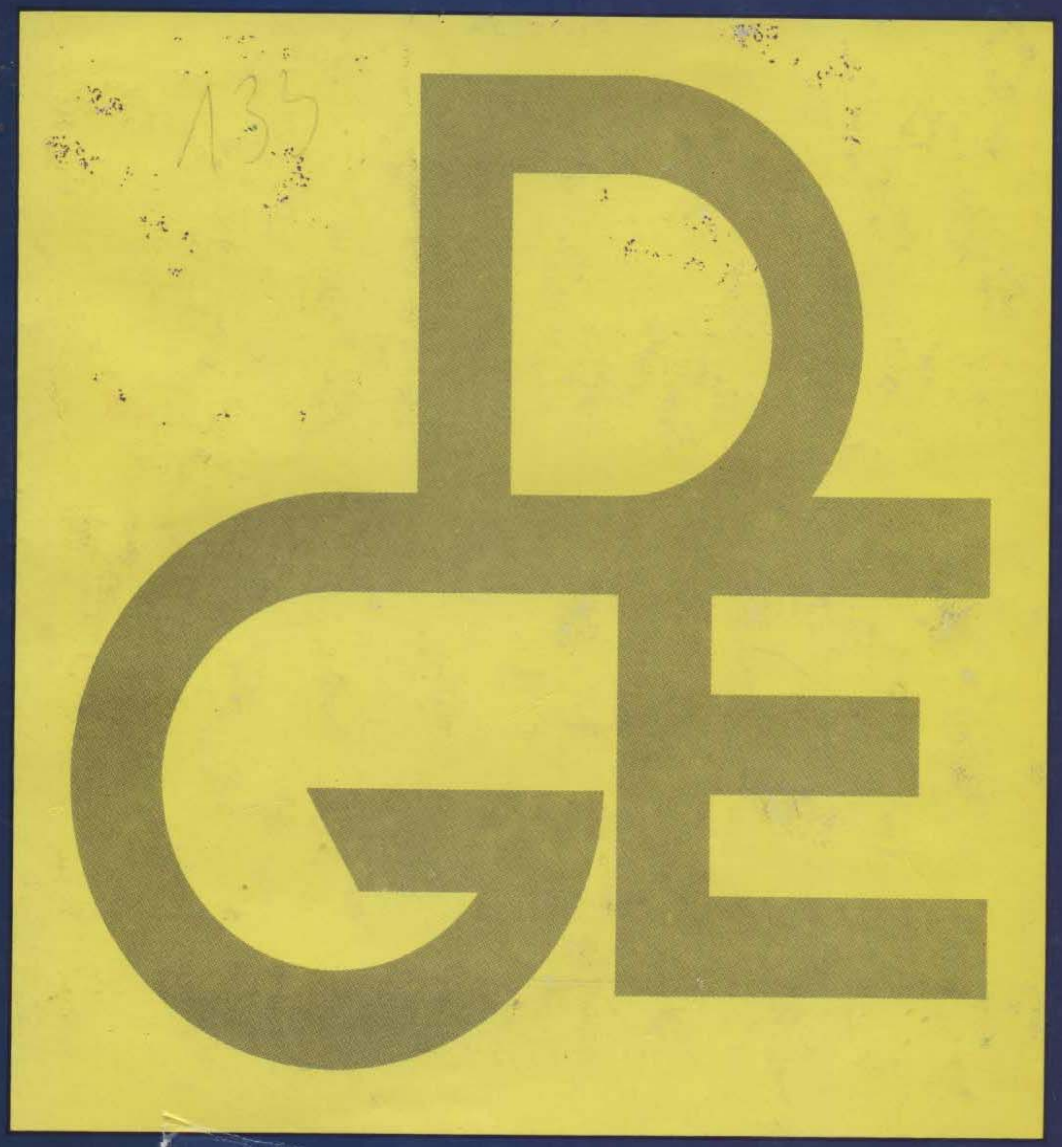
A. 55/56 - 47
EX. 1

Developments in Geotechnical Engineering, 61

Finite Element Techniques in Groundwater Flow Studies

With Applications in Hydraulic and Geotechnical Engineering

I. Kazda



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