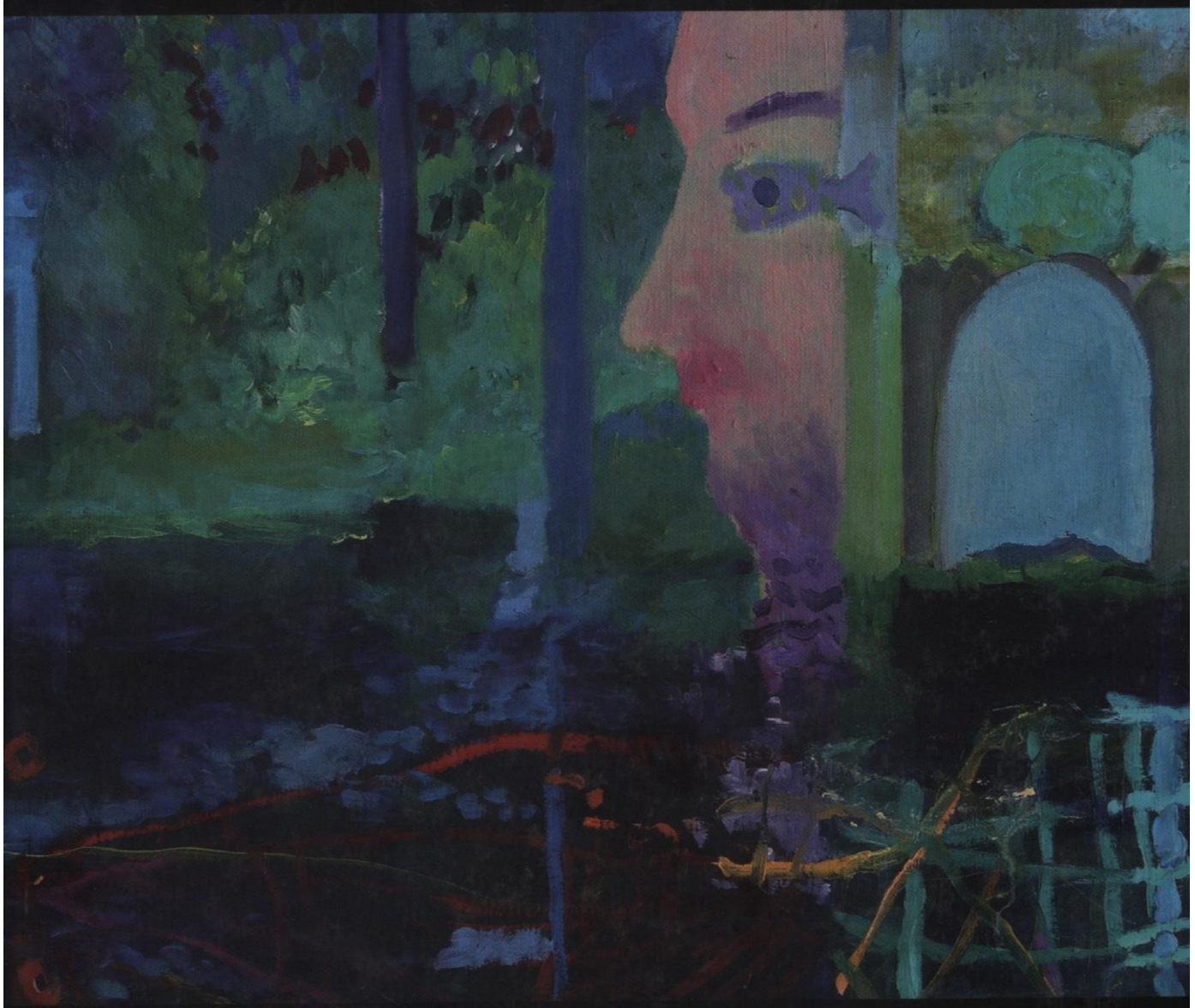
ADVANCED TOPICS IN
Quantum Field Theory

A Lecture Course



M. SHIFMAN

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ADVANCED TOPICS IN
Quantum Field Theory

Since the advent of Yang–Mills theories and supersymmetry in the 1970s, quantum field theory – the basis of the modern description of physical phenomena at the fundamental level – has undergone revolutionary developments. This is the first systematic and comprehensive text devoted specifically to aspects of modern field theory at the cutting edge of current research.

The book emphasizes nonperturbative phenomena and supersymmetry. It includes a thorough discussion of various phases of gauge theories, extended objects and their quantization, and global supersymmetry from a modern perspective. Featuring extensive cross-referencing from more traditional topics to recent breakthroughs in the field, it prepares students for independent research. The side boxes summarizing the main results, and over 70 exercises, make this an indispensable book for graduate students and researchers in theoretical physics.

M. Shifman is the Ida Cohen Fine Professor of Physics at the University of Minnesota. He was awarded the 1999 Sakurai Prize for Theoretical Particle Physics and the 2006 Julius Edgar Lilienfeld Prize for outstanding contributions to physics.

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