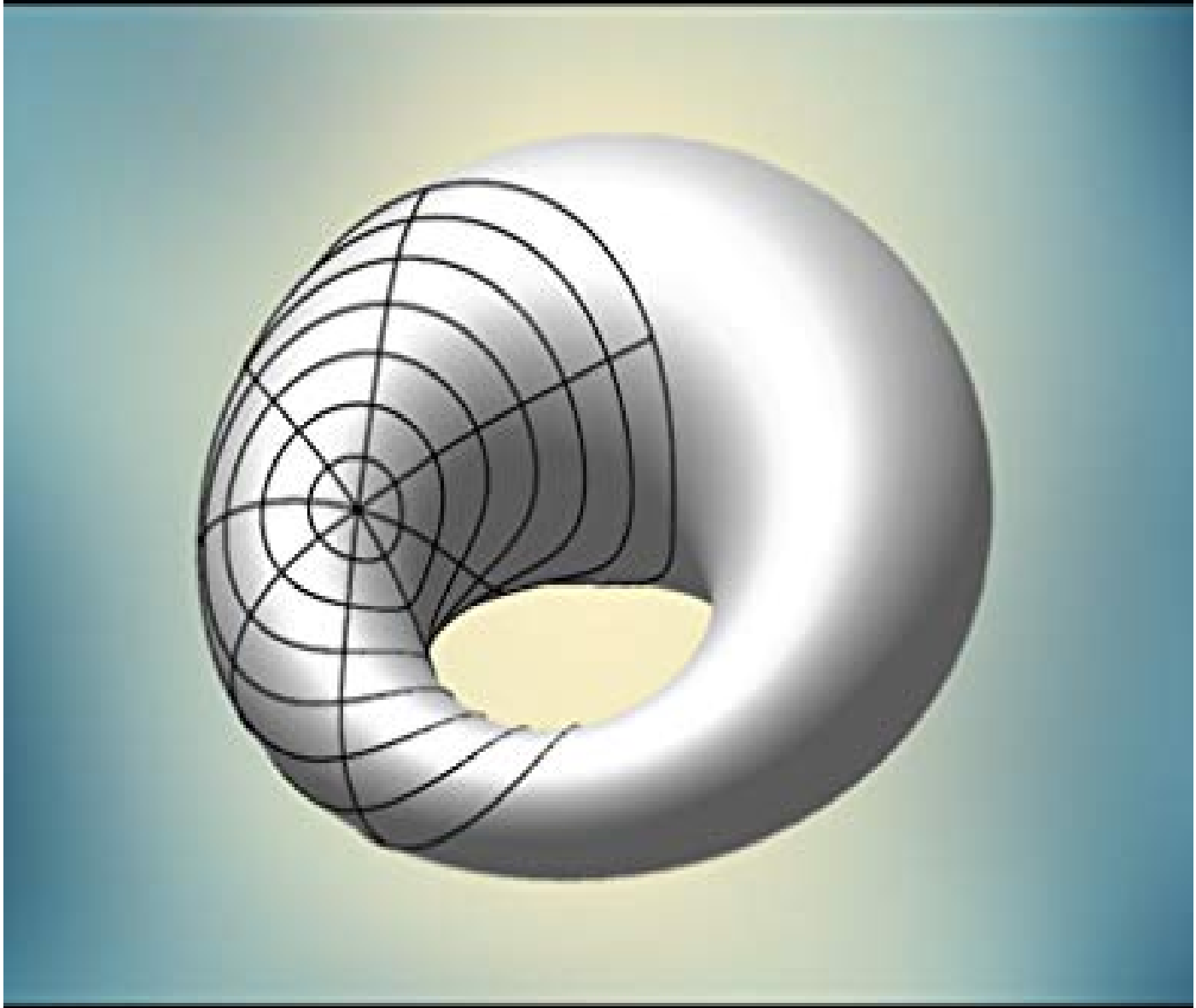
TEXTBOOKS IN MATHEMATICS

DIFFERENTIAL GEOMETRY OF MANIFOLDS

SECOND EDITION



STEPHEN LOVETT

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

Differential Geometry of Manifolds

Second Edition

Stephen Lovett

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an informa business

Contents

Preface vii

Acknowledgements xiii

1 Analysis of Multivariable Functions 1
- 1.1 Functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ . . . 1
- 1.2 Continuity, Limits, and Differentiability . . . 9
- 1.3 Differentiation Rules; Functions of Class C^r . . . 20
- 1.4 Inverse and Implicit Function Theorems . . . 28

2 Variable Frames 39
- 2.1 Frames Associated to Coordinate Systems . . . 39
- 2.2 Frames Associated to Trajectories . . . 49
- 2.3 Variable Frames and Matrix Functions . . . 57

3 Differentiable Manifolds 65
- 3.1 Definitions and Examples . . . 66
- 3.2 Differentiable Maps between Manifolds . . . 79
- 3.3 Tangent Spaces . . . 84
- 3.4 The Differential of a Differentiable Map . . . 91
- 3.5 Manifolds with Boundaries . . . 97
- 3.6 Immersions, Submersions, and Submanifolds . . . 102
- 3.7 Orientability . . . 111

4 Multilinear Algebra 119
- 4.1 Hom Space and Dual . . . 120
- 4.2 Bilinear Forms and Inner Products . . . 128
- 4.3 Adjoint, Self-Adjoint, and Automorphisms . . . 136
- 4.4 Tensor Product . . . 144
- 4.5 Components of Tensors over V . . . 151
- 4.6 Symmetric and Alternating Products . . . 160
- 4.7 Algebra over a Field . . . 172

5 Analysis on Manifolds **179**
5.1 Vector Bundles on Manifolds . . . 180
5.2 Vector and Tensor Fields on Manifolds . . . 189
5.3 Lie Bracket and Lie Derivative . . . 196
5.4 Differential Forms . . . 205
5.5 Pull-Backs of Covariant Tensor Fields . . . 216
5.6 Lie Derivative of Tensor Fields . . . 221
5.7 Integration on Manifolds - Definition . . . 225
5.8 Integration on Manifolds - Applications . . . 235
5.9 Stokes' Theorem . . . 245

6 Introduction to Riemannian Geometry **251**
6.1 Riemannian Metrics . . . 252
6.2 Connections and Covariant Differentiation . . . 267
6.3 Vector Fields along Curves; Geodesics . . . 279
6.4 Curvature Tensor . . . 291
6.5 Ricci Curvature and Einstein Tensor . . . 301

7 Applications of Manifolds to Physics **307**
7.1 Hamiltonian Mechanics . . . 308
7.2 Special Relativity; Pseudo-Riemannian Manifolds . . . 321
7.3 Electromagnetism . . . 334
7.4 Geometric Concepts in String Theory . . . 340
7.5 Brief Introduction to General Relativity . . . 348

A Point Set Topology **363**
A.1 Metric Spaces . . . 363
A.2 Topological Spaces . . . 379

B Calculus of Variations **403**
B.1 Formulation of Several Problems . . . 403
B.2 Euler-Lagrange Equation . . . 404
B.3 Several Dependent Variables . . . 409
B.4 Isoperimetric Problems and Lagrange Multipliers . . . 410

C Further Topics in Multilinear Algebra **415**
C.1 Binet-Cauchy and k-Volume of Parallelepipeds . . . 415
C.2 Volume Form Revisited . . . 418
C.3 Hodge Star Operator . . . 419

Bibliography **424**

Index **429**