

TEXTBOOKS IN MATHEMATICS

INTRODUCTION TO REAL ANALYSIS

THIRD EDITION

Manfred Stoll



CRC Press

Taylor & Francis Group

A CHATMAN & HALL BOOK

Introduction to Real Analysis

Third edition

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This classic textbook has been used successfully by instructors and students for nearly three decades. This timely new edition offers minimal, yet notable changes while retaining all the elements, presentation, and accessible exposition of previous editions. A list of updates is found in the Preface to this edition.

This text is based on the author's experience in teaching graduate courses and the minimal requirements for successful graduate study. The text is understandable to the typical student enrolled in the course, taking into consideration the variations in abilities, background, and motivation. Chapters 1 through 6 have been written to be accessible to the average student, while at the same time challenging the more talented student through the exercises.

Chapters 7 through 10 assume that the students have achieved some level of expertise in the subject. In these chapters, the theorems, examples, and exercises require greater sophistication and mathematical maturity for full understanding.

In addition to the standard topics, the text includes topics that are not always included in comparable texts.

- Chapter 6 contains a section on the Riemann-Stieltjes integral and a proof of Lebesgue's theorem providing necessary and sufficient conditions for Riemann integrability.
- Chapter 7 also includes a section on square summable sequences and a brief introduction to normed linear spaces.
- Chapter 8 contains a proof of the Weierstrass approximation theorem using the method of approximate identities.
- The inclusion of Fourier series in the text allows the student to gain some exposure to this important subject.
- The final chapter includes a detailed treatment of Lebesgue measure and the Lebesgue integral, using inner and outer measure.
- The exercises at the end of each section reinforce the concepts.
- Notes provide historical comments or discuss additional topics.

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