DIGITAL SIMULATION OF DYNAMIC SYSTEMS

A CONTROL THEORY APPROACH

TOM T. HARTLEY
GUY O. BEALE
STEPHEN P. CHICATELLI

Table of Contents

Preface xi

1. Introduction 1

1.1 Objectives of the book 1
1.2 What is digital simulation? 2
1.3 Why simulate? 3
1.4 Simulation characteristics 4
1.5 Historical perspective 5
1.6 Organization of the text 6

2. Z-Transform Methods and Simulation 8

2.1 Introduction to *z*-transform methods 8
2.2 Review of *z*-transforms 8
2.3 Difference equations and transfer functions 12
2.4 Transfer function response—Output sampling 14
2.5 Input sampling 15
2.6 Reconstruction 16
2.7 The data reconstruction simulation method 22
2.8 Conclusions 23
Problems 23

3. Operational Substitution Methods **24**

3.1 Introduction 24
3.2 Forward-looking rectangles (Euler's method) 25
3.3 General procedure 30
3.4 Other associative methods 32
3.5 Derivation of nonassociative methods 37
3.6 Matrix methods for operational substitution algebra 39
3.7 The error in operational substitution 43
3.8 Application to multielement systems 45
3.9 Simulation of closed-loop systems 47
3.10 Time-varying systems 53
3.11 Operational methods for time-varying systems 54
3.12 Operational methods for nonlinear systems 56
3.13 Other approaches for transfer function representations 57
3.14 Conclusions 59
Appendix: Families of Substitution Operator Matrices 59
Problems 64

4. Linear Multistep Methods **66**

4.1 Introduction 66
4.2 Initial-value problems 67
4.3 The integration problem 67
4.4 Introduction to linear multistep methods 68
4.5 Derivation of linear multistep methods with Taylor series 73
4.6 Control theory insight 76
4.7 Introduction to the Adams family 78
4.8 Introduction to the backward difference family 80
4.9 Properties of Linear Multistep Methods 81
4.10 Closed-loop integration 86
4.11 Control theory revisited 92
4.12 Weak stability theory (closed-loop integration) 94
4.13 Introduction to stability regions 98
4.14 The $\hat{s}$-plane 104
4.15 A design example 104
4.16 Characteristics of implicit methods 111
4.17 Introduction to predictor-corrector methods 114
4.18 Weak stability of predictor-corrector methods 116
4.19 An example predictor-corrector stability study 118
4.20 Modifiers in predictor-corrector methods 120
4.21 Weak stability for predictor-corrector methods with modifiers 124
4.22 Variable timestep and variable order methods 124
4.23 Conclusions 126
Problems 126

5. Stability Regions Revisited **129**

5.1 Introduction 129
5.2 More about frequency responses 129
5.3 Stability regions and the inverse Nyquist plot 136
5.4 Inside the λ-T-plane 141
5.5 A robustness analysis of the simulation process 148
5.6 Discussion of specific stability regions 153
5.7 Predictor-corrector stability regions 153
5.8 Predictor-corrector-modifier stability regions 153
5.9 Hardware-in-the-loop stability 154
5.10 Conclusions 160
Problems 160

6. Runge-Kutta Methods **161**

6.1 Introduction 161
6.2 Graphical derivation of Runge-Kutta methods 162
6.3 The general Runge-Kutta method 165
6.4 Linear system with piecewise constant input 166
6.5 Nonlinear system with piecewise constant input 169
6.6 Linear system with multirate input sampling 174
6.7 Transfer functions and weak stability 177
6.8 Stability regions for Runge-Kutta methods 182
6.9 Variable timestep Runge-Kutta methods 184
6.10 Conclusions 188
Problems 188

7. Stiff Systems **190**

7.1 Introduction 190
7.2 Definition and examples 191
7.3 More stability definitions 194
7.4 Stability region placement methods 202
7.5 Matrix stability region placement methods 209
7.6 Runge-Kutta methods 216
7.7 Inverse Nyquist array 224
7.8 Conclusions 238
Problems 239

8. Nonlinear Systems **241**

8.1 Introduction 241
8.2 First order continuous-time systems 242
8.3 Linearization 243
8.4 Second-order systems 244
8.5 Third-order systems 247

8.6 Higher-order systems 251
8.7 Discrete-time systems 252
8.8 Simulation of nonlimit cycling systems 258
8.9 Simulation of systems with limit cycles 266
8.10 Simulation of chaotic systems 269
8.11 Simulation of systems with discontinuous nonlinearities 270
8.12 Simulation of time-delay systems 272
8.13 Divergence preserving methods 273
8.14 Conclusions 275
Problems 275

9. Multiple Integration 276

9.1 Introduction 276
9.2 Motivating problems for double integration 276
9.3 General linear multistep double integrator 278
9.4 Weak stability for the double integrator 280
9.5 Stability regions for double integrators 280
9.6 Predictor-corrector methods for double integration 282
9.7 Triple integration 282
9.8 Higher-order integration 284
9.9 Substitution methods revisited 285
9.10 Conclusions 286
Problems 286

10. Concluding Discussion 290

10.1 Competing criteria 290
10.2 Review of basic approaches 292
10.3 System characteristics 293
10.4 Choosing a method and T 295
10.5 Available software 298
Problems 299

Appendices

A. References 300

B. Stability Region Plots 303

C. Listing of Provided Software 373

D. Collection of Dynamic Systems 379

Index 385