

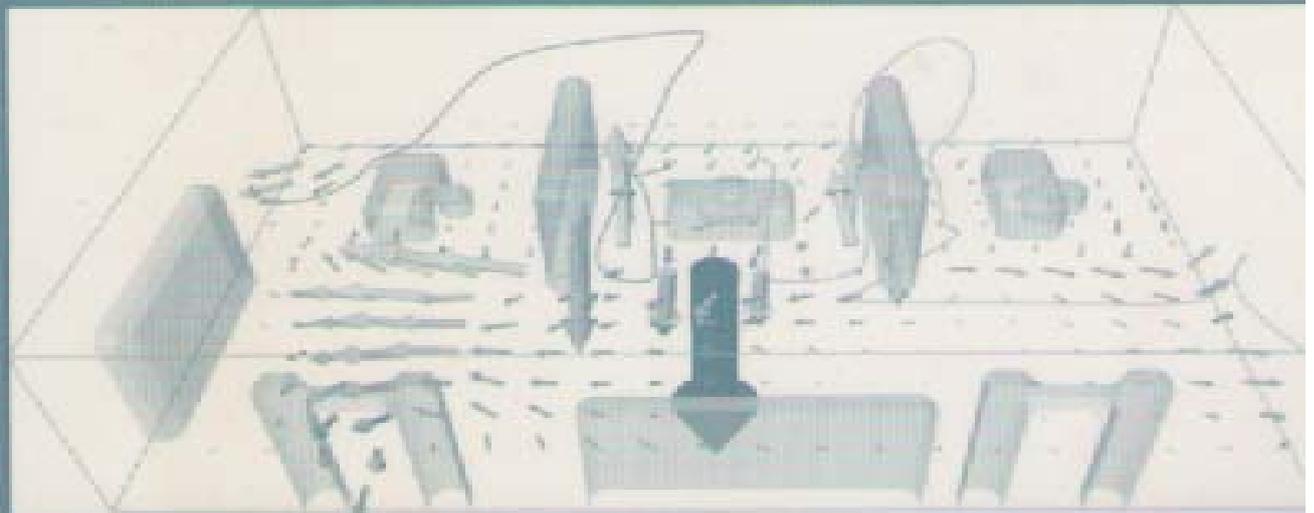
# Numerical Simulation in Fluid Dynamics

**A Practical Introduction**

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# Numerical Simulation in Fluid Dynamics

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In this translation of the German edition, the authors provide insight into the numerical simulation of fluid flow. Using a simple numerical method as expository example, the individual steps of scientific computing are presented:

- the derivation of the mathematical model,
- the discretization of the model equations,
- the development of algorithms,
- parallelization, and
- visualization of the computed data.

In addition to the treatment of the basic equations for modeling laminar, transient flow of viscous, incompressible fluids—the Navier–Stokes equations—the authors give an outlook on the simulation of

- free surface flows,
- energy and chemical transport, and
- turbulence.

Detailed hints for the implementation of the various algorithms enable readers to write their own flow simulation program from scratch. The variety of applications is shown in several simulation results, including 93 black-and-white and 17 color illustrations.

Moreover, after reading this book, readers should be able to understand more enhanced algorithms of computational fluid dynamics and to apply their new knowledge of modeling, discretization, parallelization, and visualization to other scientific fields, where numerical simulation has established itself as a new path, in addition to theoretical investigations and practical experiments, for uncovering the laws of nature. Among these fields are the examination of elastic solids, combustion, melting and coating processes, and crystal growth, as well as weather prediction.

The book is written on an advanced undergraduate level and addresses mathematicians, engineers, and natural and computer scientists who are interested in scientific computing. It is well suited for a cross-disciplinary and application-oriented training at the university level as well as for practitioners seeking basic insights into computational fluid dynamics.

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