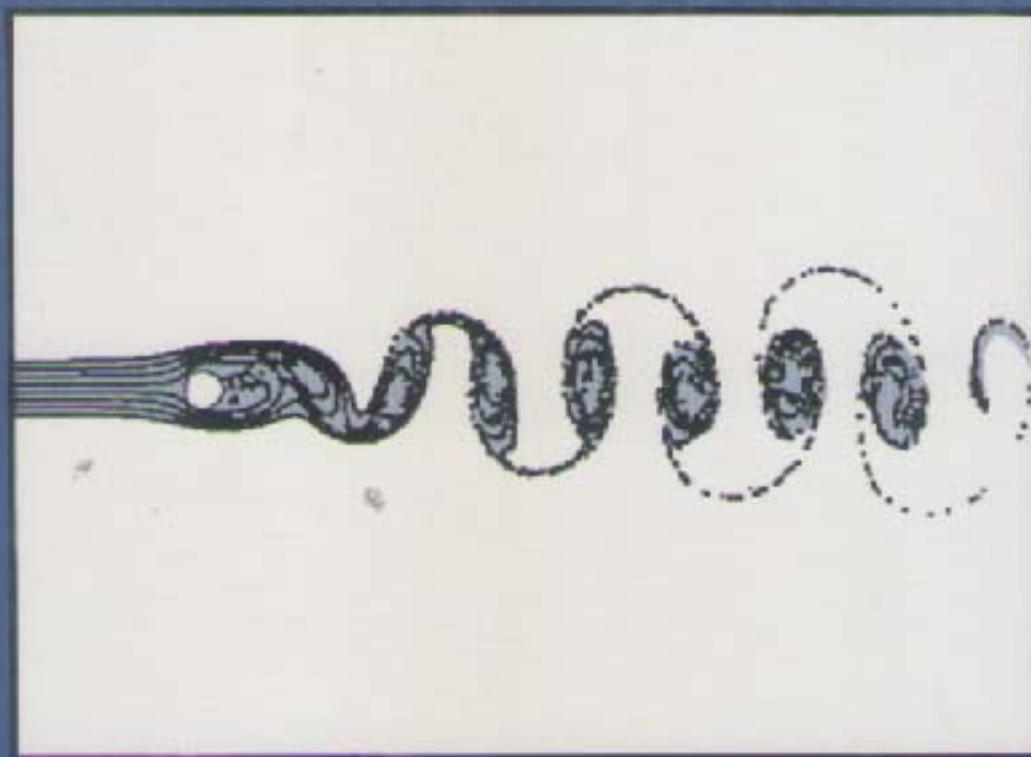


Flow Analysis Using a PC

H. Ninomiya and K. Onishi



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The aim of this book is to present a short introductory course on simple numerical modelling of fluid flows, using the elementary finite element method. Fundamental and important flows have been carefully selected and formulated in the framework of the finite element method. Comparatively low cost personal computers such as the IBM-PC, NEC-PC, or more promising recent EWS (Engineering Work Stations) are assumed to be used. BASIC programs are provided on a diskette which accompanies the text.

The topics covered are Basic Flow Properties; The Finite Element Method; Two-Dimensional Problems; Potential Flows; Transient Flows; Transient Heat Conduction; Incompressible Viscous Flow; Thermal Fluid Flow; Mass Transport; Tidal Current and Program Instructions.

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