clude NUMERICAL (a) RECIPES in C++ The Art of Scientific Computing lide political scientific Computing lide political maximum and maximum and maximum and scientific Computing lide political maximum and scientific constitution and scientific William H. Press Saul A. Teukolsky Brian P. Flannery &a) William T. Vetterling

Contents

| | Preface to the C++ Edition | |
|-----|--|----------|
| | Preface to the Second Edition | xii |
| | Preface to the First Edition | xv |
| 171 | License Information | xvii |
| | Computer Programs by Chapter and Section | xxi |
| 1 | Preliminaries Relations and Clearence Recognition (Preliminaries) | 1 |
| | Onadranic and Cubic Equations | 29.0 |
| | 1.1 Program Organization and Control Structures | 5 |
| | 1.2 Some C++ Conventions for Scientific Computing | 10 |
| | 1.3 Implementation of the Vector and Matrix Classes | 25 |
| | 1.4 Error, Accuracy, and Stability | 31 |
| 2 | Solution of Linear Algebraic Equations | 35 |
| | 2.0 Introduction | 35 |
| | 2.1 Gauss-Jordan Elimination | 39 |
| | 2.2 Gaussian Elimination with Backsubstitution | 44 |
| | 2.3 LU Decomposition and Its Applications | 46 |
| | 2.4 Tridiagonal and Band Diagonal Systems of Equations | 53 |
| | 2.5 Iterative Improvement of a Solution to Linear Equations | 58 |
| | 2.6 Singular Value Decomposition | 62 74 |
| | 2.7 Sparse Linear Systems | |
| | 2.8 Vandermonde Matrices and Toeplitz Matrices 2.9 Cholesky Decomposition | 99 |
| , | 2.9 Cholesky Decomposition | 101 |
| | 2.10 QR Decomposition 2.11 Is Matrix Inversion an N^3 Process? | 105 |
| | 2.11 Is Matrix Inversion an IV Process: | 0.0 430 |
| 3 | Interpolation and Extrapolation | 108 |
| | 3.0 Introduction | 108 |
| | 3.1 Polynomial Interpolation and Extrapolation | 111 |
| | 3.2 Rational Function Interpolation and Extrapolation | 114 |
| | 3.3 Cubic Spline Interpolation | 116 |
| | 3.4 How to Search an Ordered Table | 120 |
| | 3.5 Coefficients of the Interpolating Polynomial | 123 |
| | 3.6 Interpolation in Two or More Dimensions | 126 |

QVS 11.5 Reduction of a General Matrix to Hessenberg Fasteived miolinU 1.7 487

872 11.4 Hermitian Matrices v

vi Contents

| 4 | Integration of Functions | 133 |
|---|--|------------|
| | 4.0 Introduction | 133 |
| | 4.1 Classical Formulas for Equally Spaced Abscissas | 134 |
| | 4.2 Elementary Algorithms | 141 |
| | 4.3 Romberg Integration | 144 |
| | 4.4 Improper Integrals | 146 |
| | 4.5 Gaussian Quadratures and Orthogonal Polynomials | 152 |
| | 4.6 Multidimensional Integrals | 166 |
| 5 | Evaluation of Functions | 171 |
| | 5.0 Introduction | 171 |
| | 5.1 Series and Their Convergence | 171 |
| | 5.2 Evaluation of Continued Fractions | 175 |
| | 5.3 Polynomials and Rational Functions | 179 |
| | 5.4 Complex Arithmetic | 182 |
| | 5.5 Recurrence Relations and Clenshaw's Recurrence Formula | 184 |
| | 5.6 Quadratic and Cubic Equations 210 notice bound (| 100 |
| | 5.7 Numerical Derivatives promoted by a nonaxinagro margory. | |
| | 5.8 Chebyshev Approximation | 196 |
| | 5.9 Derivatives or Integrals of a Chebyshev-approximated Function | 201 203 |
| | 5.10 Polynomial Approximation from Chebyshev Coefficients | 203 |
| | 5.11 Economization of Power Series 5.12 Padé Approximants | 204 |
| | 5.13 Rational Chebyshev Approximation | 209 |
| | 5.14 Evaluation of Functions by Path Integration | 213 |
| 6 | Special Functions | 217 |
| | 6.0 Introduction | 217 |
| | 6.1 Gamma Function, Beta Function, Factorials, Binomial Coefficients | 218 |
| | 6.2 Incomplete Gamma Function, Error Function, Chi-Square | |
| | Probability Function, Cumulative Poisson Function | 221 |
| | 6.3 Exponential Integrals | 227 |
| | 6.4 Incomplete Beta Function, Student's Distribution, F-Distribution, | |
| | Cumulative Binomial Distribution | 231 |
| | 6.5 Bessel Functions of Integer Order | 235 |
| | 6.6 Modified Bessel Functions of Integer Order | 241 |
| | 6.7 Bessel Functions of Fractional Order, Airy Functions, Spherical | 245 |
| | Bessel Functions | 245 |
| | 6.8 Spherical Harmonics | 257 259 |
| | 6.9 Fresnel Integrals, Cosine and Sine Integrals | 264 |
| | 6.10 Dawson's Integral6.11 Elliptic Integrals and Jacobian Elliptic Functions | 265 |
| | 6.12 Hypergeometric Functions | 275 |
| | Pay 071, 22 a research rehidulation function on 10 subcattering | |
| 7 | Random Numbers | 278 |
| | 7.0 Introduction | 278 |
| | 7.1 Uniform Deviates | 279 |

Contents vii

| | 7.2 To of water Method, Europeatial and Normal Deviates | 291 |
|------|--|-----|
| | 7.2 Transformation Method: Exponential and Normal Deviates7.3 Rejection Method: Gamma, Poisson, Binomial Deviates | |
| | 7.4 Generation of Random Bits | 300 |
| | 7.5 Random Sequences Based on Data Encryption | 304 |
| | | |
| | 7.0 Simple Wionte Carlo Integration | 212 |
| | 7.7 Quasi- (that is, Sub-) Random Sequences7.8 Adaptive and Recursive Monte Carlo Methods | 320 |
| | 2 Fast Fourier Transform (FFT) Ludwig Renobil Moralloom 6 61 | |
| 8 | Sorting South Control of the Control | 332 |
| | 8.0 Introduction of Conservative Econological Conference of The THE A. | 332 |
| | 8.1 Straight Insertion and Shell's Method [858] to applicable Themsoff c. | 333 |
| 536. | 8.2 Quicksort | 336 |
| | 8.3 Heapsort | 339 |
| SAH | 8.4 Indexing and Ranking | 341 |
| | 8.5 Selecting the Mth Largest noticebound 0. | 344 |
| | 8.6 Determination of Equivalence Classes | 348 |
| 9 | Root Finding and Nonlinear Sets of Equations | 351 |
| | 9.0 Introduction FFE add and London Estimated American Provide American Pr | 351 |
| | 9.1 Bracketing and Bisection Anismp Goal Toda in guinelle lating C. | 354 |
| | 9.2 Secant Method, False Position Method, and Ridders' Method | 358 |
| | 9.3 Van Wijngaarden-Dekker-Brent Method | 363 |
| | 9.4 Newton-Raphson Method Using Derivative | 366 |
| | 9.5 Roots of Polynomials and bolgman ylmovanti to anythan A landsord & | 373 |
| | 9.6 Newton-Raphson Method for Nonlinear Systems of Equations | 383 |
| | 9.7 Globally Convergent Methods for Nonlinear Systems of Equations | 387 |
| 10 | Minimization or Maximization of Functions | 398 |
| | 10.0 Introduction | 398 |
| | 10.1 Golden Section Search in One Dimension | 401 |
| | 10.2 Parabolic Interpolation and Brent's Method in One Dimension | 406 |
| | 10.3 One-Dimensional Search with First Derivatives | 410 |
| | 10.4 Downhill Simplex Method in Multidimensions | 413 |
| | 10.5 Direction Set (Powell's) Methods in Multidimensions | 417 |
| | 10.6 Conjugate Gradient Methods in Multidimensions | 424 |
| 140 | 10.7 Variable Metric Methods in Multidimensions | 430 |
| | 10.8 Linear Programming and the Simplex Method | 434 |
| | 10.9 Simulated Annealing Methods | 448 |
| 11 | Eigensystems | 461 |
| | 11.0 Introduction | 461 |
| | 11.1 Jacobi Transformations of a Symmetric Matrix | 468 |
| | 11.2 Reduction of a Symmetric Matrix to Tridiagonal Form: | |
| | Givens and Householder Reductions | 474 |
| | 11.3 Eigenvalues and Eigenvectors of a Tridiagonal Matrix | |
| | 11.4 Hermitian Matrices Parameters Squares Squares Land Land A | |
| | 11.5 Reduction of a General Matrix to Hessenberg Form | 487 |

viii Contents

| | 11.6 The QR Algorithm for Real Hessenberg Matrices 11.7 Improving Eigenvalues and/or Finding Eigenvectors by | 491 |
|----|--|------------|
| | Generation of Random Bird Remove Brown Sequences Based on Visa Brown Increased | 498 |
| 12 | Fast Fourier Transform | 501 |
| | 12.0 Introduction Sequences (that is, Sub-) Random Sequences | 501 |
| | 12.1 Fourier Transform of Discretely Sampled Data | 505 |
| | 12.2 Fast Fourier Transform (FFT) | 509 |
| | 12.3 FFT of Real Functions, Sine and Cosine Transforms | 515 |
| | 12.4 FFT in Two or More Dimensions | 0.8 526 |
| | 12.5 Fourier Transforms of Real Data in Two and Three Dimensions | 530 |
| | 12.6 External Storage or Memory-Local FFTs | |
| 13 | 2 Eventuation of Englanded Procuping Sections | |
| | 13 O Introduction | 542 |
| | 13.1 Convolution and Deconvolution Using the FFT | 543 |
| | 13.2 Correlation and Autocorrelation Using the FFT | 550 |
| | 13.3 Optimal (Wiener) Filtering with the FFT | 334 |
| | 13.4 Power Spectrum Estimation Using the FFT | |
| | 13.5 Digital Filtering in the Time Domain | 563 |
| | 13.6 Linear Prediction and Linear Predictive Coding | 569 |
| | 13.7 Power Spectrum Estimation by the Maximum Entropy | €.0 204 |
| | (All Poles) Method Symptomic Symptomic Method Symptomic | |
| | 13.8 Spectral Analysis of Unevenly Sampled Data amonglo 10 2005 | 580 |
| | 13.9 Computing Fourier Integrals Using the FFT and novel of the second s | 589 596 |
| | 13.10 Wavelet Transforms send and about Management Vision Vision Company of the C | 611 |
| | 13.11 Numerical Use of the Sampling Theorem | |
| 14 | Statistical Description of Data | 614 |
| | 14.0 Introduction | |
| | 14.1 Moments of a Distribution: Mean, Variance, Skewness, Jodans 9 | .01 |
| | and So Forth Search with First Derivatives and To a | 615 |
| | 14.2 Do Two Distributions Have the Same Means or Variances? | 625 |
| | 14.3 Are Two Distributions Different? Many (Allews) 14.3 Are Two Distributions | |
| | 14.4 Contingency Table Analysis of Two Distributions | |
| | 14.5 Linear Correlation 14.6 Nonparametric or Rank Correlation | 644 |
| | 14.7 Do Two-Dimensional Distributions Differ? | |
| | 14.8 Savitzky-Golay Smoothing Filters | 655 |
| 15 | Modeling of Data | 661 |
| 10 | C 12 Ullimin Intervals and Japobien Ellimic Functions | 661 |
| | 15.0 Introduction 15.1 Least Squares as a Maximum Likelihood Estimator | 662 |
| | 15.2 Fitting Data to a Straight Line (15.2 Fitting Data to a Straight | 666 |
| | 15.3 Straight-Line Data with Errors in Both Coordinates | |
| | 15.4 General Linear Least Squares | |
| | 15.5 Nonlinear Models | 686 |

| Pref | Contents | ix |
|------|----------|----|

| | 15.6 Confidence Limits on Estimated Model Parameters | 694 |
|-----|---|-----|
| | 15.7 Robust Estimation | 704 |
| 16 | Integration of Ordinary Differential Equations | 712 |
| | 16.0 Introduction | 712 |
| | 16.1 Runge-Kutta Method | 715 |
| | 16.2 Adaptive Stepsize Control for Runge-Kutta | 719 |
| | 16.3 Modified Midpoint Method | 727 |
| | 16.4 Richardson Extrapolation and the Bulirsch-Stoer Method | 729 |
| | 16.5 Second-Order Conservative Equations | 737 |
| | 16.6 Stiff Sets of Equations | 739 |
| √a6 | 16.7 Multistep, Multivalue, and Predictor-Corrector Methods | 751 |
| 17 | Two Point Boundary Value Problems | 756 |
| | 17.0 Introduction | 756 |
| | 17.1 The Shooting Method | 760 |
| | 17.2 Shooting to a Fitting Point | 762 |
| | 17.3 Relaxation Methods | 765 |
| | 17.4 A Worked Example: Spheroidal Harmonics | 775 |
| | 17.5 Automated Allocation of Mesh Points | 785 |
| | 17.6 Handling Internal Boundary Conditions or Singular Points | 787 |
| 18 | Integral Equations and Inverse Theory | 790 |
| | 18.0 Introduction | 790 |
| | 18.1 Fredholm Equations of the Second Kind | 793 |
| | 18.2 Volterra Equations | 796 |
| | 18.3 Integral Equations with Singular Kernels | 799 |
| | 18.4 Inverse Problems and the Use of A Priori Information | 806 |
| | 18.5 Linear Regularization Methods | 811 |
| | 18.6 Backus-Gilbert Method | 818 |
| | 18.7 Maximum Entropy Image Restoration | 821 |
| 19 | Partial Differential Equations | 829 |
| | 19.0 Introduction | 829 |
| | 19.1 Flux-Conservative Initial Value Problems | 836 |
| | 19.2 Diffusive Initial Value Problems | 849 |
| | 19.3 Initial Value Problems in Multidimensions | 855 |
| | 19.4 Fourier and Cyclic Reduction Methods for Boundary | |
| | Value Problems | 859 |
| | 19.5 Relaxation Methods for Boundary Value Problems | 865 |
| | 19.6 Multigrid Methods for Boundary Value Problems | 873 |
| 20 | Less-Numerical Algorithms | 891 |
| | 20.0 Introduction | 891 |
| | 20.1 Diagnosing Machine Parameters | 891 |
| | 20.1 Diagnosing Watchine Latameters | 896 |
| | | |

| 20.3 Cyclic Redundancy and Other Checksums | 898 |
|--|-----|
| 20.4 Huffman Coding and Compression of Data | 906 |
| 20.5 Arithmetic Coding | 912 |
| 20.6 Arithmetic at Arbitrary Precision | 916 |
| References bottom Matthe Spring 1.31 | 927 |
| Appendix A: Table of Function Declarations | 931 |
| Appendix B: Utility Routines and Classes | 939 |
| Appendix C: Converting to Single Precision | 957 |
| Index of Programs and Dependencies | 959 |
| General Index The Shooting Method of Shooting Replacement Company | 972 |
| | |
| | |

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