Encyclopaedia of Mathematical Sciences

Volume 12

S.P. Novikov (Ed.)

Topology I

Springer

Topology

Sergei P. Novikov

Translated from the Russian
by Boris Botvinnik and Robert Burns

Contents

Introduction 4

Introduction to the English Translation 5

Chapter 1. The Simplest Topological Properties 5

Chapter 2. Topological Spaces. Fibrations. Homotopies 15

§1. Observations from general topology. Terminology 15
§2. Homotopies. Homotopy type 18
§3. Covering homotopies. Fibrations 19
§4. Homotopy groups and fibrations. Exact sequences. Examples 23

Chapter 3. Simplicial Complexes and CW-complexes. Homology and Cohomology. Their Relation to Homotopy Theory. Obstructions 40

§1. Simplicial complexes 40
§2. The homology and cohomology groups. Poincaré duality 47
§3. Relative homology. The exact sequence of a pair. Axioms for homology theory. CW-complexes 57
§4. Simplicial complexes and other homology theories. Singular homology. Coverings and sheaves. The exact sequence of sheaves and cohomology 64
§5. Homology theory of non–simply–connected spaces. Complexes of modules. Reidemeister torsion. Simple homotopy type 70

§6. Simplicial and cell bundles with a structure group. Obstructions. Universal objects: universal fiber bundles and the universal property of Eilenberg-MacLane complexes. Cohomology operations. The Steenrod algebra. The Adams spectral sequence . 79

§7. The classical apparatus of homotopy theory. The Leray spectral sequence. The homology theory of fiber bundles. The Cartan-Serre method. The Postnikov tower. The Adams spectral sequence 103

§8. Definition and properties of K-theory. The Atiyah-Hirzebruch spectral sequence. Adams operations. Analogues of the Thom isomorphism and the Riemann-Roch theorem. Elliptic operators and K-theory. Transformation groups. Four-dimensional manifolds 113

§9. Bordism and cobordism theory as generalized homology and cohomology. Cohomology operations in cobordism. The Adams-Novikov spectral sequence. Formal groups. Actions of cyclic groups and the circle on manifolds 125

Chapter 4. Smooth Manifolds 142

§1. Basic concepts. Smooth fiber bundles. Connexions. Characteristic classes 142

§2. The homology theory of smooth manifolds. Complex manifolds. The classical global calculus of variations. H-spaces. Multi-valued functions and functionals 165

§3. Smooth manifolds and homotopy theory. Framed manifolds. Bordisms. Thom spaces. The Hirzebruch formulae. Estimates of the orders of homotopy groups of spheres. Milnor's example. The integral properties of cobordisms 203

§4. Classification problems in the theory of smooth manifolds. The theory of immersions. Manifolds with the homotopy type of a sphere. Relationships between smooth and PL-manifolds. Integral Pontryagin classes 227

§5. The role of the fundamental group in topology. Manifolds of low dimension ($n = 2, 3$). Knots. The boundary of an open manifold. The topological invariance of the rational Pontryagin classes. The classification theory of non-simply-connected manifolds of dimension ≥ 5. Higher signatures. Hermitian K-theory. Geometric topology: the construction of non–smooth homeomorphisms. Milnor's example. The annulus conjecture. Topological and PL–structures 244

Concluding Remarks 273

Appendix. Recent Developments in the Topology of 3-manifolds and Knots 274

§1. Introduction: Recent developments in Topology 274
§2. Knots: the classical and modern approaches to the Alexander polynomial. Jones-type polynomials 275
§3. Vassiliev Invariants 289
§4. New topological invariants for 3-manifolds. Topological Quantum Field Theories 291

Bibliography 299

Index 311