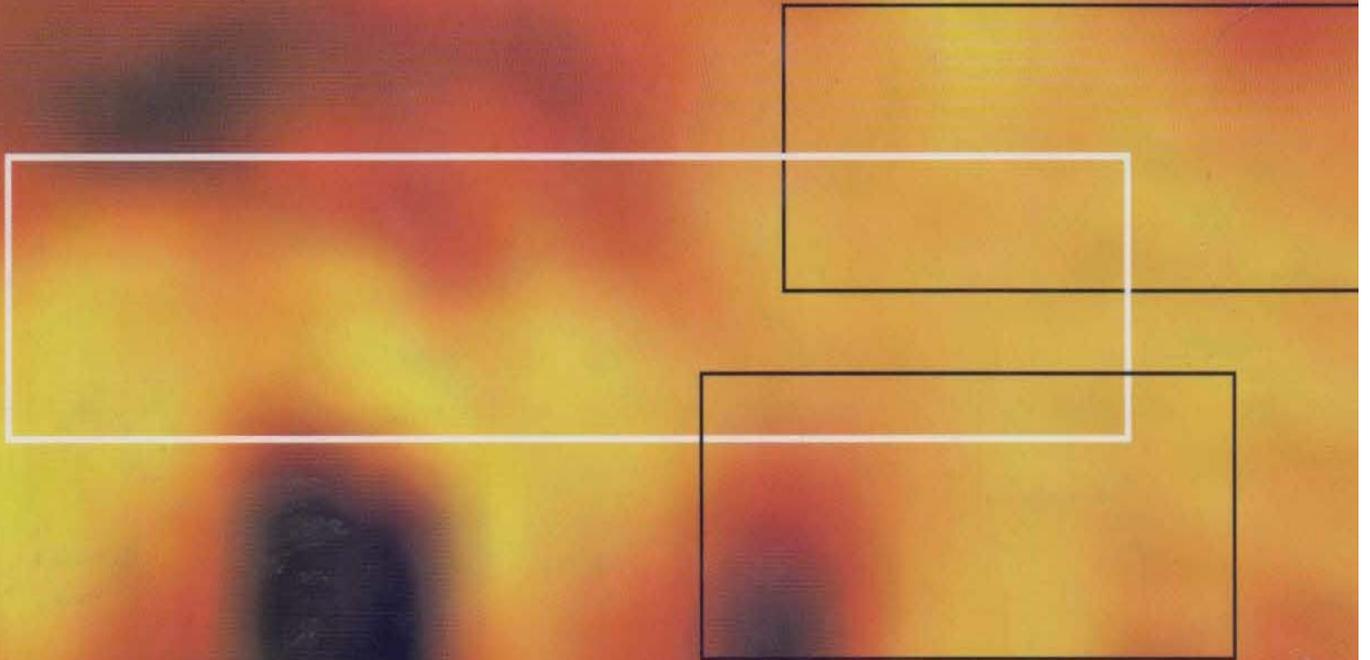


systems and control



stanislaw h. žak

CONTENTS

Preface xv

| | | |
|----------|--|-----------|
| 1 | Dynamical Systems and Modeling | 1 |
| 1.1 | What Is a System? | 1 |
| 1.2 | Open-Loop Versus Closed-Loop | 3 |
| 1.3 | Axiomatic Definition of a Dynamical System | 5 |
| 1.4 | Mathematical Modeling | 8 |
| 1.5 | Review of Work and Energy Concepts | 9 |
| 1.6 | The Lagrange Equations of Motion | 12 |
| 1.7 | Modeling Examples | 21 |
| 1.7.1 | Centrifugal Governor | 21 |
| 1.7.2 | Ground Vehicle | 22 |
| 1.7.3 | Permanent Magnet Stepper Motor | 28 |
| 1.7.4 | Stick Balancer | 35 |
| 1.7.5 | Population Dynamics | 38 |
| | Notes | 40 |
| | Exercises | 40 |
| 2 | Analysis of Modeling Equations | 48 |
| 2.1 | State-Plane Analysis | 48 |
| 2.1.1 | Examples of Phase Portraits | 49 |
| 2.1.2 | The Method of Isoclines | 53 |
| 2.2 | Numerical Techniques | 59 |
| 2.2.1 | The Method of Taylor Series | 59 |
| 2.2.2 | Euler's Methods | 62 |
| 2.2.3 | Predictor–Corrector Method | 64 |

| | | |
|----------|--|------------|
| 2.2.4 | Runge's Method | 67 |
| 2.2.5 | Runge–Kutta Method | 70 |
| 2.3 | Principles of Linearization | 71 |
| 2.4 | Linearizing Differential Equations | 74 |
| 2.5 | Describing Function Method | 77 |
| 2.5.1 | Scalar Product of Functions | 77 |
| 2.5.2 | Fourier Series | 81 |
| 2.5.3 | Describing Function in the Analysis of Nonlinear Systems | 82 |
| | Notes | 88 |
| | Exercises | 88 |
| 3 | Linear Systems | 94 |
| 3.1 | Reachability and Controllability | 94 |
| 3.2 | Observability and Constructability | 107 |
| 3.3 | Companion Forms | 111 |
| 3.3.1 | Controller Form | 112 |
| 3.3.2 | Observer Form | 117 |
| 3.4 | Linear State-Feedback Control | 120 |
| 3.5 | State Estimators | 126 |
| 3.5.1 | Full-Order Estimator | 127 |
| 3.5.2 | Reduced-Order Estimator | 130 |
| 3.6 | Combined Controller–Estimator Compensator | 131 |
| | Notes | 141 |
| | Exercises | 141 |
| 4 | Stability | 149 |
| 4.1 | Informal Introduction to Stability | 149 |
| 4.2 | Basic Definitions of Stability | 150 |
| 4.3 | Stability of Linear Systems | 154 |
| 4.4 | Evaluating Quadratic Indices | 159 |
| 4.5 | Discrete-Time Lyapunov Equation | 165 |
| 4.6 | Constructing Robust Linear Controllers | 168 |
| 4.7 | Hurwitz and Routh Stability Criteria | 171 |
| 4.8 | Stability of Nonlinear Systems | 176 |
| 4.9 | Lyapunov's Indirect Method | 188 |

CONTENTS

| | | |
|------|----------------------------------|-----|
| 4.10 | Discontinuous Robust Controllers | 194 |
| 4.11 | Uniform Ultimate Boundedness | 201 |
| 4.12 | Lyapunov-Like Analysis | 210 |
| 4.13 | LaSalle's Invariance Principle | 213 |
| | Notes | 217 |
| | Exercises | 217 |

5 Optimal Control 225

| | | |
|-----|--|-----|
| 5.1 | Performance Indices | 225 |
| 5.2 | A Glimpse at the Calculus of Variations | 227 |
| | 5.2.1 Variation and Its Properties | 229 |
| | 5.2.2 Euler–Lagrange Equation | 234 |
| 5.3 | Linear Quadratic Regulator | 244 |
| | 5.3.1 Algebraic Riccati Equation (ARE) | 244 |
| | 5.3.2 Solving the ARE Using the Eigenvector Method | 248 |
| | 5.3.3 Optimal Systems with Prescribed Poles | 251 |
| | 5.3.4 Optimal Saturating Controllers | 266 |
| | 5.3.5 Linear Quadratic Regulator for Discrete Systems on an Infinite Time Interval | 270 |
| 5.4 | Dynamic Programming | 273 |
| | 5.4.1 Discrete-Time Systems | 273 |
| | 5.4.2 Discrete Linear Quadratic Regulator Problem | 277 |
| | 5.4.3 Continuous Minimum Time Regulator Problem | 280 |
| | 5.4.4 The Hamilton–Jacobi–Bellman Equation | 283 |
| 5.5 | Pontryagin's Minimum Principle | 292 |
| | 5.5.1 Optimal Control with Constraints on Inputs | 292 |
| | 5.5.2 A Two-Point Boundary-Value Problem | 304 |
| | Notes | 307 |
| | Exercises | 307 |

6 Sliding Modes 315

| | | |
|-----|-----------------------------------|-----|
| 6.1 | Simple Variable Structure Systems | 315 |
| 6.2 | Sliding Mode Definition | 320 |
| 6.3 | A Simple Sliding Mode Controller | 324 |
| 6.4 | Sliding in Multi-Input Systems | 327 |

| | | |
|----------|--|------------|
| 6.5 | Sliding Mode and System Zeros | 328 |
| 6.6 | Nonideal Sliding Mode | 329 |
| 6.7 | Sliding Surface Design | 333 |
| 6.8 | State Estimation of Uncertain Systems | 336 |
| | 6.8.1 Discontinuous Estimators | 336 |
| | 6.8.2 Boundary Layer Estimators | 338 |
| 6.9 | Sliding Modes in Solving Optimization Problems | 340 |
| | 6.9.1 Optimization Problem Statement | 340 |
| | 6.9.2 Penalty Function Method | 342 |
| | 6.9.3 Dynamical Gradient Circuit Analysis | 343 |
| | Notes | 356 |
| | Exercises | 357 |
| 7 | Vector Field Methods | 367 |
| 7.1 | A Nonlinear Plant Model | 368 |
| 7.2 | Controller Form | 369 |
| 7.3 | Linearizing State-Feedback Control | 380 |
| 7.4 | Observer Form | 382 |
| 7.5 | Asymptotic State Estimator | 387 |
| 7.6 | Combined Controller–Estimator Compensator | 388 |
| | Notes | 389 |
| | Exercises | 389 |
| 8 | Fuzzy Systems | 393 |
| 8.1 | Motivation and Basic Definitions | 393 |
| 8.2 | Fuzzy Arithmetic and Fuzzy Relations | 396 |
| | 8.2.1 Interval Arithmetic | 396 |
| | 8.2.2 Manipulating Fuzzy Numbers | 398 |
| | 8.2.3 Fuzzy Relations | 403 |
| | 8.2.4 Composition of Fuzzy Relations | 405 |
| 8.3 | Standard Additive Model | 407 |
| 8.4 | Fuzzy Logic Control | 410 |
| 8.5 | Stabilization Using Fuzzy Models | 420 |
| | 8.5.1 Fuzzy Modeling | 421 |

CONTENTS

| | | |
|----------|---|------------|
| 8.5.2 | Constructing a Fuzzy Design Model Using a Nonlinear Model | 422 |
| 8.5.3 | Stabilizability of Fuzzy Models | 427 |
| 8.5.4 | A Lyapunov-Based Stabilizer | 431 |
| 8.6 | Stability of Discrete Fuzzy Models | 437 |
| 8.7 | Fuzzy Estimator | 440 |
| 8.7.1 | The Comparison Method for Linear Systems | 442 |
| 8.7.2 | Stability Analysis of the Closed-Loop System | 444 |
| 8.8 | Adaptive Fuzzy Control | 447 |
| 8.8.1 | Plant Model and Control Objective | 447 |
| 8.8.2 | Background Results | 447 |
| 8.8.3 | Controllers | 449 |
| 8.8.4 | Examples | 456 |
| | Notes | 463 |
| | Exercises | 464 |
| 9 | Neural Networks | 469 |
| 9.1 | Threshold Logic Unit | 470 |
| 9.2 | Identification Using Adaptive Linear Element | 478 |
| 9.3 | Backpropagation | 484 |
| 9.4 | Neural Fuzzy Identifier | 489 |
| 9.5 | Radial-Basis Function (RBF) Networks | 494 |
| 9.5.1 | Interpolation Using RBF Networks | 494 |
| 9.5.2 | Identification of a Single-Input, Single-State System | 497 |
| 9.5.3 | Learning Algorithm for the RBF Identifier | 498 |
| 9.5.4 | Growing RBF Network | 500 |
| 9.5.5 | Identification of Multivariable Systems | 503 |
| 9.6 | A Self-Organizing Network | 506 |
| 9.7 | Hopfield Neural Network | 508 |
| 9.7.1 | Hopfield Neural Network Modeling and Analysis | 508 |
| 9.7.2 | Analog-to-Digital Converter | 514 |
| 9.8 | Hopfield Network Stability Analysis | 517 |
| 9.8.1 | Hopfield Network Model Analysis | 518 |
| 9.8.2 | Single-Neuron Stability Analysis | 520 |
| 9.8.3 | Stability Analysis of the Network | 522 |

| | | |
|-----------|--|------------|
| 9.9 | Brain-State-in-a-Box (BSB) Models | 529 |
| 9.9.1 | Associative Memories | 529 |
| 9.9.2 | Analysis of BSB Models | 533 |
| 9.9.3 | Synthesis of Neural Associative Memory | 535 |
| 9.9.4 | Learning | 546 |
| 9.9.5 | Forgetting | 547 |
| | Notes | 549 |
| | Exercises | 551 |
| 10 | Genetic and Evolutionary Algorithms | 553 |
| 10.1 | Genetics as an Inspiration for an Optimization Approach | 553 |
| 10.2 | Implementing a Canonical Genetic Algorithm | 555 |
| 10.3 | Analysis of the Canonical Genetic Algorithm | 559 |
| 10.4 | Simple Evolutionary Algorithm (EA) | 561 |
| 10.5 | Evolutionary Fuzzy Logic Controllers | 563 |
| 10.5.1 | Vehicle Model and Control Objective | 563 |
| 10.5.2 | Case 1: EA Tunes Fuzzy Rules | 565 |
| 10.5.3 | Case 2: EA Tunes Fuzzy Membership Functions | 567 |
| 10.5.4 | Case 3: EA Tunes Fuzzy Rules and Membership Functions | 568 |
| | Notes | 574 |
| | Exercises | 575 |
| 11 | Chaotic Systems and Fractals | 578 |
| 11.1 | Chaotic Systems Are Dynamical Systems with Wild Behavior | 578 |
| 11.2 | Chaotic Behavior of the Logistic Equation | 579 |
| 11.2.1 | The Logistic Equation—An Example from Ecology | 579 |
| 11.2.2 | Stability Analysis of the Logistic Map | 581 |
| 11.2.3 | Period Doubling to Chaos | 588 |
| 11.2.4 | The Feigenbaum Numbers | 592 |
| 11.3 | Fractals | 593 |
| 11.3.1 | The Mandelbrot Set | 595 |
| 11.3.2 | Julia Sets | 595 |
| 11.3.3 | Iterated Function Systems | 596 |
| 11.4 | Lyapunov Exponents | 599 |
| 11.5 | Discretization Chaos | 605 |

CONTENTS

| | | |
|--------|--|-----|
| 11.6 | Controlling Chaotic Systems | 607 |
| 11.6.1 | Ingredients of Chaotic Control Algorithm | 607 |
| 11.6.2 | Chaotic Control Algorithm | 608 |
| | Notes | 619 |
| | Exercises | 620 |
| | | |
| | Appendix: Math Review | 624 |
| A.1 | Notation and Methods of Proof | 624 |
| A.2 | Vectors | 626 |
| A.3 | Matrices and Determinants | 629 |
| A.4 | Quadratic Forms | 633 |
| A.5 | The Kronecker Product | 637 |
| A.6 | Upper and Lower Bounds | 640 |
| A.7 | Sequences | 641 |
| A.8 | Functions | 648 |
| A.9 | Linear Operators | 651 |
| A.10 | Vector Spaces | 654 |
| A.11 | Least Squares | 656 |
| A.12 | Contraction Maps | 659 |
| A.13 | First-Order Differential Equation | 662 |
| A.14 | Integral and Differential Inequalities | 663 |
| | A.14.1 The Bellman-Gronwall Lemma | 663 |
| | A.14.2 A Comparison Theorem | 664 |
| A.15 | Solving the State Equations | 665 |
| | A.15.1 Solution of Uncontrolled System | 665 |
| | A.15.2 Solution of Controlled System | 666 |
| A.16 | Curves and Surfaces | 667 |
| A.17 | Vector Fields and Curve Integrals | 670 |
| A.18 | Matrix Calculus Formulas | 674 |
| | Notes | 675 |
| | Exercises | 675 |
| | | |
| | Bibliography | 679 |
| | Index | 693 |