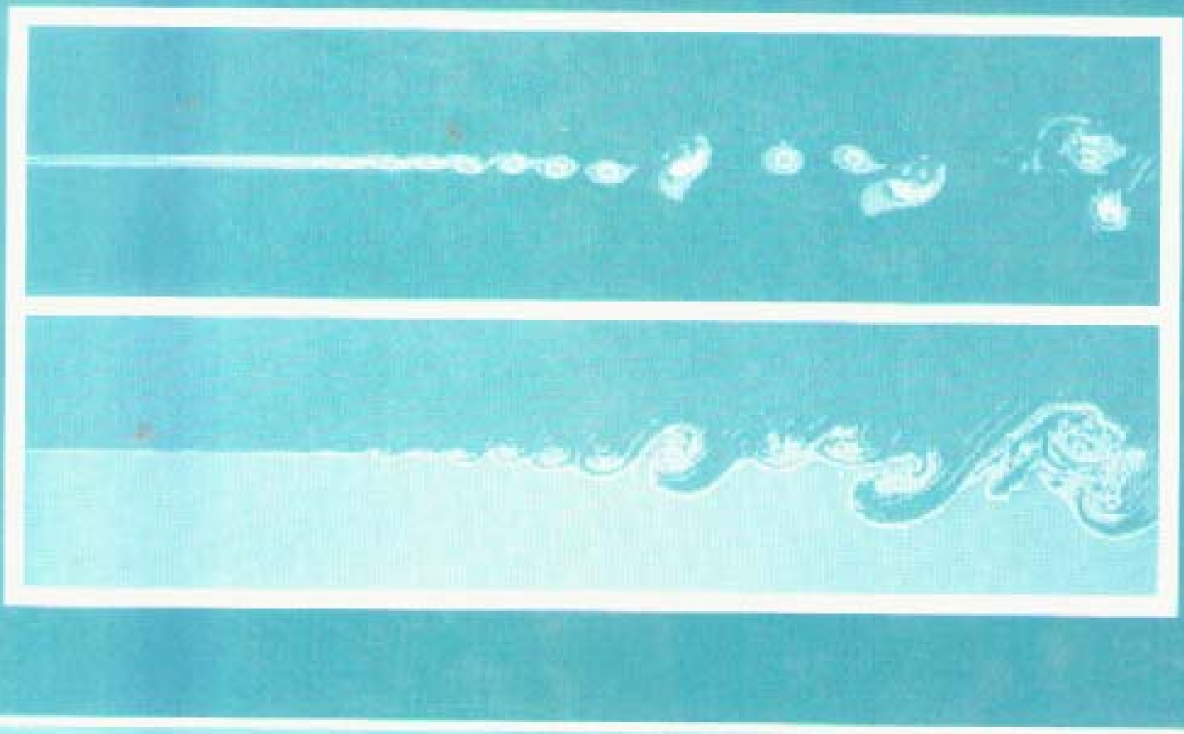
FLUID MECHANICS AND ITS APPLICATIONS

M. Lesieur

Turbulence in Fluids

Second Revised Edition



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Turbulence in Fluids

Stochastic and Numerical Modelling

by

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Turbulence in Fluids

M. Lesieur

Turbulence in Fluids reconciles the theory of turbulence, too often presented in a formal, isolated mathematical context, with the general theory of fluid dynamics. It reviews, in a unifying manner, the main characteristics and general theorems of rotational fluid dynamics, with application to density-stratified and rotating flows, such as atmospheres or oceans, and compressible flows as well. Emphasis is placed on vortex dynamic and coherent structures. Transition to turbulence in wall or free-shear flows is considered both on the basis of the hydrodynamic linear-instability theory, and of experiments or numerical simulations.

It presents the essential kinematic properties of turbulence, in particular in the Fourier space, as well as the phenomenological theories of isotropic turbulence based on Kolmogorov's law, or Prandtl's mixing-length theory applied to shear layers. It emphasizes the use of two-point closures and stochastic models, an extremely powerful tool allowing one to represent the strongly non-linear interactions between various scales of motion, and to give a firm basis to the phenomenological theories. A thorough discussion on the possibility of singularities within Euler equations is led. The role of helicity is assessed, as well as the performances of the renormalization group techniques with respect to the nonlocal interaction theory.

Particular emphasis is given to diffusion processes, numerical large-eddy-simulations and predictability, both in the three-dimensional and in the two-dimensional cases. The latter situation is the simplest approximation of large-scale atmosphere and ocean dynamics, within the context of the geostrophic theory. The concept of two-dimensional turbulence is extended to flows of engineering interest. Numerous experimental, environmental and aerodynamic examples are provided. A systematic recourse is made to direct-numerical simulations as a tool for exploring turbulence media.

This monograph is a unique tool for graduate students and researchers in mechanical and aerospace engineering, mathematics, nonlinear and statistical physics, meteorology, oceanography and astrophysics. It views the problem of turbulence in a very general way: statistical theories, intermittency, transition, coherent structures, singularities, unpredictability or deterministic chaos are only small pieces of the same puzzle, which have to be assembled.

The book contains 21 colour plates.

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