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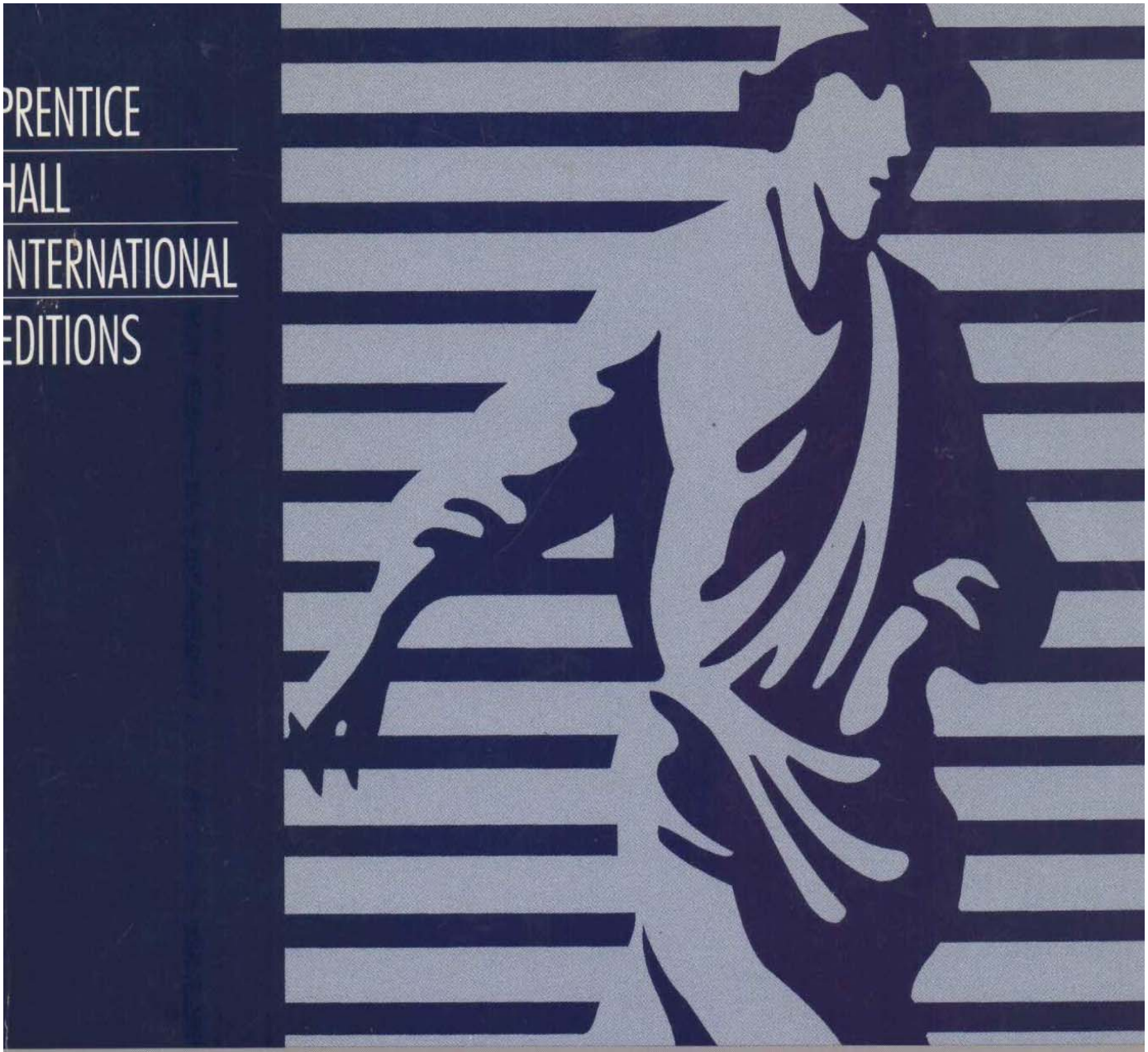

AUTOMATIC CONTROL SYSTEMS

SEVENTH EDITION

BENJAMIN C.
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