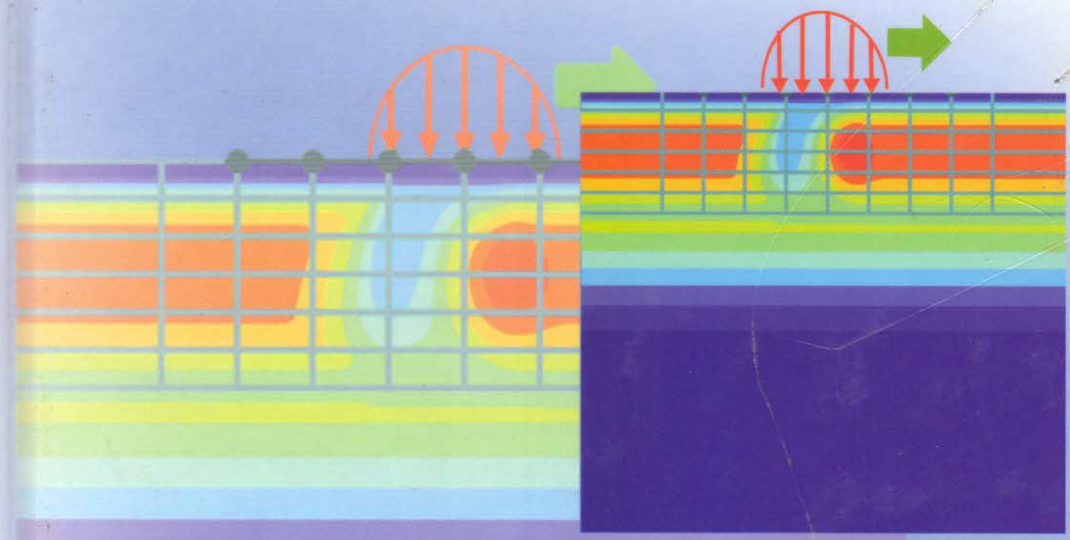
WILEY

Boundary Integral Equation Methods for Solids and Fluids



Marc Bonnet

Contents

Preface xi

Acknowledgements xiii

Introduction xv
References xix

1 Basic principles and fields of application 1
1.1 Basic principles 1
1.2 Overview of fields of application 3
1.3 Heat conduction 5
1.4 Fluid flow 6
1.5 Electromagnetism 8
1.6 Linear elastostatics 9

Part I STATICS

2 Integral equations for the Poisson equation 15
2.1 Integral representation 16
2.2 Boundary integral equation 23
2.3 Exterior problems and conditions at infinity 26
2.4 Indirect integral formulations 29

2.5 Two-dimensional problems 31
Appendix 2.A Some useful formulas 34
Appendix 2.B Derivation of the fundamental solution of the Laplacian 35
Appendix 2.C Integrability of $r_{,n}/r^2$ 37
References 38

3 Boundary element method for potential problems 39
3.1 Discretization of the boundary 39
3.2 Discretization of the unknowns 45
3.3 Assembly and solution of the discrete problem 46
3.4 Evaluation of nonsingular integrals 50
3.5 Evaluation of singular integrals 56
3.6 Evaluation of domain integrals 61
3.7 Using subregions 63
References 66

4 Integral equations and representations for elastostatics 67
4.1 Maxwell–Betti reciprocity identity 67
4.2 Fundamental solutions of linear elastostatics 68
4.3 Integral representation formulas 73
4.4 Displacement boundary integral equation 75
4.5 The boundary element method in elastostatics 80
4.6 Integral equations for exterior problems 82
4.7 The boundary element method 85
4.8 The direct approach for singular element integration 92
4.9 Elasticity in plane strain 95
Appendix 4.A The three-dimensional Kelvin solution 98
Appendix 4.B Integral of the Kelvin traction vector 102
Appendix 4.C Half-space fundamental solution 104
Appendix 4.D Fundamental solution for two bonded half-spaces 106
References 107

5 Representation of gradients and stresses on the boundary 109
5.1 Motivation 109
5.2 Definition using a limiting process 112
5.3 Second-order regularization 113
5.4 Integration by parts and first-order regularization 115
5.5 Extension to elastostatics 118
5.6 Indirect boundary integral equations 122
5.7 Comments 126
5.8 Boundary element discretization 128
5.9 The direct approach 131
Appendix 5.A Integral identities for the Kelvin solution 135

Appendix 5.B Proof of expression (5.16) of the free term 138
Appendix 5.C Proof of expression (5.28) for the elastic free term 139
References 141

6 Some classical general results 143
6.1 Fundamental problems of potential theory 143
6.2 Singular boundary integral equations of elasticity theory 144
6.3 Green tensors for bounded domains 145
6.4 Fundamental problems of elasticity theory 146
References 147

Part II WAVE AND EVOLUTION PROBLEMS

7 Scalar and elastic wave problems in the time domain 151
7.1 Wave equation and fundamental solutions 151
7.2 Integral representation formulas 154
7.3 Boundary integral equations 156
7.4 Exterior problems and conditions at infinity 159
7.5 The boundary element method in dynamics 162
7.6 Governing equations of linear elastodynamics 168
7.7 Elastodynamic fundamental solutions 170
7.8 Integral representation of an elastodynamic state 174
7.9 Boundary integral equations in elastodynamics 176
7.10 Exterior problems and conditions at infinity 178
References 181

8 Scalar and elastic wave problems in the frequency domain 183
8.1 Helmholtz equation and fundamental solutions 184
8.2 Integral equations and representations 186
8.3 Indirect integral formulations and the fictitious eigenvalue problem 188
8.4 The boundary element method for frequency-domain problems 191
8.5 Frequency-domain elastodynamics 193
8.6 Integral equations and representations for frequency-domain elastodynamics 197
8.7 Numerical examples in time-harmonic elastodynamics 199
References 208

9 Diffusion, fluid flow 209
9.1 Diffusion equation 209
9.2 Fluid flow 212
References 215

Part III ADVANCED TOPICS

10 Symmetric Galerkin boundary integral formulations 219
10.1 Basic idea 219
10.2 Direct Galerkin formulation of Dirichlet and Neumann problems 220
10.3 Indirect Galerkin formulation of Dirichlet and Neumann problems 224
10.4 Dirichlet and Neumann scalar wave problems 225
10.5 Some convergence results 226
10.6 Numerical implementation 228
10.7 Galerkin integral formulation for mixed boundary-value problems 231
10.8 Fluid–structure coupling in acoustics 234
10.9 Linear elasticity 236
10.10 Numerical examples 240
Appendix 10.A Regularized expressions of I_1 and I_2 244
References 245

11 Exploitation of geometrical symmetries 249
11.1 Introduction 249
11.2 Geometrical symmetry in boundary-value problems 250
11.3 Boundary integral equations and geometrical symmetry 257
11.4 Discretization of the integral equation 259
11.5 Integral representations at interior points 260
11.6 Exploitation of symmetry: recapitulation 262
11.7 Numerical example in elastodynamics 264
References 265

12 Domain differentiation of boundary integral equations 267
12.1 Motivation 267
12.2 Differentiation in a domain perturbation 269
12.3 Differentiation of an objective function 272
12.4 Governing boundary integral equation for the first-order domain derivative of the state 273
12.5 Governing boundary integral equation for the second-order domain derivative of the state 278
12.6 Comments about the derivative integral equations 281
12.7 Boundary element discretisation 284
12.8 Derivative boundary integral equations in elastostatics 286
12.9 The adjoint state method 288
12.10 Example: energy release rate in fracture mechanics 290
Appendix 12.A Derivative of the differential area element 290
Appendix 12.B Proof of identity (12.36) 291

Appendix 12.C Expression of $(D_{ij}f)^\star$, $(D_if)^\star$, $(D_iu_i)^\star$ 292
Appendix 12.D Symmetric expression for $K_1(x,u;\theta,\mu)$ 294
References 295

Part IV SOLID MECHANICS

13 Linear fracture mechanics **299**
13.1 Inapplicability of the displacement boundary integral equation 299
13.2 The multiregion approach 301
13.3 Displacement discontinuity approach: cracks of arbitrary shape in infinite media 302
13.4 Displacement discontinuity approach: planar cracks in infinite media 305
13.5 Displacement discontinuity approach for a bounded body 308
13.6 Evaluation of stress intensity factors 310
13.7 Numerical examples 314
13.8 Boundary element formulation for the energy approach in fracture mechanics 316
References 320

14 Initial strains and stresses – elastoplasticity **323**
14.1 Elastostatic problems with initial strain or stress distributions 323
14.2 Elastic fields generated by inclusions 327
14.3 Small-strain elastoplasticity 329
14.4 Examples 334
References 341

APPENDICES

Appendix A Differential operators and integration by parts on surfaces **345**

Appendix B Interpolation functions and numerical integration **349**
B.1 One-dimensional interpolation 350
B.2 Two-dimensional interpolation 351
B.3 Gaussian numerical integration rules 356
References 357

Appendix C Bibliography **359**
C.1 Books on BIE/BEM 359
C.2 Articles 361
C.3 General purpose books 385

Author index 387

Subject index 389