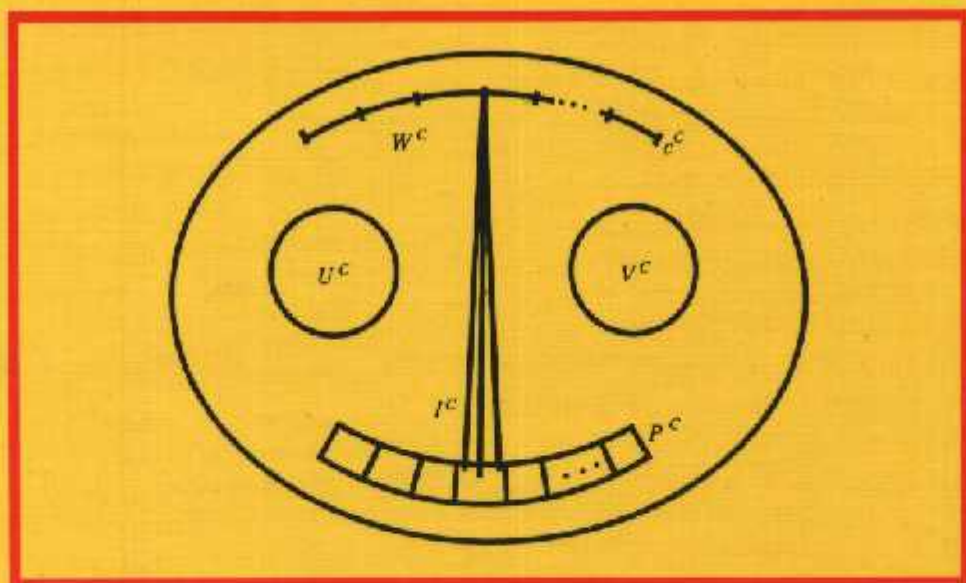


Undergraduate Texts in Mathematics

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# Mathematical Logic

Second Edition



Springer

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What is a mathematical proof? How can proofs be justified? Are there limitations of provability? To what extent can machines carry out mathematical proofs?

This junior/senior level text focuses on these questions. The book starts with a thorough treatment of first-order logic and its role in the foundations of mathematics. It covers several advanced topics, not commonly treated in introductory texts, such as Trachtenbrot's undecidability theorem, Fraïssé's characterization of elementary equivalence, Lindström's theorem on the maximality of first-order logic, and the fundamentals of logic programming.

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