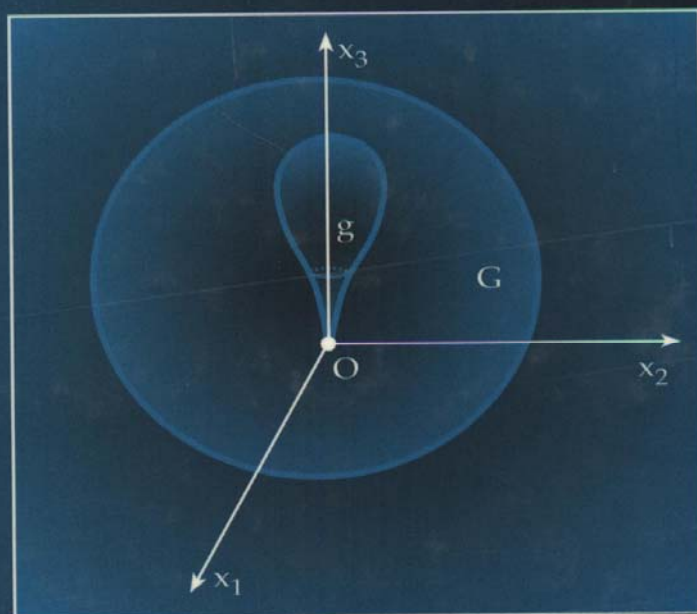


CRC Mathematical Modelling Series

Mathematical Modelling of Solids with Nonregular Boundaries



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