



TUNCER CEBECI · JEAN COUSTEIX

# Modeling and Computation of Boundary-Layer Flows



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CEBECI · COUSTEIX  
Modeling and Computation  
of Boundary-Layer Flows

This book is an introduction to computational fluid dynamics with emphasis on the modeling and calculation of boundary-layer flows. The subjects covered include laminar, transitional and turbulent boundary layers for two- and three-dimensional incompressible flows. The viscous-inviscid coupling between the boundary layer and the inviscid flow is also addressed. A large number of homework problems are included. A variety of readers including undergraduate and graduate students, teachers and scientists working in aerodynamics or hydrodynamics will gain a good understanding of the principles and applications of boundary-layer flows and related numerical methods.

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