



**NARAFEMTS**

National Agency  
for Finite Element  
Methods & Standards



## **A FINITE ELEMENT PRIMER**

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