Lecture Notes in Computer Science 848

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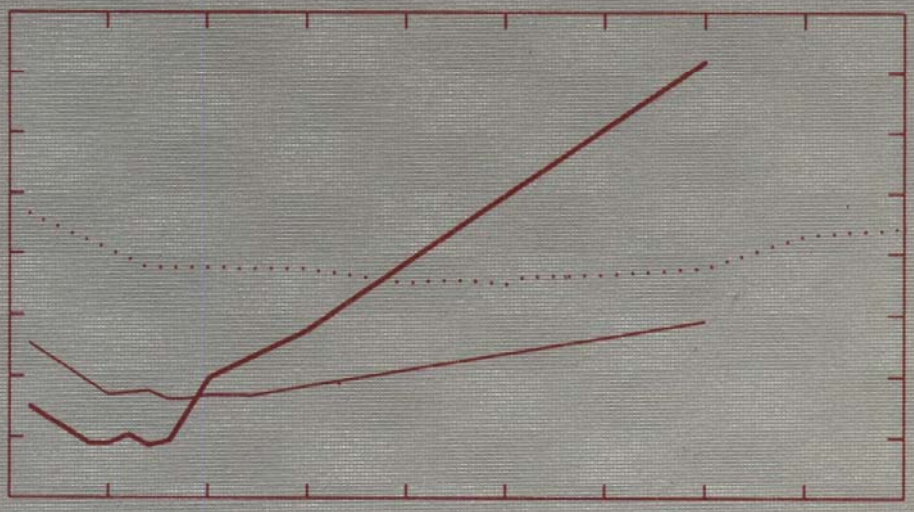
Numerical Integration

on Advanced Computer Systems



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