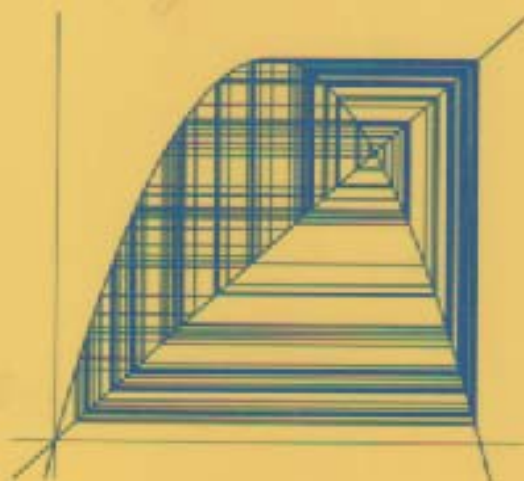


Richard A. Holmgren

# A FIRST COURSE IN DISCRETE DYNAMICAL SYSTEMS

SECOND EDITION



Springer

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# Universitext

A discrete dynamical system can be characterized as an iterated function. Given the efficiency with which computers can do iteration, it is now possible for anyone with access to a personal computer to generate beautiful images whose roots lie in discrete dynamical systems. Images of Mandelbrot and Julia sets abound in publications both mathematical and not. The mathematics behind the pictures is beautiful in its own right and is the subject of this text. The level of presentation is suitable for advanced undergraduates who have completed a year of college-level calculus. Concepts from calculus are reviewed as necessary. *Mathematica*® programs that illustrate the dynamics and that will aid the student in doing the exercises are included in the Appendix.

In this second edition, the topics covered are rearranged to make the text more flexible. In particular, the material on symbolic dynamics is now optional, and the book can easily be used for a single-semester course dealing exclusively with functions of a single real variable. Alternatively, the basic properties of dynamical systems can be introduced using functions of a real variable, and then the reader can skip directly to the material on the dynamics of complex functions. Additional changes include the simplification of several proofs, a thorough review and expansion of the exercises, and substantial improvement in the efficiency of the *Mathematica*® programs.

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