

Universitext

Dirk van Dalen

Logic and Structure

Fourth Edition



Springer

Contents

0	Introduction	1
1	Propositional Logic	5
1.1	Propositions and Connectives	5
1.2	Semantics	15
1.3	Some Properties of Propositional logic	21
1.4	Natural Deduction	30
1.5	Completeness	40
1.6	The Missing Connectives	49
2	Predicate Logic	57
2.1	Quantifiers	57
2.2	Structures	58
2.3	The Language of a Similarity Type	60
2.4	Semantics	68
2.5	Simple Properties of Predicate Logic	73
2.6	Identity	81
2.7	Examples	83
2.8	Natural Deduction	91
2.9	Adding the Existential Quantifier	96
2.10	Natural Deduction and Identity	99
3	Completeness and Applications	103
3.1	The Completeness Theorem	103
3.2	Compactness and Skolem-Löwenheim	111
3.3	Some Model Theory	118
3.4	Skolem Functions or How to Enrich Your Language	135
4	Second Order Logic	143

5	Intuitionistic Logic	153
5.1	Constructive Reasoning	153
5.2	Intuitionistic Propositional and Predicate Logic	156
5.3	Kripke Semantics	162
5.4	Some Model Theory	173
6	Normalisation	187
6.1	Cuts	187
6.2	Normalization for Classical Logic	192
6.3	Normalization for Intuitionistic Logic	198
7	Gödel's theorem	209
7.1	Primitive recursive functions	209
7.2	Partial Recursive Functions	218
7.3	Recursively enumerable sets	230
7.4	Some arithmetic	236
7.5	Representability	242
7.6	Derivability	246
7.7	Incompleteness	251
	Bibliography	257
	Index	259



DIRK VAN DALEN studied at the University of Amsterdam, where he obtained his PhD. He has taught since 1960 at Utrecht University, where he is full professor. He also taught at MIT and Oxford. His technical work is mostly in the area of intuitionistic mathematics and logic. He uses to call attention to the benefits and challenges of constructive methods. His current project is a biography of L. E. J. Brouwer and the editing of Brouwer's correspondence.

Logic and Structure

A book which efficiently presents the basics of propositional and predicate logic, van Dalen's popular textbook contains a complete treatment of elementary classical logic, using Gentzen's Natural Deduction. Propositional and predicate logic are treated in separate chapters in a leisureed but precise way. Chapter Three presents the basic facts of model theory, e.g. compactness, Skolem-Löwenheim, elementary equivalence, non-standard models, quantifier elimination, and Skolem functions.

The discussion of classical logic is rounded off with a concise exposition of second-order logic.

In view of the growing recognition of constructive methods and principles, one chapter is devoted to intuitionistic logic. Completeness is established for Kripke semantics. A number of specific constructive features, such as apartness and equality, the Gödel translation, the disjunction and existence property have been incorporated.

The power and elegance of natural deduction is demonstrated best in the part of proof theory called 'cut-elimination' or 'normalization'. Chapter 6 is devoted to this topic; it contains the basic facts on the structure of derivations, both classically and intuitionistically.

Finally, this edition contains a new chapter on Gödel's first incompleteness theorem. The chapter is self-contained, it provides a systematic exposition of primitive recursion, partial recursive functions, recursive by enumerable sets, and recursive separability. The arithmetization of Peano's arithmetic is based on the natural deduction system.

ISBN 3-540-20879-8



9 783540 208792