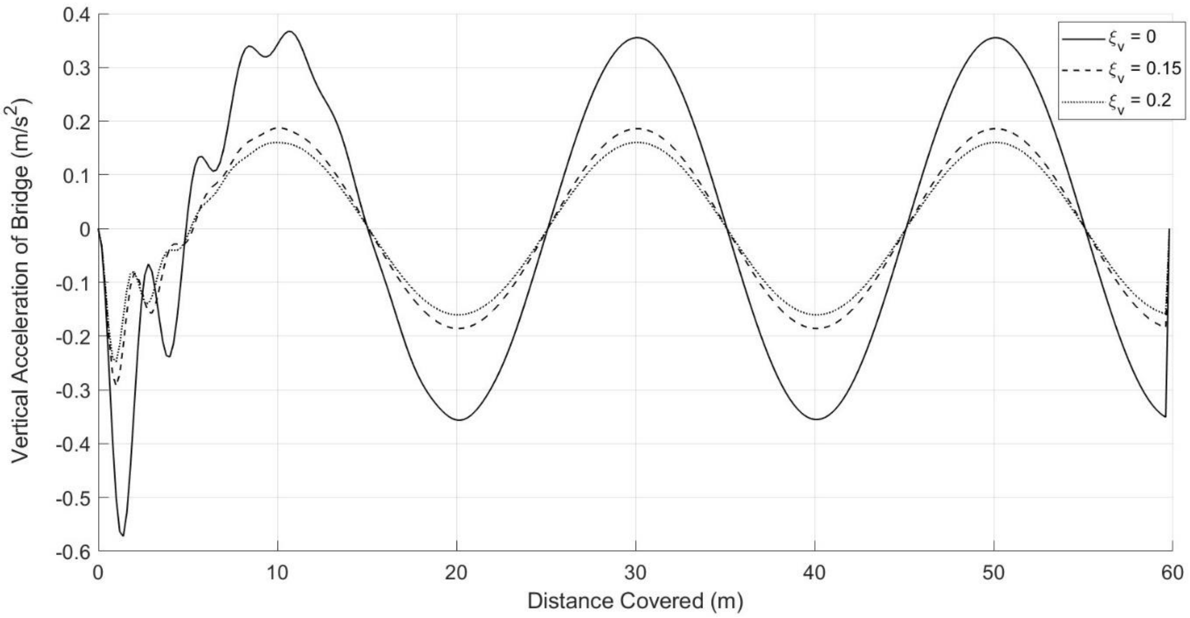

Figure IV-10: Influence of Damping Factor on Bridge Vertical Acceleration

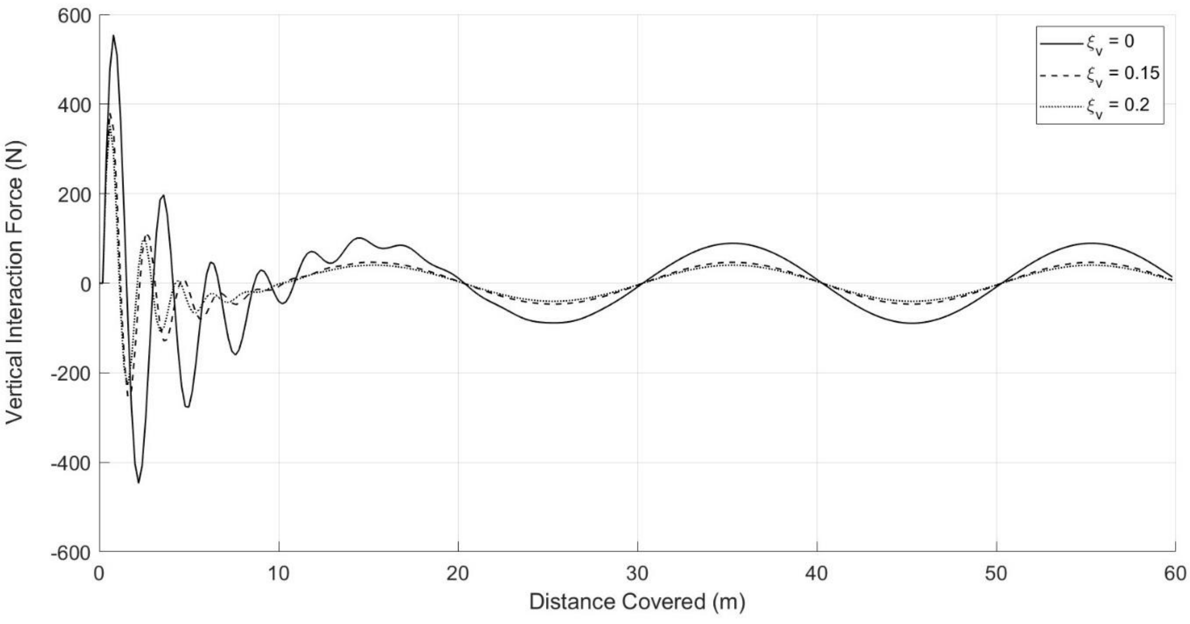

Figure IV-11: Influence of Damping Factor on Vertical Interaction Force

4-Influence of the Vehicle speed on the dynamic bridge-convoy behavior

Figs. 12-13 and 14 represent the influence of the speed of the vehicle on the vertical displacement of the bridge under the wheel of the vehicle respectively for three Vehicle speeds of the convoy (40, 80 and 120 km/h), $\xi_v = 0, N = 0$

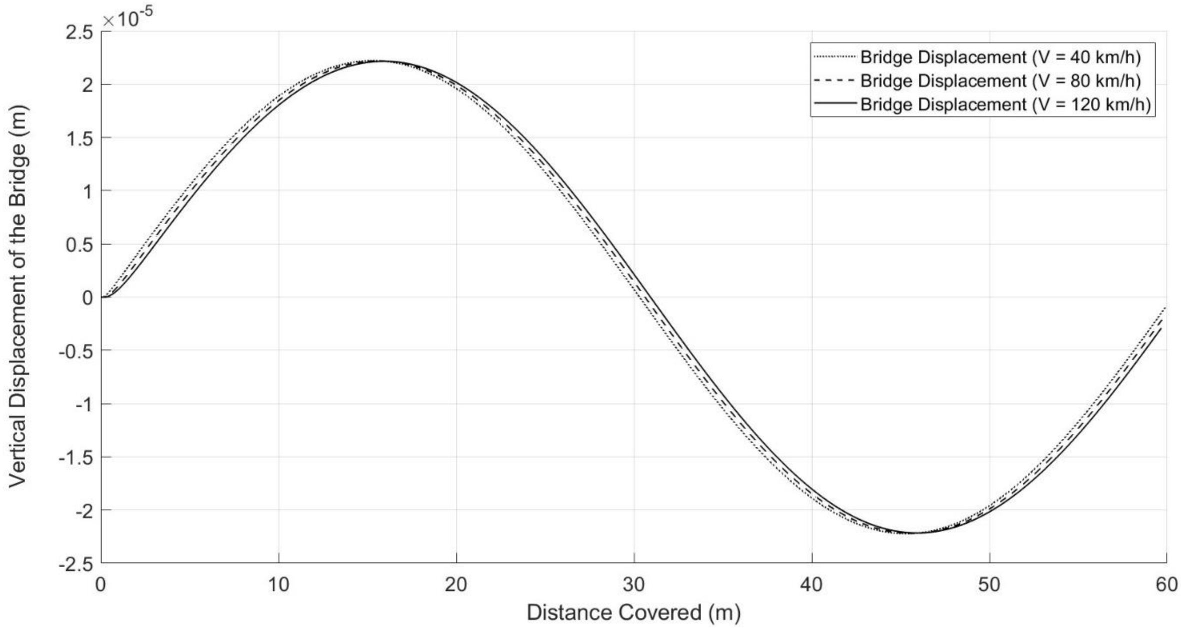

Figure IV-12: Influence of Vehicle Speeds on the Vertical Displacement of the Bridge

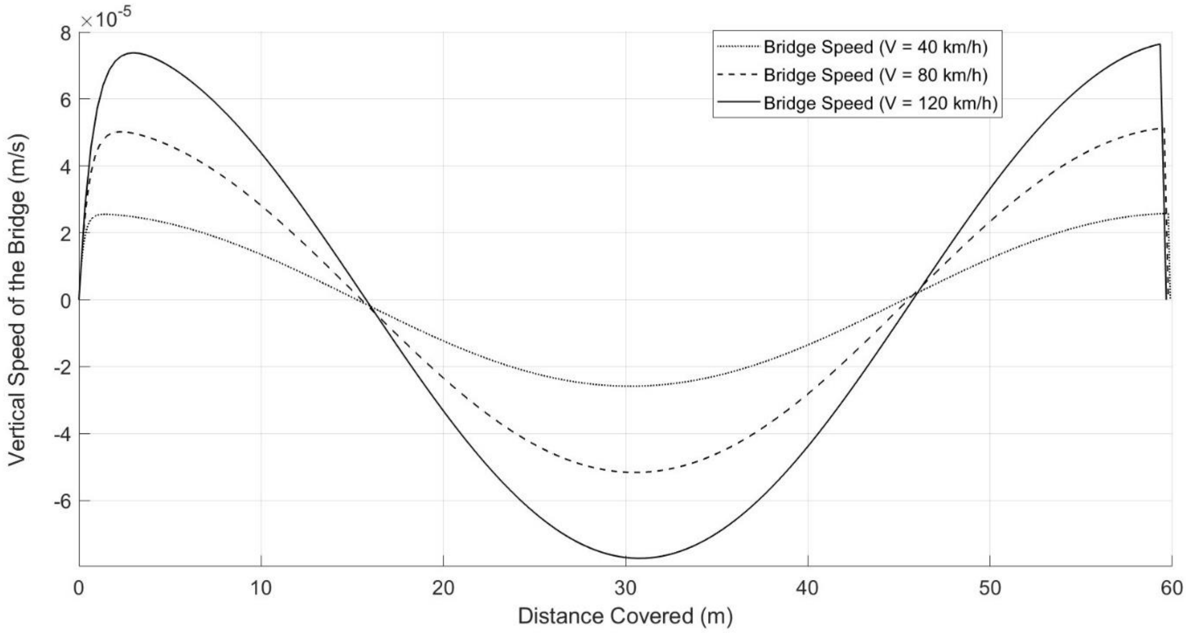

Figure IV-13: Influence of Vehicle Speeds on the Vertical speed of the Bridge

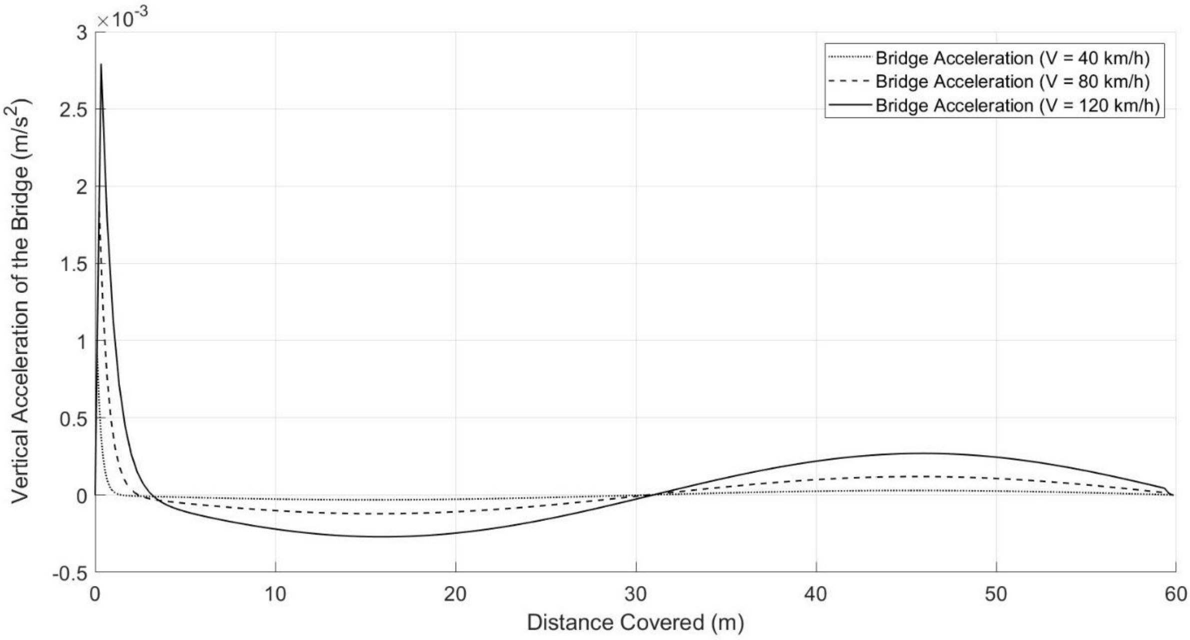

Figure IV-14: Influence of Vehicle Speeds on the Vertical Acceleration of the Bridge

5-Influence of the number of vehicles on the dynamic bridge-convoy behavior

We respectively present in Figs. 6-8 the vertical displacement bridge-convoy as well as the force of interaction between them for a bridge of 60 m in length, without intermediate supports $(N = 0)$ and for a damping coefficient $\xi_v = 0.15$

The number of iterations that allows the convergence of the dynamic bridge-convoy interaction increases with the increase in the number of vehicles on the bridge and decreases with the increase in the running speed of the convoy

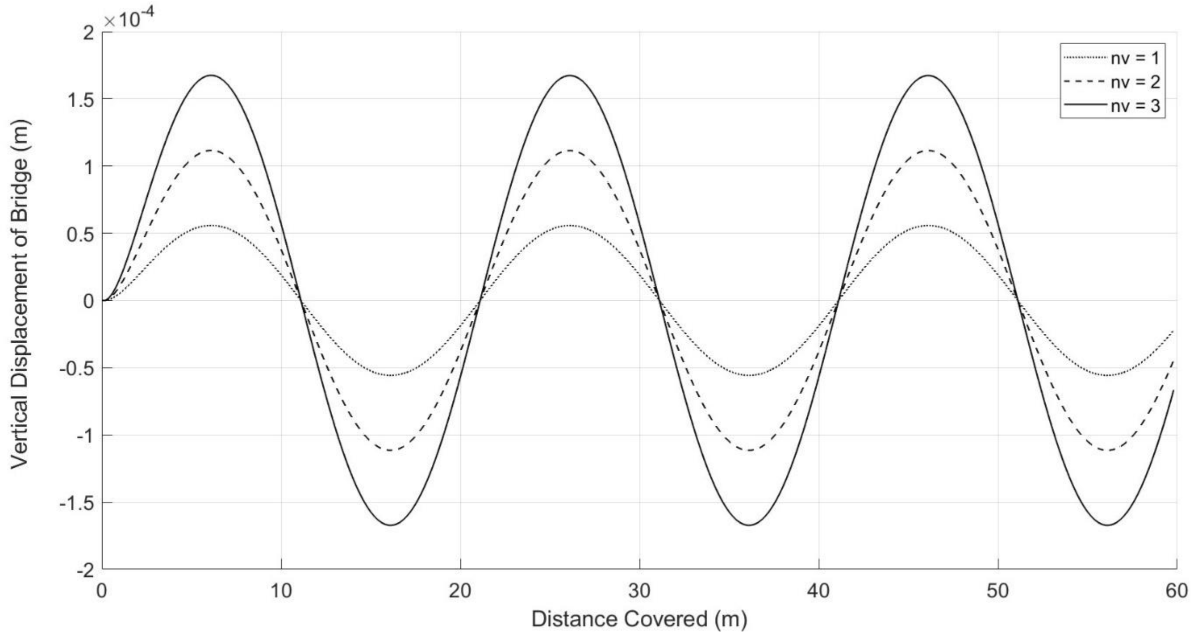

Figure IV-15: Influence of Number of Vehicles on Vertical Displacement of the Bridge

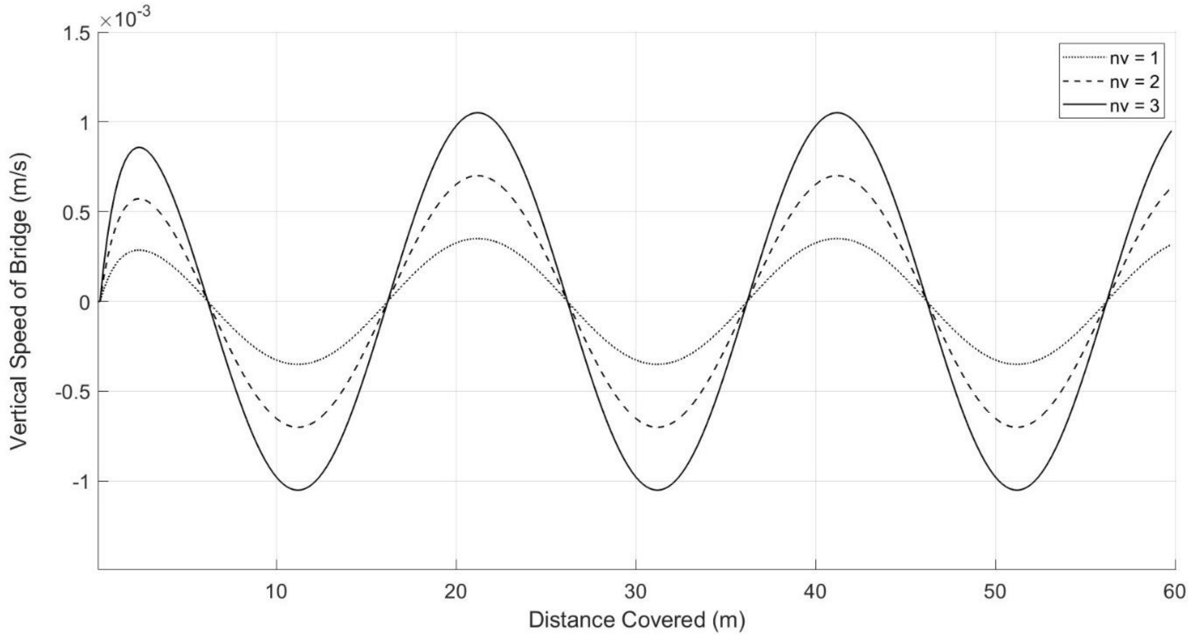

Figure IV-16: Influence of Number of Vehicles on Vertical Speed of the Bridge

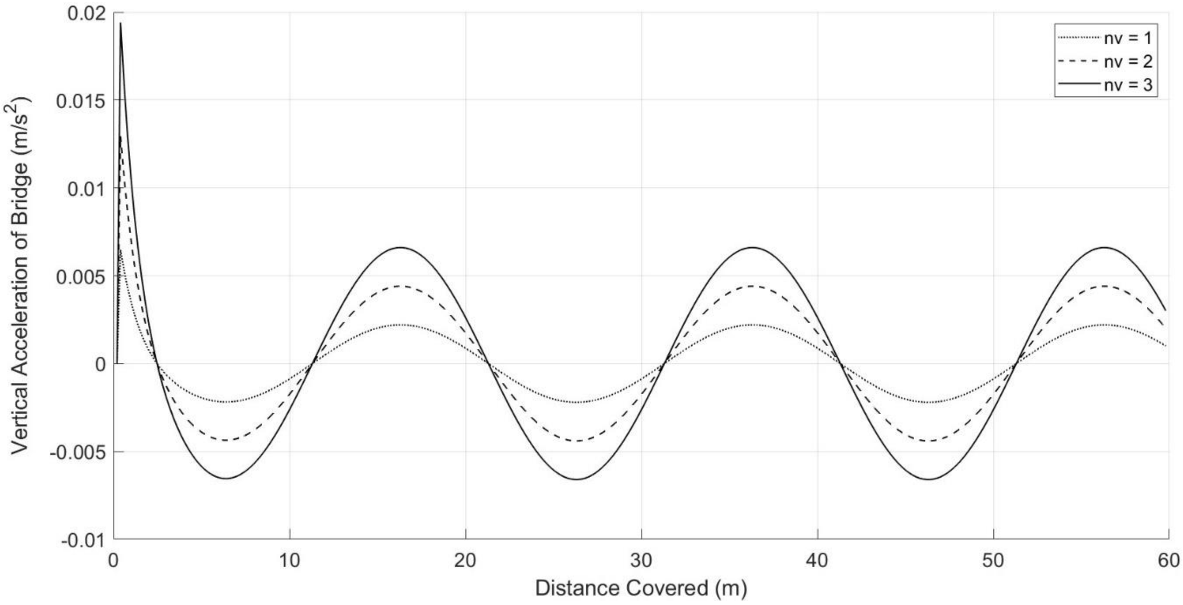

Figure IV-17: Influence of Number of Vehicles on Vertical Acceleration of the Bridge

Conclusion

In this chapter, the dynamic behavior of a multi-span road bridge under convoy passage is studied through numerical simulations using MATLAB. after establishing the mathematical tools necessary in the previous chapter, several physical parameters were studied, such as the displacement, the vertical acceleration and speed, the bridge-vehicle interaction forces and the damping factor, the focus is on understanding how varying the damping factor and convoy speed influence structural response. Results show that increasing damping reduces bridge displacement, vertical speed, acceleration, and interaction forces, indicating its crucial role in mitigating structural vibrations and enhancing durability. these results hightlights how different factors can dictate the dynamic response of a structure.

Conclusion general

this thesis has given both a general overview and a detailed overview with regards to conducting a study to look at the dynamic behavior of bridges loaded with moving traffic loads, after defining the phenomenon created as a result of the movements of the vehicle or vehicles and taking a look at the factors at play, understanding the relationship between the dynamic behavior of the structure and the factors coupled with the change in parameters showcases the patterns with which we can safely predict the behavior of the structure, in the process of doing so the complexity of the study required a unified model which can be used for different elements in the study to practically introduce mathematical and numerical tools to simulate the influence of the factors at hand. the mathematical and numerical tools made it possible for us to effectively reduce the complexity and integrate real life elements to a computerized simulation, The findings underscore the significant impact of increased damping, which effectively minimizes bridge displacement, vertical speed, acceleration, and interaction forces. This highlights the critical role of damping in reducing structural vibrations and bolstering durability. Moreover, these results illuminate how various factors influence the dynamic response of bridge structures, providing valuable insights into optimizing their performance under dynamic loading conditions.

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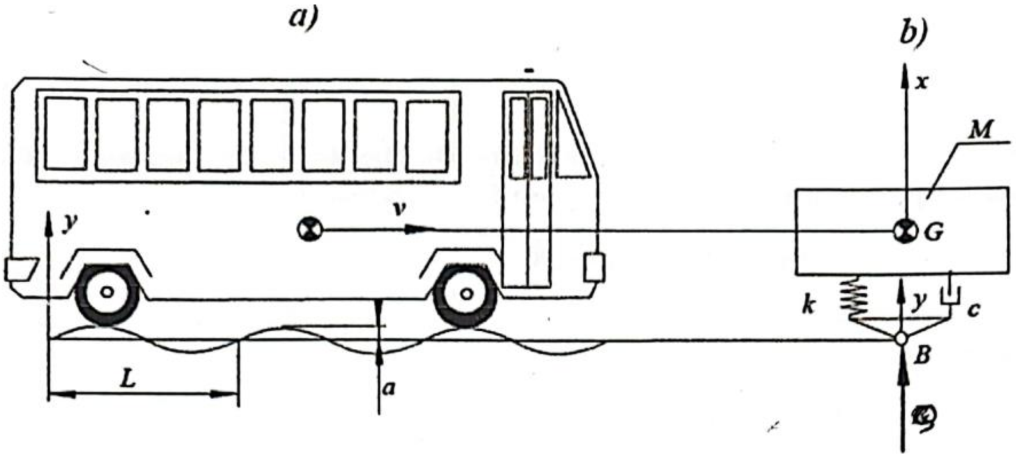
Annexe 1

CINEMATIC EXCITATION

The figure shows an example of a physical model of cinematic excitation.



the vertical movement of point **B** along the $\boldsymbol{y}$ axis generates vibrations in the vertical mass block **M**. this model is used to analyze the vertical vibrations of a road vehicle moving on a road in the profile of the surface condition possibly approached by a harmonic function. (in the form of a sine or cosine)

the bus is represented by the mass block **M** and the suspension elements of the bus have the following characteristics:

K: stiffness constant

C: damping coefficient

The surface condition of the road presents undulations of period **L** and amplitude **a**, the bus is moves with a constant speed **v**

Putting into equation:

From the period of the road profile, the corresponding pulse is sought, the length **L** is given by:

$$L = vT \ \text{ from } T = \frac{L}{v} \tag{1}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi v}{L} \tag{2}$$

And the movement of point **B** along the y axis can be represented by:

$$y = a \sin \omega t \tag{3}$$

The equation of the movement of the bus is given by:

$$M\ddot{x} = -kx - c\dot{x} + ky + c\dot{y} \tag{4}$$

Introducing the expression (3) into (4) :

$$M\ddot{x} + kx + c\dot{x} = ka\sin\omega t + ca\omega\cos\omega t \tag{5}$$

Let's divide by the mass M,

$$\ddot{x} + 2\eta\omega_0\dot{x} + \omega_0^2 x = \omega_0^2 a\sin\omega t + 2\omega\eta\omega_0 a\cos\omega t = q\sin(\omega t + \alpha) \tag{6}$$

Or

$$q = a\omega_0^2\sqrt{1 + 4\eta^2\left(\frac{\omega}{\omega_0}\right)^2} \tag{7}$$

Without the generalization of the formulation being affected, we can adopt the model next mathematical by neglecting the phase α in equation (6).

$$\ddot{x} + 2\eta\omega_0\dot{x} + \omega_0^2 x = q\sin\omega t \tag{8}$$

The movement of the block **M** along the axis is done according to the law already studied in the case of a damped forced 1 dof system:

$$x = A\sin(\omega t + \varphi) \tag{9}$$

Or

$$A = \frac{\frac{q}{\omega_0^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + 4\eta^2\left(\frac{\omega}{\omega_0}\right)^2}} \qquad et \qquad \tan\varphi = \frac{2\eta\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2} \tag{10}$$

By introducing the expression of (7) giving **q** in that of the vibration amplitude **A** on find:

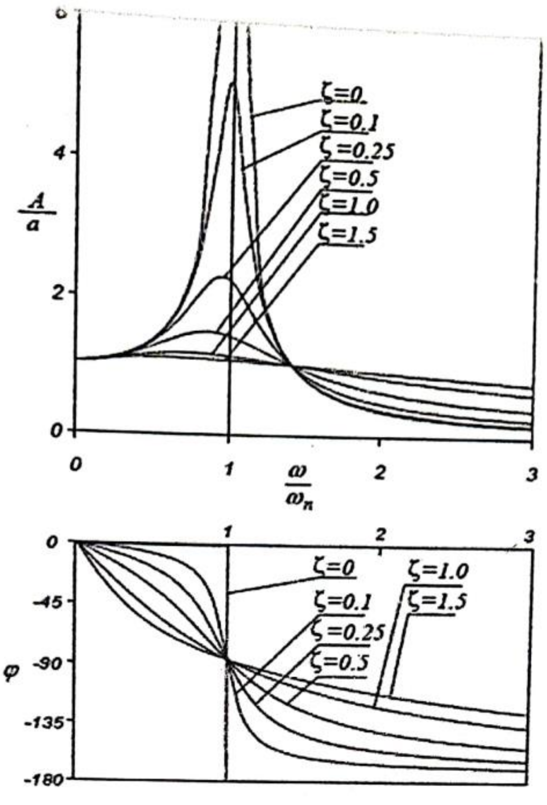
$$A = \frac{a\sqrt{1 + 4\eta^2\left(\frac{\omega}{\omega_0}\right)^2}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + 4\eta^2\left(\frac{\omega}{\omega_0}\right)^2}} \qquad et \qquad \tan\varphi = \frac{2\eta\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2} \tag{11}$$

If we introduce **TR** The transmissibility factor,

$$TR = \frac{A}{a};$$

$$TR = \frac{\sqrt{1+4\eta^2\left(\frac{\omega}{\omega_0}\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+4\eta^2\left(\frac{\omega}{\omega_0}\right)^2}} \tag{12}$$

The transmissibility factor **TR** which represents the ratio of the amplitude of the force transmitted on the amplitude of the excitation force as well as the phase φ are represented in the following graphs as a function of the ratio of the pulses $\tau = \frac{\omega}{\omega_0}$



For values of $\omega \geq 1.4\omega_0$, it is possible to have vibration amplitudes of the which are smaller than those of the cinematic excitation.

The expression of the force transmitted to the foundation is written from (4):

$$\begin{aligned} Q &= kx + cx + k\dot{y} - k\dot{y} \\ &= kA\sin(\omega t+\varphi) + c\omega A\cos(\omega t+\varphi) - ka\sin\omega t - c\omega a\cos\omega t \qquad (13) \\ &= |R|\sin(\omega t+\gamma) \end{aligned}$$

The problem of minimizing the reaction force or amplitude A is called: **Vibro-insulation**

Annexe 2

When a bridge is subjected to dynamic loading from a moving mass, the resulting mathematical representation of its motion is expressed through the equation of motion

$$[M^*]\{\ddot{q}\} + [C^*]\{\dot{q}\} + [K^*]\{q\} = \{P^*\}$$

With:

$$[q] = \{q_{11} \ldots q_{1m} \ldots q_{n1} \ldots q_{nm}\}^T$$

$$[M^*] = Diag(M_{11} \ldots M_{1m} \ldots M_{n1} \ldots M_{nm})$$

$$+M\begin{bmatrix} \phi_{11}^2 \cdots \phi_{11}\phi_{1m} \cdots \phi_{11}\phi_{n1} \cdots \phi_{11}\phi_{nm} \\ \vdots \\ \phi_{1m}\phi_{11} \cdots \phi_{1m}^2 \cdots \phi_{1m}\phi_{n1} \cdots \phi_{1m}\phi_{nm} \\ \vdots \\ \phi_{n1}\phi_{11} \cdots \phi_{n1}\phi_{1m} \cdots \phi_{n1}^2 \cdots \phi_{n1}\phi_{nm} \\ \vdots \\ \phi_{nm}\phi_{11} \cdots \phi_{nm}\phi_{1m} \cdots \phi_{nm}\phi_{n1} \cdots \phi_{nm}^2 \end{bmatrix}$$

$$[C^*] = 2Diag(\xi_{11}\omega_{11}M_{11} \ldots \xi_{1m}\omega_{1m}M_{1m} \ldots \xi_{n1}\omega_{n1}M_{n1} \ldots \xi_{nm}\omega_{nm}M_{nm})$$

$$+2Mv_x\begin{bmatrix} \phi_{11}\phi_{11}' \cdots \phi_{11}\phi_{1m}' \cdots \phi_{11}\phi_{n1}' \cdots \phi_{11}\phi_{nm}' \\ \vdots \\ \phi_{1m}\phi_{11}' \cdots \phi_{1m}\phi_{1m}' \cdots \phi_{1m}\phi_{n1}' \cdots \phi_{1m}\phi_{nm}' \\ \vdots \\ \phi_{n1}\phi_{11}' \cdots \phi_{n1}\phi_{1m}' \cdots \phi_{n1}\phi_{n1}' \cdots \phi_{n1}\phi_{nm}' \\ \vdots \\ \phi_{nm}\phi_{11}' \cdots \phi_{nm}\phi_{1m}' \cdots \phi_{nm}\phi_{n1}' \cdots \phi_{nm}\phi_{nm}' \end{bmatrix}$$

Annexe 3

The Newmark method is a one-step numerical integration method. On y calculate the state of the system at a given moment $t + \Delta t$ as a function of the state known at time t by the Taylor's formula:

$$\{q\}_{t+\Delta t} = \{q\}_t + \Delta t\{\dot{q}\}_t + \frac{\Delta t^2}{2}\{\ddot{q}\}_t + \cdots + \frac{\Delta t^n}{n!}\{q^{(n)}\}_t + \{R_{\mathrm{n}}\}$$

With the $\{Rn\}$ is the rest of the development of order n

$$\{R_n\} = \frac{1}{n!}\int_t^{t+\Delta t}\{q^{(n+1)}\}_\tau (t + \Delta t - \tau)^n\, d\tau$$

The formula makes it possible to calculate the speed and the displacement at the instant t

$$\{\dot{q}\}_{t+\Delta t} = \{\dot{q}\}_t + \int_t^{t+\Delta t}\{\ddot{q}\}_\tau\, d\tau$$

$$\{q\}_{t+\Delta t} = \{q\}_t + \Delta t\{\dot{q}\}_t + \int_t^{t+\Delta t}(t + \Delta t - \tau)\{\ddot{q}\}_\tau\, d\tau$$

The approximation therefore consists in calculating the integrals of the acceleration. For to do this, let's express the following: in the interval $\{\ddot{q}\}_\tau$, $[t, t + \Delta t]$as a function of $\{\ddot{q}\}_t$ and $\{\ddot{q}\}_{t+\Delta t}$ that’s at the terminals of the interval:

$$\{\ddot{q}\}_t = \{\ddot{q}\}_\tau + \{q^{(3)}\}_\tau (t - \tau) + \{q^{(4)}\}_\tau \frac{(t-\tau)^2}{2} + \cdots$$

$$\{\ddot{q}\}_{t+\Delta t} = \{\ddot{q}\}_\tau + \{q^{(3)}\}_\tau (t + \Delta t - \tau) + \{q^{(4)}\}_\tau \frac{(t + \Delta t - \tau)^2}{2} + \cdots$$

Let's multiply by $1 - \gamma$ and by γ respectively we obtain:

$$(1-\gamma)\{\ddot{q}\}_t = (1-\gamma)\{\ddot{q}\}_\tau + (1-\gamma)\{q^{(3)}\}_\tau (t-\tau) + (1-\gamma)\{q^{(4)}\}_\tau \frac{(t-\tau)^2}{2} + \cdots$$

$$\gamma\{\ddot{q}\}_{t+\Delta t} = \gamma\{\ddot{q}\}_\tau + \gamma\{q^{(3)}\}_\tau (t + \Delta t - \tau) + \gamma\{q^{(4)}\}_\tau \frac{(t+\Delta t-\tau)^2}{2} + \cdots$$

We find:

$$\{\ddot{q}\}_\tau = (1-\gamma)\{\ddot{q}\}_t + \gamma\{\ddot{q}\}_{t+\Delta t} + \{q^{(3)}\}_\tau (\tau - \gamma\Delta t - t) + \mathrm{O}(\Delta t^2\{q^{(4)}\}_\tau)$$

Annexe 4

Resolution Algorithm

1. Initialize Parameters

- Define the bridge parameters: length (L), mass per unit length (m), moment of inertia (I), Young's modulus (E).
- Define vehicle parameters: mass of the wheel (M_{v1}), mass of the vehicle (M_{v2}), stiffness of the suspension (k_v), damping factor (ξv).
- Calculate the damping of the suspension (c_v).
- Define convoy parameters: speed (V), Newmark parameters (gamma, beta), distance between vehicles.

2. Calculate Derived Parameters

- Natural frequency of the vehicle: $\omega_v = \sqrt{\left(\frac{k_v}{M_{v2}}\right)}$
- First natural frequency of the bridge: $\omega_{P,1} = \pi^2 \sqrt{\left(\frac{EI}{ml_1^4}\right)}$
- Time step: $dt = 0.01\ s$
- End time: $t_{end} = \frac{L}{V}$
- Number of time steps: $num\_steps = \frac{t_{end}}{dt}$

3. Set Up External Force Vector

- Define an example external force vector F_v that varies with time.

4. Loop Over Number of Vehicles

- Define the number of vehicles (nv) and iterate over different values (e.g., 1, 2, 3).

5. Initialize Displacement and Velocity Arrays

- Initialize arrays to store the vertical displacements of the bridge and each vehicle at each time step.

6. Calculate Initial Conditions

- Determine the initial positions of the vehicles based on the distance between them.
- Calculate the corresponding initial times.

7. Time Integration Using Newmark Method

- For each time step, compute the effective stiffness (K_{eff}) and damping (C_{eff}) for each vehicle.

- Update the displacement of each vehicle using the effective force.

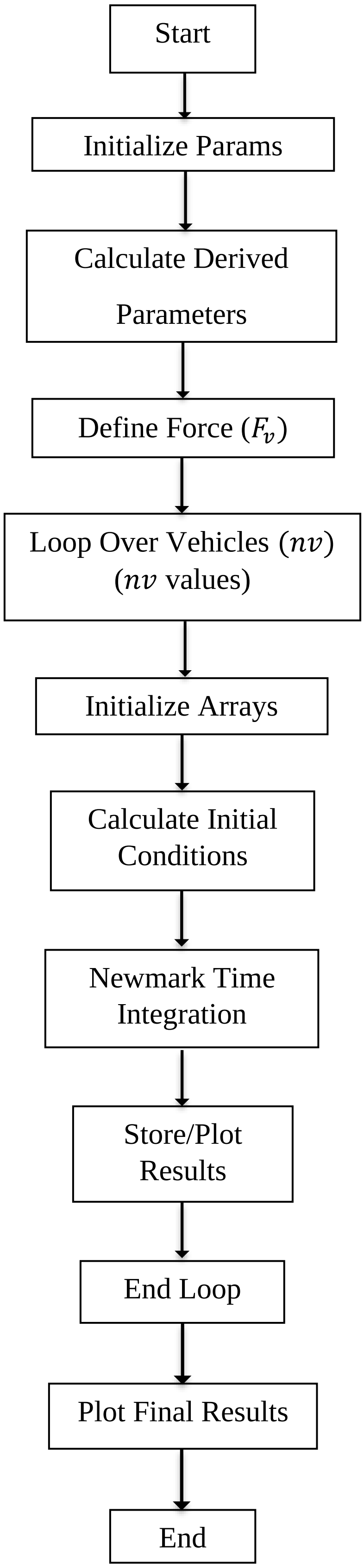
8. Calculate Total Displacement of Bridge

- Sum the displacements of all vehicles to get the total displacement of the bridge at each time step.

9. Plot Results

Plot the vertical displacement of the bridge as a function of the distance covered by the vehicles for each case.

Annexe 5


Start
Initialize Params
Calculate Derived Parameters
Define Force (F_v)
Loop Over Vehicles (nv) (nv values)
Initialize Arrays
Calculate Initial Conditions
Newmark Time Integration
Store/Plot Results
End Loop
Plot Final Results
End